PREFACE

The real world is complex and complexity in the world arises from uncertainty. Knowledge, which describes the real world, is usually imprecise and vague. The mathematical tools available to represent the real world are suitable to describe rigorous and precise information. Thus there is always some difference between vagueness of reality and its rigorousness of mathematical model.

In an attempt to bridge this gap fuzzy set theory was initiated by L. A. Zadeh in 1960's. Since a membership in a fuzzy set theory is a matter of degree, we can represent the gradual membership of an elements of the set describing the fuzzy attributes like cold, hot, short, tall etc in a better way. Thus fuzzy sets are more capable of expressing the vague concepts of natural languages. Due to this, fuzzy sets are widely used in control theory, robotics and other complex engineering systems. While applying fuzzy set theory in these fields as well as in related fields, we frequently come across fuzzy (relation) equations, which are based on crisp sets as well as fuzzy sets. In order to get better results it is important to find out solutions of these equations.

In this dissertation we have compiled available literature from various research papers on fuzzy relation equations and presented in more cohesive form.

The theory of fuzzy relation equations was introduced by Elie Sanchez in 1976. Fuzzy relation equations and fuzzy relational equations are used interchangeably. The notion of fuzzy relation equations based upon max-min composition was first proposed and investigated by Sanchez. Since then many papers dealing with such equations have appeared, but in all of them the fuzzy relations are defined on crisp sets. The necessity of defining fuzzy relations over fuzzy subsets was

first noticed by M. Delgado, J. L Verdegay, M. Vila, E. Trillas and L. A. Zadeh. Fuzzy relation equations based on fuzzy subsets are first introduced by M. J. Fernandez, F. Suarez and P. Gill.

This dissertation consists of five chapters. Chapter 1 consists of basic definitions and results of fuzzy set theory, which are used in the remaining part of this dissertation.

In chapter 2, we discuss fuzzy relations defined on crisp sets, their matrix representation, and their different compositions. In Section 2.1 definition of fuzzy relation and matrix representation of fuzzy relation defined on finite crisp set are given. In Section 2.2 different compositions of fuzzy relations are studied. In Section 2.3 fuzzy relation equation based on above compositions are given and their solutions are discussed, in particular method for obtaining minimal solutions of the fuzzy relation equation P o Q = R for P is elaborated.

Chapter 3 deals with fuzzy relation equations defined on fuzzy subsets. In section 3.1 definition of fuzzy relations on fuzzy sets is given. In this section composition of fuzzy relations viz. sup-T, inf-S, inf-w_T, sup-ω_S are defined and their properties are discussed. In section 3.2 fuzzy relation equations are defined and their solutions are discussed.

Chapter 4 consists of union and intersection preserving operators and fuzzy relation equations based on these operators. In section 4.1 definitions of union and intersection preserving operators are given and their inter-relationship is verified. In section 4.2 fuzzy relation equations with union preserving operator are discussed. Method for calculation all minimal solutions of these equations are given. In section 4.3 fuzzy relation equations with intersection preserving operator are discussed. Method for obtaining all maximal solutions of these equations is given,

Chapter 5 deals with equations of fuzzy numbers. In Section 5.1 fuzzy numbers, addition, subtraction, multiplication and division of fuzzy numbers are defined. In section 5.2 linear equations of fuzzy numbers $A + X = B$ is discussed. The necessary and sufficient condition for the solution of this equation is given. In section 5.3 linear equation of fuzzy numbers $A \cdot X = B$ is discussed. The necessary and sufficient condition for the solution of this equation is given. In section 5.4 triangular fuzzy numbers are given. The necessary and sufficient conditions of the equations of fuzzy triangular numbers $A + X = C$ and $A \cdot X = C$ are given. In section 5.5 fuzzy complex numbers and the linear fuzzy equation of these fuzzy complex numbers are discussed.