

## P R E F A C E

The dissertation entitled "The Study of Null Congruences in General Relativity" is devoted to the understanding of the Newman-Penrose spin coefficient formalism. The work in the present dissertation has been divided into three chapters.

A brief exposition of the Newman-Penrose formalism (NP-formalism) is presented in the Chapter 1. No original results are claimed in the first chapter. The investigations are given in second and third chapters.

The power of the NP spin coefficient formalism cannot be utilised fully without the exploitation of Bianchi identities. In the Chapter-2, we have exploited fully the Bianchi identities to characterise the source term for the null and non-null electromagnetic field when coupled with different types of Petrov-type free-gravitational field. It is shown that when the non null electromagnetic field interacting with the free gravitational field of Petrov-type N the current vector  $J^a$  is necessarily a zero vector. It is proved to be space like or null accordingly as purely real ( $\text{Im}\phi_1 = 0$ ) or purely imaginary ( $\text{Re}\phi_1 = 0$ ) non-null electromagnetic field interacts with the Petrov type III field. It is also claimed that when purely real non-null field interacts with Petrov type D field then the current vector is either space like or a zero vector; however; it is observed to be time-like when purely imaginary non-null electromagnetic field interacts with Petrov-type D field. In case of null electromagnetic field of type B ( $\phi_2 = 0$ ) coupled with various

types of free-gravitational fields (except with type N with propagation vector  $l^a$   $\psi_4$  ( $l^a = 0$ ) it is computed that the current vector  $J^a$  is necessarily null. While for NN fields (null electromagnetic field and null free gravitational field with the common propagation vector  $l^a$ )  $J^a$  is space-like. The similar results are claimed for null electromagnetic field of type C ( $\phi_0 = 0$ ) when coupled with different type of free-gravitational field.

In the ~~third~~ chapter, we have propounded the concept of a ray strain variation equation and a ray vorticity equation. In particular we have obtained the (1-1), (1-m), (1-n) ray strain variation equations, vorticity variation equations and the variation equations of shear. The first vector  $l^a$  in the ordered pair (1-1) stands for the vector field of which the complete optical tensor is used whereas the second vector field  $l^a$  is the null vector field with respect to which the Lie derivative is defined. It is observed that the free gravitational field  $\psi_0$  of Petrov type N with propagation vector  $n^a$  and  $\psi_2$  of type D with propagation vector  $l^a$  or  $n^a$  and the matter fields  $\phi_{00}$ ,  $\phi_{11}$  have increasing effects on the (1-1) ray strain variation equation, whereas the free-gravitational field  $\psi_1$  of Petrov type III with propagation vector  $n^a$  and matter field  $\phi_{10}$  have decreasing effects.

It is observed that the (1-1) vorticity equation vanishes identically but not the (1-m) and (1-n) ray vorticity equation. In the latter case the necessary and sufficient conditions are obtained

for the Lie invariance of ray vorticity equation. Similar results are also discussed for the (1-m) and (1-n) ray strain variation equation.

References to the literature are arranged alphabetically towards the end.

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