CHAPTER 1

WEAKLY PSEUDO-IDEALS IN NEAR-RINGS

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§ 1.0 Introduction

Throughout this chapter N denotes a right near-ring. Pseudo – right ideal in a left near-ring is introduced by Gerald Berman and Robert J. Silver-man [2]. In the same way we have defined a pseudo-left ideal in a right near-ring N as, "A Pseudo-left ideal <I, +, .> is a normal subnear-ring of <N, +, .> such that n.i. – n.0 $\in$ I, for each i $\in$ I and for each n $\in$ N."

Generalization of the concept of pseudo-left ideal in a right near-ring N is done in this chapter, we name it weakly pseudo-left ideal. For defining weakly pseudo-left (right) ideals in N, a help is taken of the paper, 'On pseudo ideals of semigroups', by M.K. Sen [4].

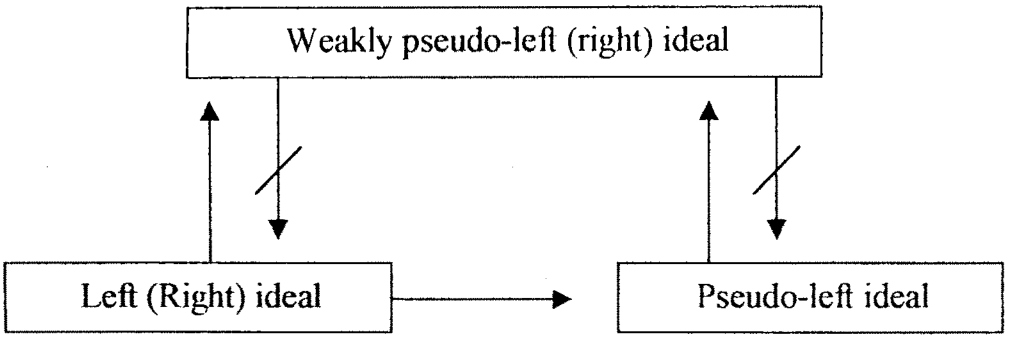
In this chapter we have studied some properties of weakly pseudo-left (right) ideals in near-rings. We have shown that every left (right) ideal in N is a weakly pseudo-left (right) ideal, but converse is not true. Also we have shown that in a Boolean near-ring, every weakly pseudo-right ideal is a right ideal.

We have proved the following result,

Result 1: N_0 is a weakly pseudo-left ideal but not generally weakly pseudo-right ideal.

Result 2: Intersection of subnear-ring S and weakly pseudo-ideal A of N is a weakly pseudo-ideal of S.

The relationship between ideal, pseudo-left ideal and weakly pseudo-ideals in N indicated in the following diagram.


Weakly pseudo-left (right) ideal
Left (Right) ideal
Pseudo-left ideal

§ Weakly pseudo-left ideal in a near-ring

§ 1.1 Definition and examples :

In this article we first define weakly pseudo left ideal in a near-ring N and give some examples of weakly pseudo-left ideals.

As a generalization of a pseudo-left ideal (Def .0.1.11) we define weakly pseudo-left ideal in a near-ring N as

Definition 1.1.1 :

Let <N,+,.> be a near-ring . A non-empty subset I of N is called a weakly pseudo-left ideal in N if it satisfies the following conditions.

(1) < I,+> is a normal subgroup af <N,+> and

(2) $n^2.i-n^2.0 \in I$, $\forall$ $i \in I$ and $\forall$ $n \in N$.

Some examples of weakly pseudo-left ideals in near-ring are given below.

Example 1.1.2: (Clay)

Consider the near-ring N ={0,a,b,c} with addition and multiplication as given by the following tables.

+	0	a	b	c
0	0	a	b	c
a	a	0	c	b
b	b	c	0	a
c	c	b	a	0

.	0	a	b	c
0	0	0	0	0
a	0	0	a	a
b	0	a	b	c
c	0	a	b	c

Let I={0,b}, I is a weakly pseudo -left ideal in N.

Example 1.1.3: (Clay, 2.2,13)

Consider the near-ring N ={0,a,b,c} with addition and multiplication defined by the following tables.

+	0	a	b	c
0	0	a	b	c
a	a	0	c	b
b	b	c	0	a
c	c	b	a	0

.	0	a	b	c
0	0	0	0	0
a	0	a	b	c
b	0	0	0	0
c	0	a	b	c

The subsets {0,a }, {0,b} and {0,c}are weakly pseudo-left ideals in N.

□

Example 1.1.4: (Clay, 2.2,2)

Consider the near-ring N={0,a,b,c} under the addition and multiplication defined by the following tables.

+	0	a	b	c
0	0	a	b	c
a	a	0	c	b
b	b	c	0	a
c	c	b	a	0

.	0	a	b	c
0	0	0	0	0
a	0	0	a	a
b	0	a	b	b
c	0	a	c	c

Let I ={0,c} .Here, $b^2.c-b^2.0 = b.c-0 = b \notin I$, for $c \in I$ and $b \in N$. Therefore I is not a weakly pseudo-left ideal. From this example we say that every non -empty subset of a near-ring need not be a weakly pseudo-left ideal.

□

In any near-ring every pseudo-left ideal is a weakly pseudo-left ideal. It is proved in the following result.

Result 1.1.5 : Every pseudo-left ideal in a near-ring is a weakly pseudo-left ideal.

Proof : Let < N, +, .> be a near-ring . Let I be a pseudo-left ideal in N . Therefore < I, +> is a normal subgroup of < N, +> and $n.i-n.0 \in I$, $\forall\, i \in I$ and $\forall\, n \in N$.(See Def. 0.1.11).If $n \in N$ then $n^2 \in N$.

Hence $n^2.i - n^2.0 \in I$, $\forall\, i \in I$ and $\forall\, n \in N$.

Thus I is a weakly pseudo-left in N.

||

Converse of the result 1.1.5 need not be true. This we establish by the following example.

Example 1.1.6 :- (Clay)

Consider the near-ring N={0,a,b,c} with addition and multiplication as given by the following tables.

+	0	a	b	c
0	0	a	b	c
a	a	0	c	b
b	b	c	0	a
c	c	b	a	0

.	0	a	b	c
0	0	0	0	0
a	0	0	a	a
b	0	a	b	c
c	0	a	b	c

Let I ={0,b}. Here I is weakly pseudo-left ideal of N. But as, $a.b-a.0= a - 0 = a \notin I$, for $b \in I$ and $a \in N$. Hence I is not a pseudo-left ideal.

||

Every left ideal in a near-ring is a weakly pseudo-left ideal in N. It is established in the following result.

Result 1.1.7 : Every left ideal in a near-ring is a weakly pseudo-left ideal.

Proof : Let < N, +, . > be a near-ring . Let I be a left ideal in a near-ring N. Therefore < I, +> a normal subgroup of < N,+> and n(n'+ I) - n.n' ∈ I for all n, n'∈N and for all i∈I.

If n∈N then n^2∈N.

Therefore $n^2.i - n^2.0 = n^2(0+i) - n^2.0 \in I$. [Since I is a left ideal.]

Hence $n^2.i - n^2.0 \in I$, ∀ n∈N and ∀ i∈ I.

Therefore , I is a weakly pseudo-left ideal in N.

| |

Converse of the result 1.1.7 need not be true. This we establish by the following example.

Example 1.1.8 : (Clay)

Consider the near-ring N ={0,a,b,c} under the addition and multiplication as given by the following tables.

+	0	a	b	c
0	0	a	b	c
a	a	0	c	b
b	b	c	0	a
c	c	b	a	0

.	0	a	b	c
0	0	0	0	0
a	0	0	a	a
b	0	a	b	c
c	0	a	b	c

Let I ={0,b}. Here I is a weakly pseudo-left ideal in N. But as a (c+b) -a.c = a.a - a = 0+a = a $\notin$ I, for b $\in$ I and a,c $\in$N. Hence I is not a left ideal in N.

[ ]

§ 1.2 Properties of weakly pseudo-left ideals

In this article we study some properties of weakly pseudo-left ideals in near-rings.

Result 1.2.1 : Intersection of any collection of weakly pseudo-left ideals in a near-ring N is a weakly pseudo-left ideal.

Proof : Take $I = \cap \{ I_i / I_i$ is a weakly pseudo left ideal in N$\}$. To prove that I is a weakly pseudo-left ideal in N.

Since $I_i \neq \emptyset$ $\forall$ i and $0 \in I_i$ $\forall$ i Hence $0 \in \cap_i I_i = I$.

Therefore $I \neq \emptyset$.

(1) Intersection of any collection of normal subgroups in N being normal (see Result 0.2.2)

we get < I,+> is a normal subgroup in < N, +>.

(2) Let x$\in$I and let n$\in$N . Therefore $x \in I_i$ where I_i is a weakly pseudo-left ideal in N.

By definition of weakly pseudo-left ideal, $n^2.x - n^2.0 \in I_i$, $\forall$ I_i .

Hence $n^2.x - n^2.0 \in \cap_i I_i$, $\forall$ $x \in I_i$ and $\forall$ n $\in$N.

Therefore $n^2.x - n^2.0 \in I$, $\forall$ x$\in$I and $\forall$ n$\in$N.

This proves that I is a weakly pseudo-left ideal in N.

[ ]

By the definition of a Moore family of subsets of a given set (see Def. 0.1.15) we get,

Corollary 1.2.2 : Set of all weakly pseudo-left ideals in a near-ring N forms a Moore family of subsets of N.

Proof : (1) Every near-ring N is a weakly pseudo-left ideal (By Def. 1.1.1)

(2) Intersection of any collection of weakly pseudo-left ideals in a near-ring N is a weakly pseudo-left ideal. (by Result 1.2.1)

Hence from (1) and (2) set of all weakly pseudo left ideals in N forms a Moore family of subsets of N.

| |

Union of any two weakly pseudo-left ideals need not be a weakly pseudo-left ideal. For this consider the following example .

Example 1.2.3 (Clay 2.2 , 13)

Consider the near-ring N ={0,a,b,c}under the addition and multiplication defined by the following tables.

+	0	a	b	c
0	0	a	b	c
a	a	0	c	b
b	b	c	0	a
c	c	b	a	0

.	0	a	b	c
0	0	0	0	0
a	0	a	b	c
b	0	0	0	0
c	0	a	b	c

Here A={0,a} and B={ 0,b} are weakly pseudo-left ideals in N.

Thus AUB ={ 0, a, b}.

As, a+b=c $\notin$ AUB, for a,b $\in$ AUB.

Therefore AUB is not a weakly pseudo-left ideal in N.

§ Weakly pseudo-right ideal in a near-ring

§ 1.3 Definition and examples :

In this article our aim is to define weakly pseudo-right ideal in a near-ring and to provide some examples of weakly pseudo-right ideal.

Definition 1.3.1 :

Let <N,+ ,.> be a near-ring. A non-empty subset I of N is called weakly pseudo-right ideal in N if it satisfies the following conditions .

(1) <I,+>is a normal subgroup of <N,+> and

(2) $i.n^2 \in I$, $\forall\, i \in I$ and $\forall\, n \in N$.

Examples of weakly pseudo-right ideals in near-rings are given below.

Example 1.3.2 : (Clay,2.1,10)

Consider the near-ring N={0,a,b,c} with addition and multiplication defined by the following tables.

+	0	a	b	c
0	0	a	b	c
a	a	b	c	0
b	b	c	0	a
c	c	0	a	b

.	0	a	b	c
0	0	0	0	0
a	0	a	b	a
b	0	b	0	b
c	0	c	b	c

Let I = { 0,b} . Here I is weakly pseudo-right ideal in N.

Example 1.3.3 (Clay, 2.2, 2)

Consider the near-ring N={0,a,b,c} under the addition and multiplication defined by the following tables.

+	0	a	b	c
0	0	a	b	c
a	a	0	c	b
b	b	c	0	a
c	c	b	a	0

.	0	a	b	c
0	0	0	0	0
a	0	0	a	a
b	0	a	b	b
c	0	a	c	c

Let I = {0,c}. Here I is a weakly pseudo-right ideal in N.

But $b^2.c - b^2.0 = b.c - 0 = b \notin I$, for $b \in N$ and $c \in I$.

Therefore I is not a weakly pseudo-left ideal in N.

□

Example 1.3.4: (Clay, 2.2, 13)

Consider the near-ring N={0,a,b,c}with addition and multiplication defined by the following tables.

+	0	a	b	c
0	0	a	b	c
a	a	0	c	b
b	b	c	0	a
c	c	b	a	0

.	0	a	b	c
0	0	0	0	0
a	0	a	b	c
b	0	0	0	0
c	0	a	b	c

Let I ={ 0,c}. Here I is weakly pseudo-left ideal in N but not a weakly pseudo-right ideal. Because $c.a^2 = c.a = a \notin I$, for $c \in I$ and for $a \in N$.

□

From the above two examples we say that weakly pseudo-left ideal and weakly pseudo-right ideal in a near-ring are independant concepts.

□

A relation between right ideal and weakly pseudo-right ideal is established in the following result.

Result 1.3.5 : Every right ideal in a near-ring is a weakly pseudo-right ideal.

Proof : Let < N, +, . > be a near-ring . Let I be a right -ideal in N. Therefore < I,+> is a normal subgroup of < N,+> and i.n∈I, ∀ i ∈ I and ∀ n∈N . If n∈N then n^2∈N.

Therefore i.n^2 ∈ I, ∀ i∈I and ∀ n∈N.

Hence I is a weakly pseudo-right ideal in N.

□

But every weakly pseudo-right ideal in a near-ring need not be a right ideal . This is established in the following example.

Example 1.3.6 : (Clay, 2.2, 2)

Consider the near-ring N {0,a,b,c} with addition and multiplication is defined by the following tables.

+	0	a	b	c
0	0	a	b	c
a	a	0	c	b
b	b	c	0	a
c	c	b	a	0

.	0	a	b	c
0	0	0	0	0
a	0	0	a	a
b	0	a	b	b
c	0	a	c	c

Consider I = {0,c}. I is weakly pseudo-right ideal in N. But as c.a=a$\notin$ I, for c$\in$I and a$\in$N. Therefore I is not a right ideal.

□

§1.4 Properties of weakly pseudo-right ideals:

In this article we study some properties of weakly pseudo-right ideals in near-rings.

When a near-ring N is a Boolean near-ring (see Def.0.1.12) the converse of the result 1.3.5 holds . This is established in the following result.

Result 1.4.1 : Every weakly pseudo-right ideal in a Boolean near-ring is a right ideal.

Proof : Let <N, +, .> be a Boolean near-ring . Therefore $n^2 = n$,$\forall$ n $\in$ N (see Def.0.1.12). Let I be a weakly pseudo-right ideal in N. Then < I,+> is a normal subgroup of < N,+> and i.$n^2\in$ I, $\forall$ i$\in$I and $\forall$n$\in$N.

Thus i.n =i.$n^2\in$ I, $\forall$ i$\in$I [Since n^2=n, $\forall$n$\in$N].

i.e. i.n$\in$I, $\forall$ i$\in$ I and $\forall$n$\in$N.

Therefore I is a right ideal in N.

□

Result 1.4.2 : Intersection of any collection of weakly pseudo-right ideals in a near-ring N is a weakly pseudo-right ideal .

Proof : Take I =$\cap${I_i /I_i is a weakly pseudo-right ideal in N}. To prove that I is a weakly pseudo-right ideal in N.

(1) As intersection of any collection of a normal subgroups in N is a normal subgroup in N (see Result 0.2.2) We get < I,+> is a normal subgroup < N,+>.

(2) Let $x \in I$ and let $n \in N$. Therefore $x \in I_i$ where I_i is a weakly pseudo-right ideal in N. By definition of weakly pseudo-right ideal in N, $x.n^2 \in I_i$, $\forall$ I_i, Therefore $x.n^2 \in \cap_i I_i$

Hence $x.n^2 \in I$, $\forall$ $x \in I$ and $\forall$ $n \in N$.

This proves that I is a weakly pseudo-right ideal in N.

□

As N itself is a weakly pseudo-right ideal, by definition of Moore family of subsets of a given set (see Def 0.1.15) and by result 1.4.2 we get the following corollary.

Corollary 1.4.3 : Set of all weakly pseudo-right ideals in a near-ring N forms a Moore family of subsets of N.

□

Union of any two weakly pseudo-right ideals need not be a weakly pseudo-right ideal . For this consider the following example

Example 1.4.4 : (Clay , 2.2,2)

Consider the near-ring N={0,a,b,c} under the addition and multiplication defined by the following tables.

+	0	a	b	c
0	0	a	b	c
a	a	0	c	b
b	b	c	0	a
c	c	b	a	0

.	0	a	b	c
0	0	0	0	0
a	0	0	a	a
b	0	a	b	b
c	0	a	c	c

Let $A = \{0,a\}$ and $B=\{0,c\}$ be any two weakly pseudo-right ideals in N. $A \cup B =\{0,a,c\}$ as, $a+c =b \notin A \cup B$ for $a,c \in A \cup B$

Therefore $A \cup B$ is not a weakly pseudo-right ideal in N.

□

In a commutative near-ring the two concepts of weakly pseudo-left ideal and of weakly pseudo-right ideal coincide. This we prove in the following result.

Result1.4.5 : In a commutative near-ring N, a non-empty subset A is weakly pseudo-left ideal in N iff A is weakly pseudo-right ideal in N.

Proof : Let $< N,+,. >$ be a commutative near-ring . First suppose A is weakly pseudo-left ideal in N. Therefore $< A,+ >$ is a normal subgroup of $< N,+ >$ and $n^2.a - n^2.0 \in A$, $\forall\ a \in A$ and $\forall\ n \in N$. Since N is commutative , therefore $n^2.a - n^2.0 = a.n^2 - 0.n^2 = a.n^2 - 0 = a.n^2$ [see Def. 0.1.3 and Result 0.2.1] Hence . $a.n^2 = n^2.a - n^2.0 \in A$, $\forall\ a \in A$ and $\forall\ n \in N$. Thus A is weakly pseudo-right ideal.

Conversely suppose A is weakly pseudo-right ideal in N. Therefore $< A,+>$ is a normal subgroup of $< N,+>$ and $a.n^2 \in A$, $\forall\ a \in A$ and $\forall\ n \in N$. Therefore $n^2.a - n^2.0= a.n^2 - 0 n^2 = a.n^2 - 0 = a.n^2 \in A$ [Since N is commutative, see Def . 0.1.3 and result 0.2.1] Hence $n^2.a - n^2.0 \in A$ $\forall\ a \in A$ and $\forall\ n \in N$ .Thus A is weakly pseudo-left ideal in N.

□

If A and B are two non-empty subsets of a near-ring N, then we define $A+B = \{ a+b \ / \ a \in A \text{ and } b \in B\}$. When A and B are weakly pseudo-right ideals we get,

Result 1.4.6 : Let $< N,+,. >$ be a near-ring. Let A and B two weakly pseudo-right ideals in N .Then A+B is the smallest weakly pseudo-right ideal containing both A and B.

Proof : Here $A+B = \{ a+b \ / \ a \in A \text{ and } b \in B\}$. Since A and B are weakly pseudo-right ideals in N.

Therefore $< A , + >$ and $< B, +>$ are normal subgroups of $< N,+>$ and $a.n^2 \in A$ and $b.n^2 \in B$, $\forall$ $a \in A$, $\forall$ $b \in B$ and $\forall$ $n \in N$. As A and B are normal subgroups of $<N,+>$, therefore $< A+B, + >$ is a normal subgroup of $< N, +>$ [See result 0.2.3] ---------- (1)

Let $x \in A+B$ and let $n \in N$.

Therefore $x = a+b$, for some $a \in A$ and $b \in B$.

Therefore $x.n^2 = (a+b) .n^2 = a.n^2 + b.n^2 \in A+B$ [See Def. 0.1.1. and $a.n^2 \in A$, $\forall$ $a \in A$ and $\forall$ $n \in N$, $b.n^2 \in B$, $\forall$ $b \in B$ and $\forall$ $n \in N$]

Hence $x.n^2 \in A+B$, $\forall$ $x \in A+B$ and $\forall$ $n \in N$ --------------(2)

Therefore from (1) and (2) , A+B is a weakly pseudo-right ideal in N.

Since $0 \in B$. For any $a \in A$, $a = a+0 \in A+B$

Therefore $A \subseteq A+B$. Similary $B \subseteq A+B$. Let C be any weakly pseudo-right ideal in N such that $A \subseteq C$ and $B \subseteq C$.

To prove that $A-B \subseteq C$.

Let $x \in A+B$.

Therefore $x = a+b$ for some $a \in A$ and $b \in B$.

Since $A \subseteq C$, therefore $a \in C$ and $B \subseteq C$, therefore $b \in C$. Hence $x = a+b \in C$ [Since C is weakly pseudo-right ideal of N]. Therefore $A+B \subseteq C$. Hence A+B is the smallest weakly pseudo-right ideal containing both A and B.

□

Result 1.4.7 : Let $< N, +, . >$ be a near-ring . Zero-symmetric subnear-ring N_o is a weakly pseudo-left ideal of N but not generally weakly pseudo-right ideal.

Proof : Let $< N, +, . >$ be a near-ring . let N_o be a zero-symmetric subnear-ring of N . $N_o = \{ n \in N / n.0 = 0\}$.

To prove that N_o is weakly pseudo-left ideal of N .

(1) To prove that $< N_o,+ >$ is normal subgroup of $< N,+ >$.

Let $a,b \in N_o$

Therefore $a.0 = 0$ and $b.0 = 0$

Therefore $(a-b).0 = a.0 - b.0 = 0-0 = 0$

Therefore $(a-b) \in N_o \ \forall \ a,b \in N_o$

Let $n \in N$, $a \in N_o$

Therefore $(n+a-n).0 = n.0 + a.0 - n.0 = n.0 + 0 - n.0 = n.0 - n.0 = 0$.

Therefore $(n+a-n) \in N_o$, $\forall \ n \in N$ and $\forall \ a \in N_o$.

Hence $< N_o,+ >$ is a normal subgroup of $< N , + >$

(2) Let $n \in N$ and $a \in N_o$

Therefore $(n^2.a - n^2.0).0 = (n^2.a).0 - (n^2.0).0$

$$= n^2.(a.0) - n^2.(0.0) = n^2.0 - n^2.0 =0$$

Hence $(n^2.a - n^2.0) \in N_o \ \forall \ n \in N$ and $\forall \ a \in N_o$

Therefore N_o is weakly pseudo-left ideal of N.

But N_0 is not a weakly pseudo-right ideal of N. This we prove in the following example.

Let R be a ring. Let $N = \{ f / f : R \to R$ be a function $\}$

Define '+' and 'o' on N as follows,

$(f + g)(x) = f(x) + g(x)$

$(fog)(x) = f[g(x)] \quad \forall\, x \in R$ and $\forall\, f, g \in N$

Therefore, $< N, +, o >$ is a near-ring .

Let $N_0 = \{ f \in N / fo0 = 0\}$

Where $0 : R \to R$ is a zero function.

Therefore $0(x) = 0 , \forall\, x \in R$.

Here N_0 is a weakly pseudo - left ideal of N.

Identity map $i:R \to R$ [defined by $i(x) = x$, $\forall\, x \in R$] is an element in N_0

Consider $g : R \to R$ defined by $g(x) = 1 \;\; \forall\, x \in R$.

Therefore $g \in N$

Now consider (iog^2)

Therefore $(iog^2)(x) = i[g^2(x)] = i\,[gog\,(x)]$

$= (iog)\,[g(x)]$

$= iog(1)$

$= i[g(1)]$

$= i(1) = 1$

Hence $(iog^2)(x) = 1, \quad \forall\, x \in R$

Therefore $(iog^2)(0) = 1$

Hence $(iog^2) \notin N_0$ for $i \in N_0$ and $g \in N$.

Thus N_0 is not a weakly pseudo-right ideal.

§ Weakly Pseudo-ideal in a near-ring

§ 1.5 Definition and examples :

In this section we define weakly pseudo-ideal in a near-ring N and give some examples of a weakly pseudo-ideal in a near-ring.

We know every weakly pseudo-left ideal in a near-ring need not be a weakly pseudo-right ideal and every weakly pseudo-right ideal in a near-ring need not be a weakly pseudo-left ideal (see Example 1.3.4 and Example 1.3.3). This motivates us to define

Definition 1.5.1 :

Let $< N, +, . >$ be a near-ring . A non-empty subset I of N is called a weakly pseudo-ideal in N if it satisfies the following conditions

(1) $< I,+ >$ is a normal subgroup of $< N,+ >$.

(2) $n^2.i - n^2.0 \in I$, $\forall\, i \in I$ and $\forall\, n \in N$.

(3) $i.n^2 \in I$, $\forall\, i \in I$ and $\forall\, n \in N$.

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Some examples of weakly pseudo-ideals in near-rings are given below.

Example 1.5.2 : (Clay , 2.1, 10)

Consider the near-ring N = { 0, a, b,c} with addition and multiplication as given by the following tables

+	0	a	b	c
0	0	a	b	c
a	a	b	c	b
b	b	c	0	a
c	c	0	a	b

.	0	a	b	c
0	0	0	0	0
a	0	a	b	a
b	0	b	0	b
c	0	c	b	c

Let I = {0,b} . I is both weakly pseudo-left ideal as well as weakly pseudo - right ideal in N. Thus I is weakly pseudo-ideal in N.

□

Example 1.5.3 : (Clay , 2.2,13)

N = { 0, a, b, c} is a near-ring under the addition and multiplication defined by the following tables.

+	0	a	b	c
0	0	a	b	c
a	a	0	c	b
b	b	c	0	a
c	c	b	a	0

.	0	a	b	c
0	0	0	0	0
a	0	a	b	c
b	0	0	0	0
c	0	a	b	c

The subsets { 0,a} , {0,b} and {0,c} are weakly pseudo-left ideals of N , whereas {0,b} is its only weakly pseudo-ideal.

□

§ 1.6 Properties of weakly pseudo-ideals

Using the result 1.1.7 and result1. 1.3.5 we get the following result.

Result 1.6.1 : Every ideal in a near-ring N is a weakly pseudo-ideal in N.

□

Converse of the result 1.6.1 need not be true. This is established in the following example

Example 1.6.2 : (Pilz, page 408)

Consider the near-ring N={ 0, a, b, c} with addition and multiplication as g iven by the following tables.

+	0	a	b	c
0	0	a	b	c
a	a	0	c	b
b	b	c	0	a
c	c	b	a	0

.	0	a	b	c
0	0	0	0	0
a	0	b	0	b
b	0	0	0	0
c	0	b	0	b

Let I ={0,a}. I is weakly pseudo-left ideal as well as weakly pseudo-right ideal in N. Therefore I is weakly pseudo-ideal in N.But as a.a = b$\notin$I for a$\in$I and a$\in$N . Hence I is not a right ideal of N. Therefore I is not an ideal.

□

Union of any two weakly pseudo-ideal in a near-ring N need not be a weakly pseudo-ideal. This is established in the following example.

Example 1.6.3 : (Pilz, page 408)

Consider the near-ring N={ 0, a, b, c} with addition and multiplication as given by the following tables.

+	0	a	b	c
0	0	a	b	c
a	a	0	c	b
b	b	c	0	a
c	c	b	a	0

.	0	a	b	c
0	0	0	0	0
a	0	b	0	b
b	0	0	0	0
c	0	b	0	b

Let A= {0,a} and B={0,b} be two weakly pseudo-ideals in N AUB = { 0 , a, b}. But as, a+b = c ∉ AUB, for a,b ∈ AUB. Therefore AUB is not a weakly pseudo-ideal in N.

□

Intersection of weakly pseudo-ideal A and a subnear-ring S of N is weakly pseudo- ideal of S. This we prove in the following result.

Result 1.6.4 : If A is a weakly pseudo-ideal of a near-ring N and S is a subnear-ring of N then A∩S is a weakly pseudo-ideal of S.

Proof : Let < N, +, . > be a near-ring. Let A be a weakly pseudo-ideal of N. Let S be a subnear-ring of N.

To prove that < A∩S , + > is a normal subgroup of <N,+>.

Let x ∈ A∩S, n ∈ S⊆N

Therefore x∈A and x∈S, n∈S

Therefore n+x-n ∈ A and n+x-n ∈ S [Since < A ,+ > is normal subgroup of < N, +> and < S,+> is a subgroup of < N,+>]

Hence n+x-n ∈ A∩S, ∀ x ∈ A∩S and ∀ n∈S.

Therefore < A∩S , + > is normal subgroup of < N, +>. ---- (1)

Now to prove that $n^2.x - n^2.0 \in A\cap S$, ∀ x ∈ A ∩S and ∀ n∈S

Let x ∈ A∩S and let n ∈ S⊆N.

Therefore x∈A and x∈S, n∈S.

Therefore $n^2.x - n^2.0 \in A$ and $n^2.x - n^2.0 \in S$. [Since A is weakly pseudo-ideal in N and S is a subnear-ring of N].

Therefore $n^2.x - n^2.0 \in A \cap S$, $\forall$ $x \in A \cap S$ and $\forall$ $n \in N$ -----(2)

To prove that $x.n^2 \in A \cap S$, $\forall$ $x \in A \cap S$ and $\forall$ $n \in S$.

Let $x \in A \cap S$ and let $n \in S \subseteq N$.

Therefore $x \in A$ and $x \in S$, $n \in S$

Therefore $x.n^2 \in A$ and $x.n^2 \in S$. [Since A is weakly pseudo-ideal in N and S is a subnear-ring of N.]

Therefore $x.n^2 \in A \cap S$, $\forall$ $x \in A \cap S$ and $\forall$ $n \in S$ .... (3)

Hence from (1) , (2) a nd (3) , $A \cap S$ is a weakly pseudo-ideal in S.

□

For any non-empty subset A of N, we define

$xAy = \{x.a.y \,/\, a \in A\}$ where $x,y \in N$.

Result 1.6.5 : If A is a weakly pseudo-ideal of commutative near-ring N then $xAx \subseteq A$, $\forall$ $x \in N$.

Proof : Let $< N,+, .>$ be a commutative near-ring . Let A be a weakly pseudo-ideal of N. Therefore $< A,+>$ is a normal subgroup of $< N,+>$ and $x^2.a - x^2.0 \in A$, $\forall$ $a \in A$ and $\forall$ $x \in N$ and $a.x^2 \in A$, $\forall$ $a \in A$ and $\forall$ $x \in N$. Since N is commutative near-ring (see Def 0.1.3) Therefore $x.a.x = (x.a).x = (a.x).x = a.(x.x) = a.x^2$. Therefore $x.a.x = a.x^2 \in A$, $\forall$ $a \in A$ and $\forall$ $x \in N$. Therefore $xAx \subseteq A$, $\forall$ $x \in N$.

□