CHAPTER - V

A NEW TRANSPORT OPERATOR D_u

1. MOTIVATION FOR DEFINING THE NEW TRANSPORT OPERATOR D_u

The decomposition of the tensor gradient of the flow field :

The condition

$$u^a u_a = 1$$

on u^a, (a = 1, 2, 3, 4), implies that there are only 3 independent components of u^a. Hence $u_{a;b}$ are 3 x 4 = 12 independent components. Thus the tensor gradient of the timelike flow vector field $u_{a;b}$ has 12 independent components which have been decomposed into

i) 5 independent components of the shear tensor field σ_{ab}

ii) 3 independent components of the rotation tensor field ω_{ab}

iii) 3 independent components of the acceleration field $\dot{u}_a$

iv) 1 independent component of the expansion scalar field Θ.

due to the relations

$$\sigma^a_a = 0,\ \sigma^b_a u_b = 0,\ \omega_{ab} = -\omega_{ba}$$

$$\omega^a_b u^b = 0,\ \dot{u}_a u^a = 0,\ \Theta = u^a_{;a},\ \sigma_{ab} = \sigma_{ba}.$$

The flow gradient is partitioned as follows

$$u_{a;b} = \sigma_{ab} + \omega_{ab} + \tfrac{1}{3} h_{ab}\Theta + \dot{u}_a u_b,\quad h_{ab} = g_{ab} - u_a u_b .$$

The new operator which we propose is general in the sense that it will be a combination of all these kinematical parameters σ_{ab}, ω_{ab}, Θ, $\dot{u}_a$, and also it reduces to known operator under some conditions.

THE NEW TRANSPORT OPERATOR :

We introduce a new transport in relativistic continuum mechanics as follows :

$$D_u x^a = f\dot{x}^a - (\alpha\sigma^a_c + \beta\omega_c{}^a + \gamma_1\delta^a_c\Theta + \gamma_2 u^a u_c\Theta + \chi\dot{u}^a u_c + \psi u^a \dot{u}_c)x^c \quad .$$

where $f, \alpha, \beta, \gamma_1, \gamma_2, \chi, \psi$ are seven arbitrary scalar fields.

The negative sign is chosen for convenience. Here "transport" is used in the sense that the derivative operator is chosen along the time-like flow vector. It is a generalization of many famous transports in relativistic continuum mechanics, like

1) the material transport : (Radhakrishna, 1976; Katkar, 1982; Gumaste, 1984).
2) the Jaumann transport: (Rahdkrishna, Katkar and Date, 1981).
3) the Fermi transport : (Synge, 1962; Radhakrishna and Bhosale, 1975-76).
4) The Oldroyd (convective) transport : (Carter and Quintana, 1972).
5) the Truesdell transport : (Radhakrishna and Walwadkar, 1982).
6) the Lie transport : (Narlikar, 1978; Stephani, 1982; Van Dantzig, 1932).

To appreciate the operator D as a general transport, we cite below the conditioins on the 7 scalar fields to correspond to the well known operators.

1) <u>the Material transport</u> :

The material transport of the contravariant vector field x^a is defined by

$$\frac{\delta x^a}{\delta s} = (x^a)_{;b} u^b , \quad u^b = \frac{dx^b}{ds}$$
$$= \dot{x}^a$$

comparing this with the definition of $D_u x^a$, we find that D reduces to $\frac{\delta}{\delta s}$ when

$$f = 1, \ \alpha = \beta = \gamma_1 = \gamma_2 = \chi = \psi = 0$$

Here $\frac{\delta}{\delta s}$ represents covariant derivative along the flow of the material continuum.

2) <u>the Jaumann transport</u> :

The Jaumann transport of the contravariant vector field x^a is defined by

$$J_u x^a = \dot{x}^a + x^k \omega_{\cdot k}^{a}$$

since ω_{ka} is a skew symmetric tensor.

Comparing this with the definition of $D_u x^a$, we find that D reduces to J when

$$f = 1, \ \beta = +1, \ \alpha = \gamma_1 = \gamma_2 = \chi = \psi = 0.$$

3) <u>the Fermi transport</u> :

The Fermi transport of the contravariant vector field x^a is defined by

$$F_u x^a = \dot{x}^a + x^k (\dot{u}^a u_k - \dot{u}_k u^a) \ .$$

Comparing this with the definition of $D_u x^a$ we find that, D reduces to F when

$$f = 1,\ \chi = -1,\ \psi = 1$$
$$\alpha = \beta = \gamma_1 = \gamma_2 = 0 .$$

4) <u>the Oldroyd (convective) transport</u> :

The Oldroyd derivative developed by Carter and Quintana for the contravariant vector field x^a is given by

$$C_u x^a = \dot{x}^a - x^k (u^a_{;k} - u^a \dot{u}_k)$$
$$= \dot{x}^a - x^k (\sigma^a_k + \omega^a_k + \tfrac{1}{3}\theta\gamma^a_k + \dot{u}^a u_k - u^a \dot{u}_k) .$$

Comparing this with the definition of $D_u x^a$, we find that D reduces to c when

$$f=1,\ \psi=-1, \alpha=1,\ \beta=1,\ \gamma_1=\tfrac{1}{3},\ \gamma_2=0,\ \chi=+1 .$$

5) <u>the Truesdell transport</u> :

The Truesdell transport of the contravariant vector field x^a is defined as

$$T_u x^a = \dot{x}^a - x^k u^a_{;k} + \tfrac{1}{2} x^a \theta$$
$$= \dot{x}^a - x^k (\sigma^a_k + \omega^a_k + \tfrac{1}{3}\delta^a_k \theta - \tfrac{1}{3} u^a u_k \theta + \dot{u}^a u_k) + \tfrac{1}{2} x^a \theta$$
$$= \dot{x}^a - x^k (\sigma^a_k + \omega^a_k - \tfrac{1}{3} u^a u_k \theta + \dot{u}^a u_k) - \tfrac{1}{3} x^a \theta + \tfrac{1}{2} x^a \theta .$$

comparing this with the definition of $D_u x^a$, we find that D reduces to T^- when

$$f=1,\ \alpha=1,\ \beta=1,\ \gamma_1= -\frac{1}{6}, \gamma_2= \frac{-1}{3},\ \chi=1,\ \psi = 0 .$$

6) the Lie transport :

The Lie transport of the contravariant vector field x^a is defined by

$$\pounds_u x^a = \dot{x}^a - x^k u^a_{;k}$$

$$= \dot{x}^a - x^k (\sigma^a_{\ k} + \omega^a_{\cdot k} + \tfrac{1}{3} h^a_k \Theta + \dot{u}^a u_k)$$

$$= \dot{x}^a - x^k (\sigma^a_{\ k} + \omega^a_{\ \cdot k} + \tfrac{1}{3} \delta^a_k \Theta - \tfrac{1}{3}\Theta u^a u_k + \dot{u}^a u_k) ,$$

since $h^a_k = \delta^a_k - u^a u_k$.

Comparing this with the definition of $D_u x^a$, we find that, D reduces to £ when

$$f=1, \ \alpha=1, \ \beta=1, \ \gamma_1 = \tfrac{1}{3}, \gamma_2 = -\tfrac{1}{3}, \ \chi=1, \ \psi=0.$$

The general transport of a covariant vector field :

Till now we have studied the general transport for a contravariant vector field. We now develop the theory for a covariant vector field.

We tentatively propose the relation

$$D_u x_a = f' \dot{x}_a + \Omega'^c_a x_c$$

where $\Omega'^c_a = \alpha' \sigma^c_a + \beta' \omega^{\ \cdot c}_b + \gamma'_1 \delta^c_a \Theta + \gamma'_2 u_a u^c \Theta + \chi' \dot{u}_a u^c$

with $f', \alpha', \beta', \gamma_1', \gamma_2', \psi', \chi'$ are arbitrary scalars.

II) In the next section we will determine the relationship between f, f', α, α', β, β', γ', γ, etc. so that certain standard properties for any operator (derivative along flow) are valid.

2. SPECIAL LEIBNITZ PROPERTY :

The general transport of a scalar function must be the material transport of the same scalar function (Eringen, 1962). This will be true when $D_u h = \dot{h}$ for every $h(x^k)$. It follows that we should have

$$D_u (x^a y_a) = (x^a y_a)^{\cdot}$$

which is referred here as Special Leibnitz Property.

For instance

$$D_u(x^a y_b) = (D_u x^a) y_b + x^a (D_u y_b)$$

is the well-known Leibnitz property.

We establish the following.

CLAIM : $D_u(x^a y_a) = (x^a y_a)^{\cdot}$ implies $f = f' = 1$,

$\alpha + \alpha' = \beta + \beta' = \gamma_1 + \gamma'_1 = \gamma_2 + \gamma'_2 = \chi + \chi' = \psi + \psi' = 0.$

Proof : Suppose

$$D_u x^a = f\dot{x}^a + \Omega^a_c x^c \quad \ldots \quad (2.1)$$

$$D_u x_a = f'\dot{x}_a + \Omega'^c_a x_c \quad \ldots \quad (2.2)$$

where

$$\Omega^a_c = -(\alpha\sigma^a_c + \beta\omega^{\cdot a}_c + \gamma_1 \delta^a_c \Theta + \gamma_2 u^a u_c \Theta + \chi' u^a u_c)$$

$$\Omega'^c_a = -(\alpha'\sigma^c_a + \beta'\omega^c_{\cdot a} + \gamma'_1 \delta^c_a \Theta + \gamma'_2 u_a u^c \Theta + \chi' \dot{u}_a u^c)$$

The special Leibnitz property

$$(Dx^a)y_a + xa\,(Dy_a) = (x^a y_a)^{\cdot}$$

implies on using (2.1) and (2.2)

$$f\dot{x}^a y_a + f' x^a \dot{y}_a + \Omega^a_c x^c y_a + \Omega'^c_a y_c x^a = \dot{x}^a y_a + x^a \dot{y}_a \quad .$$

Comparing the co-efficients of like terms, we have,

$f = 1,\ f' = 1,\ \Omega^a_c x^c y_a + \Omega'^c_a y_c x^a = 0$

or

$\Omega^a_c x^c y_a + \Omega'^a_c y_a x^c = 0,$

changing dummies in second term, we get

$$(\Omega^a_c + \Omega'^a_c) x^c y_a = 0 \quad . \text{ Hence } \Omega^a_c + \Omega'^a_c = 0 \ ,$$

since x^a, y^a are arbitrary independent vector fields.

This implies that $\alpha + \alpha' = 0,\ \beta + \beta' = 0,\ \gamma_1 + \gamma_1' = 0,$

$\gamma_2 + \gamma_2' = 0, \chi + \chi' = 0,\ \psi + \psi' = 0$

since the kinematical parameters $\sigma_{ab}, \omega_{ab}, \Theta,\ \dot{u}_a$ are independent.

We infer that

$$D_u x^a = \dot{x}^a + \Omega^a_c x^c \qquad \dots \qquad (2.3)$$

$$D_u x_a = \dot{x}_a - \Omega^c_a x_c \qquad \dots \qquad (2.4)$$

This establishes the formulae for the D-transport of contravariant vector field (vide 2.3) and D-transport of the covariant vector fields (vide 2.4).

3. THE FORMULA FOR $D_u A^a_b$:

We consider the outer product of x^a and y_b and impose the condition that the Leibnitz property should be satisfied.

$$D_u(x^a y_b) = (D_u x^a) y_b + x^a (D_u Y_b) \ .$$

Proof : By the definition of the general transport, we have

$$D_u x^a = \dot{x}^a + (\alpha\sigma^a_c + \beta\omega^{\cdot a}_{c} + \gamma_1 \delta^a_c \Theta + \gamma_2 u^a u_c \Theta + \chi \dot{u}^a u_c + \psi u^a \dot{u}_c) x^c$$

$$D_u y_b = \dot{y}_b - (\alpha\sigma^c_b + \beta\omega^{\cdot c}_{b} + \gamma_1 \delta^c_b \Theta + \gamma_2 u_b u^c \Theta + \chi \dot{u}_b u^c + \psi u_b \dot{u}^c) y_c$$

We know that the Leibnitz product rule is given by

$$D_u(x^a y_b) = (D_u x^a) y_b + x^a (D_u y_b) .$$

$$\text{R.H.S.} = [\dot{x}^a + (\alpha\sigma^a_c + \beta\omega^{\cdot a}_c + \gamma_1 \delta^a_c \Theta + \gamma_2 u^a u_c \Theta + \chi \dot{u}^a u_c + \psi u^a \dot{u}_c) x^c] y_b$$

$$+ x^a [\dot{y}_b - (\alpha\sigma^c_b + \beta\omega_b^{\cdot c} + \gamma_1 \delta^c_b \Theta + \gamma_2 u_b u^c \Theta + \chi \dot{u}_b u^c + \psi u_b \dot{u}^c) y_c]$$

$$= \dot{x}^a y_b + x^a \dot{y}_b + \alpha\sigma^a_c y_b x^c - \alpha\sigma_b^{\ c} y_c x^a + \beta\omega^{\cdot a}_c y_b x^c - \beta\omega_b^{\cdot c} y_c x^a$$

$$+ \gamma_1 \delta^a_c \Theta y_b x^c - \gamma_1 \delta^c_b \Theta y_c x^a + \gamma_2 u^a u_c \Theta y_b x^c - \gamma_2 u_b u^c \Theta y_c x^a$$

$$+ \chi \dot{u}^a u_c y_b x^c - \chi \dot{u}_b u^c x^a y_c + \psi u^a \dot{u}_c x^c y_b - \psi u_b \dot{u}^c x^a y_c .$$

$$= \dot{x}^a y_b + x^a \dot{y}_b + \alpha\sigma^a_c y_b x^c - \alpha\sigma^c_b y_c x^a + \beta\omega^{\cdot a}_c y_b x^c - \beta\omega_b^{\cdot c} y_c x^a$$

$$+ \gamma_2 u^a u_c \Theta y_b x^c - \gamma_2 u_b u^c \Theta y_c x^a + \chi \dot{u}^a u_c y_b x^c - \chi \dot{u}_b u^c x^a y_c$$

$$+ \psi u^a \dot{u}_c x^c y_b - \psi u_b \dot{u}^c x^a y_c ,$$

since, $\gamma_1 \delta^a_c \Theta y_b x^c - \gamma_1 \delta^c_b \Theta y_c x^a = \gamma_1 \Theta y_b x^a - \gamma_1 \Theta y_b x^a = 0 .$

Put $x^a y_b = A^a_{\cdot b}$, $x^c y_b = A^c_{\cdot b}$, $y_c x^a = A^a_{\cdot c}$

$$DA^a_{\cdot b} = (A^a_{\cdot b})^{\cdot} + \alpha\sigma^a_c A^c_{\cdot b} - \alpha\sigma_b^{\ c} A^a_{\cdot c} + \beta\omega^{\cdot a}_c A^c_{\cdot b} - \beta\omega_b^{\cdot c} A^a_{\cdot c}$$

$$+ \gamma_2 \Theta u^a u_c A^c_{\cdot b} - \gamma_2 \Theta u_b u^c A^a_{\cdot c} + \chi \dot{u}^a u_c A^c_{\cdot b} - \chi \dot{u}_b u^c A^a_{\cdot c} + \psi \dot{u}^c (u^a A_{cb} - u_b A^a_{\cdot c}$$

$$D_u A^a_{\cdot b} = (A^a_{\cdot b})^{\cdot} + \alpha (\sigma^a_c A^c_{\cdot b} - \sigma^c_b A^a_{\cdot c}) + \beta (\omega^{\cdot a}_c A^c_{\cdot b} \omega_b^{\cdot c} A^a_{\cdot c})$$

$$+ \gamma_2 \Theta (u^a u_c A^c_{\cdot b} - u_b u^c A^a_{\cdot c}) + \chi (\dot{u}^a u_c A^c_{\cdot b} - \dot{u}_b u^c A^a_{\cdot c}) . \qquad \text{...(3.1)}$$

$$+ \psi \dot{u}^c (u^a A_{cb} - u_b A^a_{\cdot c})$$

We can analogously write the formulae for the general transport of an arbitrtary tensor.

4. D - TRANSPORT OF MATERIAL CONTRAVARIANT VECTOR FIELDS :

Introduction :

In this dissertation we have studied several material vector fields, viz. P^a, Q^a, R^a (i.e. $u_aP^a = 0$, $u_aQ^a = 0$, $u_aR^a = 0$). In relativistic magneto hydrodynamics the magnetic field vector is a material vector field,

$$u_aH^a = 0 \quad \text{(vide Lichnerowicz, 1967)} .$$

The vorticity vector field ω^a, the Poynting vector field Ω^a are all material vector fields. It is shown in Chapter III that the Lie transport of a material contravariant vector is not in general a material vector but the OLDROYD transport of a material vector is again a material vector.

Hence it is necessary to investigate the properties of D-transport of material vector fields. We observe that, in general, the D-transport of a material vector does not produce a material vector.

Theorem : 1. Let x^a be a material vector field.

The following are equivalent (TFAE).

i) $D_u x^a$ is a material vector field.

ii) $1 + \Psi = 0$.

Proof : The definition of the D operator gives

$$D_u x^a = \dot{x}^a - [\alpha\sigma^a_{\ c} + \beta\omega^{\cdot a}_{\ c} + \gamma_1\delta^a_c\Theta + \gamma_2 u^a u_c\Theta + \chi\dot{u}^a u_c + \Psi u^a\dot{u}_c]\, x^c$$

Consider, the inner product of $D_u x^a$ with u_a .

$$u_a D_u x^a = u_a[\,\dot{x}^a - (\alpha\sigma^{\ a}_{c} + \beta\omega^{\cdot a}_{\ c} + \gamma_1\delta^a_c\Theta + \gamma_2 u^a u_c\Theta + \chi\dot{u}^a u_c + \psi u^a\dot{u}_c)x^c\,]$$

$$= u_a\dot{x}^a - \alpha\sigma^a_c u_a x^c - \beta\omega^{\cdot a}_{c} u_a x^c - \gamma_1\delta^a_c\Theta u_a x^c - \gamma_2 u^a u_c\Theta u_a x^c$$

$$-\chi\dot{u}^a u_c u_a x^c - \psi u^a\dot{u}_c u_a x^c, \quad \text{on expansion.}$$

$$= u_a\dot{x}^a - \psi\dot{u}_c x^c, \quad \text{since} \quad x^a, \sigma^b_a, \omega^{ab} \text{ are all Material tensors.}$$

$$= -\dot{u}_a x^a - \psi\dot{u}_a x^a, \quad \text{since} \quad u_a\dot{x}^a = -\dot{u}_a x^a .$$

$$u_a D_u x^a = -(1+\psi)\,\dot{u}_a x^a$$

since, x^a is an arbitrary Material tensor $\dot{u}_a x^a \neq 0$,

we get, $u_a D_u x^a = 0$ iff $1 + \psi = 0$.

The new transport D_u shares with Oldroyd transport C_u the property of producing material tensors from material tensors only when $1 + \psi = 0$.

5. NON-COMMUTATIVITY OF D_u WITH RAISING/LOWERING OF SUFFIXES:

We evaluate the general transport of the gravitational potentials as follows :

$$D_u g_{ab} = \dot{g}_{ab} - \alpha\sigma_a^{\ c} g_{cb} - \alpha\sigma_b^{\ c} g_{ac} - \beta\omega_{\dot{a}}^{\ c} g_{cb} - \beta\omega_{\dot{b}}^{\ c} g_{ac} + \gamma_2\theta(-u_a u^c g_{cb}) - 2\gamma_1\Theta g_{ab}$$

$$+ \gamma_2(-u_b u^c g_{ac}) - \chi(\dot{u}_a u^c g_{cb}) - \chi(\dot{u}_b u^c g_{ac}) - \psi\dot{u}^c u_a g_{cb} - \psi\dot{u}^c u_b g_{ac}$$

$$= -2(\alpha\sigma_{ab} + \gamma_2\theta u_a u_b) - (\dot{u}_a u_b + \dot{u}_b u_a)(\chi + \psi) - 2\gamma_1\Theta g_{ab}$$

$$\text{Since} \quad \dot{g}_{ab} = 0, \ \omega_{ab} + \omega_{ba} = 0, \ u^a\dot{u}_a = 0$$

$u^a D_u g_{ab} = -2\gamma_2 \theta u_b - (\chi+\psi)\dot{u}_b - 2\gamma_1 \theta u_b$, since $u^a \sigma_{ab} = 0$.

$u^a \dot{u}^b D_u g_{ab} = -(\dot{u}^b \dot{u}_b)[\chi+\psi]$

$= k_1^2 (\chi+\psi)$

$u^a u^b D_u g_{ab} = -2\theta(\gamma_1+\gamma_2)$

It follows that

$D_u g_{ab} = 0$ implies $\gamma_1+\gamma_2 = 0$, $\alpha = 0$, $\chi+\psi = 0$

We observe that $D_u g_{ab} \neq 0$ in general.

We also note that

$$D_u g_{ab} X^a \neq g_{ab} D_u x^a$$

and so the D-operator does not commute with raising and lowering of indices.

6. ON THE D-TRANSPORT OF THE RELATIVISTIC SERRET-FRENET TETRAD:

i) $D_u u^a = \dot{u}^a - [\alpha\sigma_c{}^a + \beta\omega_c{}^a + \gamma_1 \delta_c{}^a \theta + \gamma_2 u^a u_c \theta + \chi \dot{u}^a u_c + \psi u^a \dot{u}_c] u^c$, by definition.

$$D_u u^a = -(\gamma_1+\gamma_2) u^a \theta + (1-\chi) k_1 p^a, \text{ on simplification.}$$

We conclude that ,

$$D_u u^a = 0 \text{ iff } \gamma_1+\gamma_2 = 0, \ 1-\chi = 0, \text{ since } \theta \neq 0, \ k_1 \neq 0 .$$

ii) $D_u p^a = \dot{p}^a - [\alpha\sigma_c{}^a + \beta\omega_c{}^a + \gamma_1 \delta_c{}^a \theta + \gamma_2 u^a u_c \theta + \chi \dot{u}^a u_c + \psi u^a \dot{u}_c] p^c$, by definition.

$= (1-\psi) k_1 u^a + k_2 Q^a - \alpha\sigma_c{}^a p^c - \beta\omega_c{}^a p^c + \gamma_1 \theta p^a$, by (RSF-1).

$= (1-\psi+\beta) k_1 u^a + [k_2 - \alpha(\gamma_{132}+\gamma_{123}) + \beta/2(\gamma_{123}-\gamma_{132})] Q^a$

$+ [\alpha/6(7\gamma_{122}) - \alpha/3(\gamma_{133}+\gamma_{144}) - \gamma_1(\gamma_{122}+\gamma_{133}+\gamma_{144})] p^a$

$- [\alpha(\gamma_{142}+\gamma_{124}) - \beta/2(\gamma_{124} - \gamma_{142})]\, R^a$, by computational aids.

$D_u p^a = 0$, iff, the co-efficients of u^a, p^a, Q^a, R^a are separately zero i.e.

It follows that $D_u p^a = 0$, iff, $1 - \psi + \beta = 0$.

$$k_2 - \alpha(\gamma_{132} + \gamma_{123}) + \beta/2(\gamma_{123} - \gamma_{132}) = 0 .$$

$$\alpha/6(7\gamma_{122}) - \alpha/3(\gamma_{133}+\gamma_{144}) - \gamma_1(\gamma_{122} + \gamma_{133} + \gamma_{144}) = 0 .$$

$$\alpha(\gamma_{142} + \gamma_{124}) - \beta/2(\gamma_{124} - \gamma_{142}) = 0 .$$

iii) $D_u Q^a = \dot{Q}^a - [\alpha\sigma_c{}^a + \beta\omega_c{}^a + \gamma_1\delta_c{}^a\theta + \gamma_2 u^a u_c\theta + \chi\, \dot{u}^a u_c + \psi\, u^a \dot{u}_c] Q^c$

$= - k_2 p^a - \gamma_1\theta Q^a + k_3 R^a - \alpha\sigma_c{}^a Q^c - \beta\,\omega_c{}^a Q^c$

since $u_c Q^c = 0$, $P_c Q^c = 0$,

$= [-k_2 + \alpha/2(\gamma_{132} + \gamma_{123}) + \beta/2(\gamma_{132}-\gamma_{123})] P^a - \gamma_1(\gamma_{122}+\gamma_{133}+\gamma_{144}) + \alpha/6[(\gamma_{122}$

$+ 7\gamma_{133}+\gamma_{144})] Q^a + [k_3 - \alpha/2(\gamma_{143}+\gamma_{134}) + \beta/2(\gamma_{134} - \gamma_{143})] R^a$

$D_u Q^a = 0$ iff $-k_2 + \alpha/2(\gamma_{132}+\gamma_{123}) + \beta/2(\gamma_{132}-\gamma_{123}) = 0$

$$\gamma_1(\gamma_{122} + \gamma_{133} + \gamma_{144}) + \alpha/6(\gamma_{122} + 7\gamma_{133} + \gamma_{144}) = 0$$

$$k_3 - \alpha/2(\gamma_{143} + \gamma_{134}) + \beta/2(\gamma_{134} - \gamma_{143}) = 0.$$

iv) $D_u R^a = \dot{R}^a - [\alpha\sigma_c{}^a + \beta\omega_c{}^a + \gamma_1\delta_c{}^a\theta + \gamma_2 u^a u_c\theta + \chi\, \dot{u}^a u_c + \psi\, u^a \dot{u}_c] R^c$,

by definition.

$= -k_3 Q^a - [\alpha\sigma_c{}^a R^c + \beta\omega_c{}^a R^c + \gamma_1\theta R^a]$, since $u_c R^c = 0$

$= [\alpha/2(\gamma_{142} + \gamma_{124}) - \beta/2(\gamma_{123} - \gamma_{132})]\, P^a$

$+ [\alpha/2(\gamma_{143} + \gamma_{134}) - \beta/2(\gamma_{143} - \gamma_{134})] Q^a$

$+ [\alpha\{\gamma_{144} + 1/6(\gamma_{122} + \gamma_{133} + \gamma_{144})\} - \gamma_1(\gamma_{122} + \gamma_{133} + \gamma_{144})] R^a$

, by computational aids.

$D_u R^a = 0$, iff, the co-efficients of u^a, P^a, Q^a, R^a are separately zero i.e.

$D_u R^a = 0$, iff, $\alpha/2\,(\gamma_{142} + \gamma_{124}) - \beta/2(\gamma_{123} - \gamma_{132}) = 0$

$$\alpha/2\,(\gamma_{143} + \gamma_{134}) - \beta/2\,(\gamma_{143} - \gamma_{134}) = 0$$

$$\alpha(\gamma_{144} + 1/6\,(\gamma_{122} + \gamma_{133} + \gamma_{144}) - \gamma_1\,(\gamma_{122} + \gamma_{133} + \gamma_{144}) = 0$$

The tetrad will be D-transported iff all the following 12 conditions are satisfied.

$\gamma_1 + \gamma_2 = 0,\ (1-\chi) = 0,\ \theta \neq 0,\ \ k_1 \neq 0$.

$1 - \psi + \beta = 0,\ k_2 - \alpha(\gamma_{132} + \gamma_{123}) + \beta/2\,(\gamma_{123} - \gamma_{132}) = 0,$

$\alpha/6\,(7\gamma_{122}) - \alpha/3\,(\gamma_{133} + \gamma_{144}) - \gamma_1(\gamma_{122} + \gamma_{133} + \gamma_{144}) = 0.$

$\alpha\,(\gamma_{142} + \gamma_{124}) - \beta/2\,(\gamma_{124} - \gamma_{142}) = 0.$

$-k_2 + \alpha/2\,(\gamma_{132} + \gamma_{123}) = 0.$

$\gamma_1(\gamma_{122} + \gamma_{133} + \gamma_{144}) + \alpha/6\,(\gamma_{122} + 7\gamma_{133} + \gamma_{144}) = 0,$

$k_3 - \alpha\,(\gamma_{143} + \gamma_{134}) = 0,$

$\alpha/2\,(\gamma_{142} + \gamma_{124}) - \beta/2\,(\gamma_{123} - \gamma_{132}) = 0,$

$\alpha/2\,(\gamma_{143} + \gamma_{134}) - \beta/2\,(\gamma_{143} - \gamma_{134}) = 0,$

$\alpha\{\gamma_{144} + 1/6\,(\gamma_{122} + \gamma_{133} + \gamma_{144})\} - \gamma_1(\gamma_{122} + \gamma_{133} + \gamma_{144}) = 0.$

We observe that these are 12 equations in 18 unknowns.

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