CHAPTER-I
ROUGH SETS

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I:1 INTRODUCTION

Pawlak [P_1] has developed rough set theory to describe indiscernibility mathematically. Stefan Chanas and Kuchta [C] have defined rough sets slightly in a different way. As we shall see in this chapter, second approach is more general than the first approach. The two concepts coincide, if the rough set have a generator.

Throughout this work U stands for the universe and R for an equivalence relation on U.

I:2 ROUGH SETS

Definition (I:2:1) [P_1]

An approximation space is an ordered pair K=(U,R) where U is a nonempty set called universe and R is an equivalence relation on U called an indiscernibility relation.

Hereafter we assume that K = (U,R) is an approximation space. We shall denote $U/R = \{E_\lambda | E_\lambda \varepsilon \Lambda\}$, the set of equivalence classes of U, induced by R.

Definition (I:2:2) [P_1]

For each subset $X \subseteq U$, the lower approximation of X with respect to the equivalence relation R, is the set $\underline{R}X = \cup\{Y \in U/R \mid Y \subseteq X\} = \{x \in U \mid [x]_R \subseteq X\}$; where $[x]_R$ denotes equivalence class of $x \in U$ with respect to R.

Definition (I:2:3) [P_1]

For each subset $X \subseteq U$, the upper approximation of X with respect to R, is the set

$$\bar{R}X = \cup\{Y \in U/R \mid Y \cap X \neq \Phi\} = \{x \in U \mid [x]_R \cap X \neq \Phi\}.$$

Note (I:2:4)

From above two definitions (I:2:2) and (I:2:3) it is easy to see that $\underline{R}X \subseteq X \subseteq \bar{R}X$.

Definition (I:2:5) [C] [P_1] [P_2]

Let $\mathcal{E} = \{\Phi\} \cup \{E_\lambda \mid \lambda \in \Lambda\}$ be the class of R-equivalence classes together with empty set. The elements of $\mathcal{E}$ are called R-elementary sets.

Definition (I:2:6) [P_1,P_2]

A subset $Y \subseteq U$ is R-exact or R-composed set if Y is a union of R elementary sets in the approximation space $K = (U,R)$, otherwise Y is R-rough or R-undefinable set.

Note (I:2:7)

Hereafter we shall denote R-rough set as rough set and R-composed set as composed set.

The following proposition is immediate

Proposition (I:2:8)

(i) $\underline{R}X$ is the maximal exact set included in X.

(ii) $\bar{R}X$ is the minimal exact set containing X.

Proposition (I:2:9)

Let $X \subseteq U$ and R be an equivalence relation on U in the approximation space $K = (U,R)$.

(i) X is <u>exact set</u> if and only if $\underline{R}X = \bar{R}X$.

(ii) X is <u>rough set</u> if and only if $\underline{R}X \neq \bar{R}X$.

Proof : (i) Suppose that X is exact set.

Let $X = \cup\{ E_i \mid i \in I \}$, where $I \subseteq \Lambda$

Then $\underline{R}X = \cup\{ E_i \mid E_i \subseteq X,\ i \in \Lambda \}$

$\supseteq \cup\{ E_i \mid E_i \subseteq X,\ i \in I \}$

$\supseteq \cup\{ E_i \mid i \in I \}$

$= X$

$\supseteq \underline{R}\ X$

and $\underline{R}X = \cup\{ E_i \mid E_i \cap X \neq \Phi,\ i \in \Lambda \}$

$= \cup\{ E_i \mid i \in I\}$

(Since $E_i \cap E_j = \Phi\ \forall\ i \neq j,\ i; j \in \Lambda$)

$= X$

Therefore $\underline{R}X = X = \bar{R}X$

conversely, suppose that $\underline{R}X = \bar{R}X$

Let $Y = \cup\{ E_i \mid E_i \subseteq X,\ i \in I\}$

$= \cup\{ E_i \mid E_i \cap X \neq \Phi,\ i \in I\}$

Our claim is $X = Y$

Clearly $Y \subseteq X$

Let $x \in X$. Then for $[x]_R$, $[x]_R \cap X \neq \Phi$

Hence $[x]_R \subseteq U\{ E_i \mid E_i \cap X \neq \Phi, i \in I \} = Y$

Therefore $x \in Y$

Thus $X \subseteq Y$ and the claim $X = Y$. Hence X is exact set.

(ii) Obvious.

Definition (I:2:10) [P_1,P_2,K]

Let $X \subseteq U$ be a rough set in an approximation space $K = (U,R)$.

i) The lower approximation $\underline{R}X$ of X is also called positive region of X. It is denoted as $POS_R(X)$.

ii) The negative region of X, in symbol $NEG_R(X)$, is defined as,

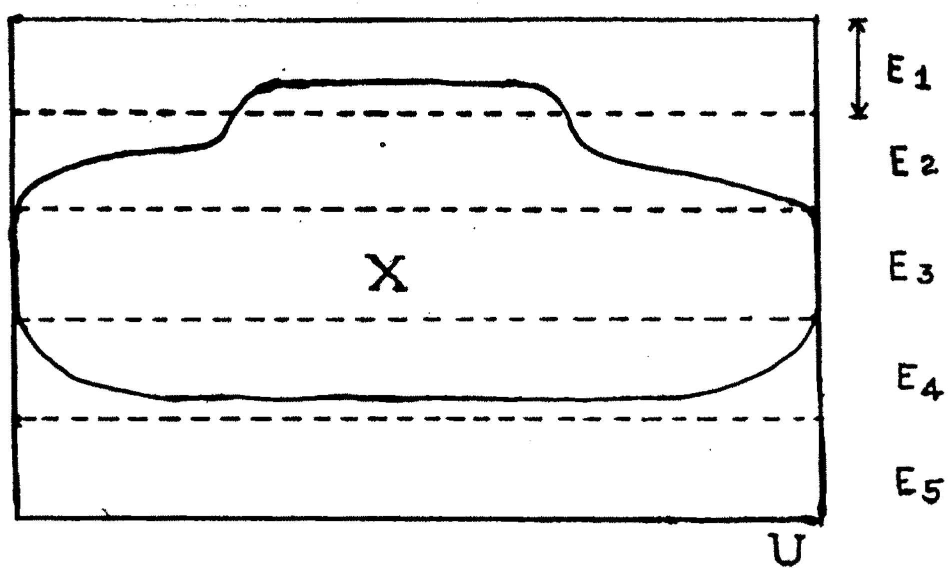
$NEG_R(X) = U - \bar{R}X$, it is the complement of an upper approximation of X with respect to U.

iii) The boundary region of X, in symbol $BON_R(X)$, is defined as

$BON_R(X) = \bar{R}X - \underline{R}X$. It is the set difference of an upper and lower approximation.

Example (I:2:11)

Let K = (U,R) be the approximation space and $U/R = \{E_1, E_2, E_3, E_4, E_5\}$.



DIG (I:2:1)

If X is a subset of U given in Dia. (I:2:1) then clearly

$\underline{R}X = \{ E_3 \} = POS_R(X)$

$\bar{R}X = \{ E_1, E_2, E_3, E_4 \}$

$U-\bar{R}X = U - \{ E_1, E_2, E_3, E_4 \} = \{ E_5 \} = NEG_R(X)$

$\bar{R}X-\underline{R}X = \{ E_1, E_2, E_3, E_4 \} - \{ E_3 \} = \{ E_1, E_2, E_4 \}$

$= BON_R(X)$.

Remarks (I:2:12)

Let $X \subseteq U$ be a rough set in the approximation space K = (U,R).

i) The elements in the positive region of X are definitely in X.

ii) The elements in the negative region of X are definitely not in X.

iii) The elements in the boundary region of X are posibly the member's of X.

I:3 PROPERTIES OF ROUGH SETS

The following results are obvious [P_1, P_2, P_3, C]; [K, N_1, N_2, W_1, W_2].

Let $X, Y \subseteq U$ be rough sets in the space K. The followings are properties of rough sets.

i) $\underline{R}\,X \subseteq X \subseteq \bar{R}\,X$

ii) $\bar{R}(X \cup Y) = \bar{R}\,X \cup \bar{R}Y$

iii) $\underline{R}(X \cup Y) \supseteq \underline{R}\,X \cup \underline{R}Y$

iv) $\underline{R}(X \cap Y) = \underline{R}\,X \cap \underline{R}\,Y$

v) $\bar{R}(X \cap Y) \subseteq \bar{R}\,X \cap \bar{R}Y$

vi) If $X \subseteq Y$ then

(a) $\underline{R}X \subseteq \underline{R}Y$

(b) $\bar{R}X \subseteq \bar{R}Y$

vii) $\underline{R}(-X) = -\bar{R}(X)$

viii) $\bar{R}(-X) = -\underline{R}(X)$

ix) $\underline{R}\,\underline{R}(X) = \underline{R}X = \bar{R}\underline{R}X$

x) $\bar{R}\bar{R}X = \bar{R}X = \underline{R}\bar{R}X$

Where -X denote the complement of X with respect to U.

The following examples shows that the strict containtment holds in properties (iii) and (v).

Example (I:3:1)

In an approximation space $K = (U,R)$, let $U = \{ x_1, x_2, \ldots, x_9 \}$ and equivalence relation R have the following equivalence classes,

$E_1 = \{ x_1, x_7, x_9 \}$

$E_2 = \{ x_2, x_8 \}$

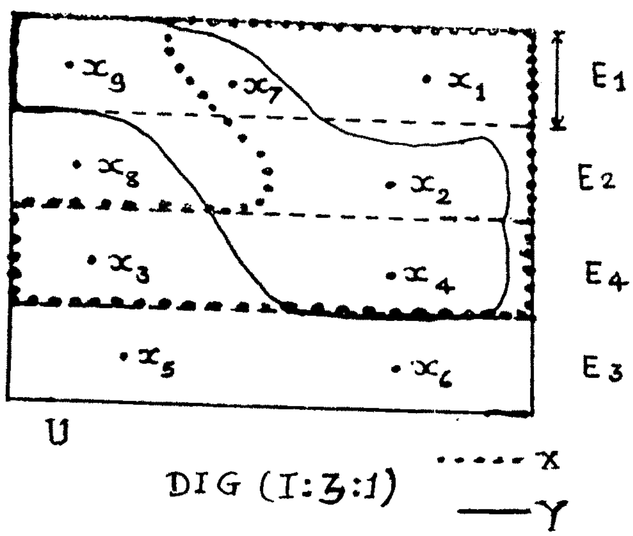
$E_3 = \{ x_5, x_6 \}$

$E_4 = \{ x_3, x_4 \}$

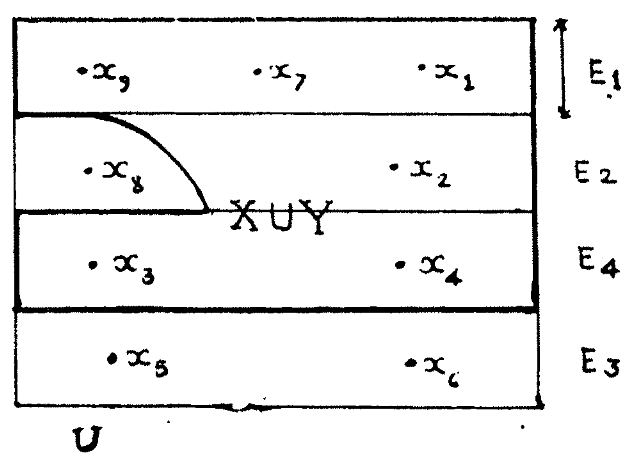
Consider $X, Y \subseteq U$ as follows

$X = \{ x_1, x_2, x_3, x_4, x_7 \}$

$Y = \{ x_2, x_4, x_7, x_9 \}$



DIG (I:3:1)



DIG (I:3:2)

From Diag.(I:3:1), we have

$\underline{R}X = \{ E_4 \}$

$\underline{R}Y = \Phi$

and $X \cup Y = \{ x_1, x_2, x_3, x_4, x_7, x_9 \}$

Using Diag.(I:3:2), it is clear that

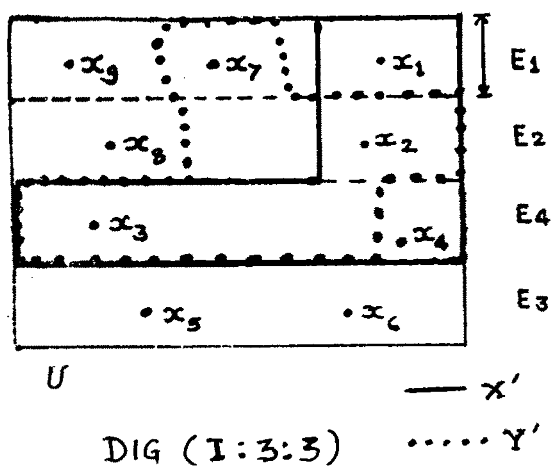
$$\underline{R}(X \cup Y) = E_1 \cup E_4$$

Hence $\underline{R}(X \cup Y) \supseteq \underline{R}X \cup \underline{R}Y$. But $\underline{R}(X \cup Y) \neq R\_X \cup \underline{R}Y$.

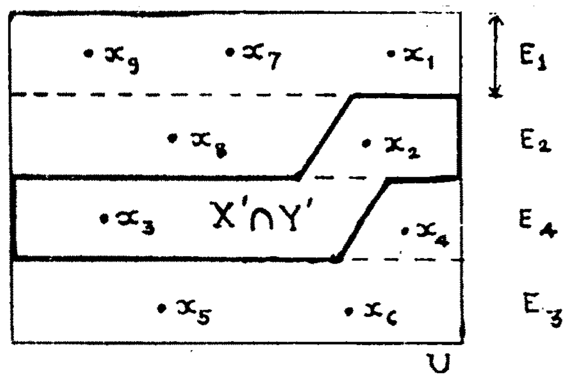
Now consider,

$$X' = \{ x_1, x_2, x_3, x_4 \}$$

$$Y' = \{ x_2, x_3, x_7 \}$$



DIG (I:3:3)



DIG(I:3:4)

From Diag.(I:3:3)

$$\bar{R}X' = E_1 \cup E_2 \cup E_4$$

$$\bar{R}Y' = E_1 \cup E_2 \cup E_4$$

Therefore, $\bar{R}X' \cap \bar{R}Y' = E_1 \cup E_2 \cup E_4$

$$X' \cap Y' = \{ x_2, x_3 \}$$

Using Diag.(I:3:4), we have

$$\bar{R}(X' \cap Y') = E_1 \cup E_4$$

Hence $\bar{R}(X' \cap Y') \subseteq \bar{R}X' \cap \bar{R}Y'$.

But $\bar{R}(X' \cap Y') \neq \bar{R}X' \cap \bar{R}Y'$

I:4 ANOTHER APPROACH TO ROUGH SETS

The rough sets defined in the previous article assume a prori knowledge of subsets of the universel set U. This is a severe constraint. Therefore a rough set is redefined by Chanas and Kuchta [C]. This definition does not presuppose the knowledge of subsets of U. We shall discuss this approach in this article.

Let $\mathcal{E} = \{E_\lambda \mid \lambda \in \Lambda\}$ be a partition of U.

i.e. $\bigcup_{\lambda \in \Lambda} E_\lambda = U$ & $E_\lambda \cap E_\mu = \Phi, \quad \forall \lambda \neq \mu; \ \lambda, \mu \in \Lambda$.

We assume that $\Phi \in \mathcal{E}$

Definition (I:4:1)

An ordered pair $K = (U, \mathcal{E})$ is called an <u>approximation space</u>. The elements of $\mathcal{E}$ are called <u>elementary sets</u>. The union of elementary sets are called <u>composed sets</u>.

Note (I:4:2)

The approximation space defined in Def.(I:2:1) and the above definition (I:4:1) are essentially the same concepts.

If $K = (U,R)$ is an approximation space in the sense of Def.(I:2:1) then $É = \{\Phi\} \cup \{ E_\lambda | \lambda \in \Lambda \}$ where E_λ are equivalence classes induced by R. On the other hand if $(U,É)$ is an approximation space in the sense of above Def.(I:4:1) then R is an equivalence relation induced by the partition É.

For this reasons we shall use both Definition of approximation spaces interchangeablly; where the relationship between É and R is obvious.

Definition (I:4:3) [C]

A pair (A_1, A_2) of subsets of U is a <u>rough set</u> in an approximation space $K = (U,É)$, if

(i) $A_1 \subseteq A_2$

(ii) A_1, A_2 are both composed sets in the approximation space K.

Definition (I:4:4)

A rough set (A_1,A_2) is called <u>exact</u> if $A_1 = A_2$.

Definition (I:4:5) [C]

A subset $X \subseteq U$ is a <u>generator</u> for a rough set (A_1, A_2) in $K = (U,É)$ if,

(i) $A_1 = \cup\{ Y \in É \mid Y \subseteq X \} = \{ x \in U \mid [x]_R \subseteq X \}$

(ii) $A_2 = \cup\{ Y \in É \mid Y \cap X \neq \Phi \} = \{x \in U \mid [x]_R \cap X \neq \Phi \}$

where R is the equivalence relation induced by É.

Note (I:4:6)

(i) If R is an equivalence relation induced by Ɛ, then in Def. (I:4:5).

$$A_1 = \cup\{ Y \in U/R \mid Y \subseteq X\} = \{ x \in U \mid [x]_R \subseteq X\} = \underline{R}X.$$

and

$$A_2 = \cup\{ Y \in U/R \mid Y \cap X = \Phi\} = \{x \in U \mid [x]_R \cap X \neq \Phi \}$$
$$= \bar{R}X.$$

(ii) There is some difference in the concepts of rough (exact) sets given in Def.(I:2:9) and Def.(I:4:3); Def.(I:4:4). If X is a rough (exact) set according to Def.(I:2:9) then $\underline{R}X \neq \bar{R}X$ ($\bar{R}X = \underline{R}X$), is a rough (exact) set according to Def.(I:4:3); (I:4:4), in this case X is a generator of the rough set ($\underline{R}X$, $\bar{R}X$). On the other hand if (A_1,A_2) is a rough set according to Def.(I:4:3); then X is its generator, then X is a rough set according to Definition (I:2:9). If (A_1,A_2) is exact then $A_1 = A_2 = X$. However, since a rough set in Def.(I:4:3) can have more than one generators, a rough set X according to (I:2:9) produces unique rough set (A_1,A_2) according to Def.(I:4:3) and (I:4:4) but not conversely.

The following example depict the concept of rough set :

Example (I:4:7) :

Let K = (U, Ɛ) be an approximation space,

Where U = { X_1, X_2, ---, X_{10} } and Ɛ = { E_1, E_2, ---, E_5 }

E_1 = { X_1, X_4 }

E_2 = { X_3, X_7 }

E_3 = { X_5, X_9 }

E_4 = { X_8, X_{10} }

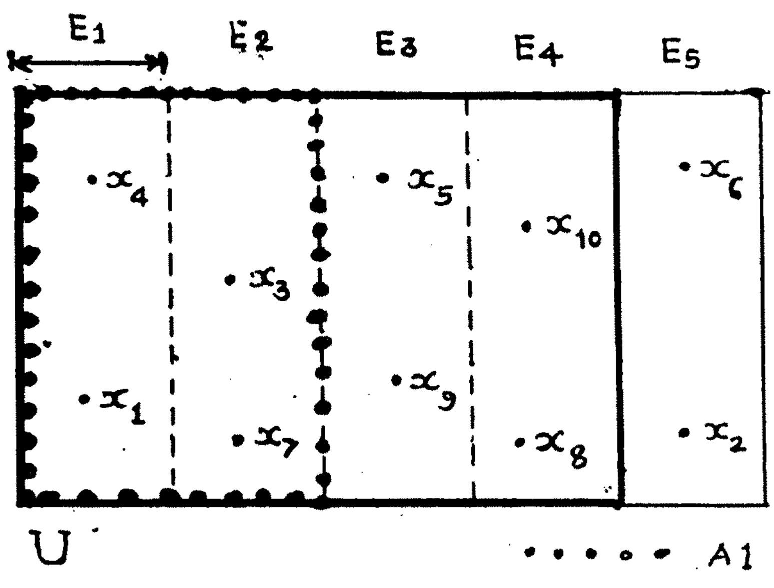
E_5 = { X_2, X_6 }

Consider,

A_1 = { X_1, X_3, X_4, X_7 }

A_2 = { X_1, X_3, X_4, X_7, X_8, X_9, X_{10} }

Then (A_1, A_2) be rough set in the space K.



DIG (I:4:1)

Now consider,

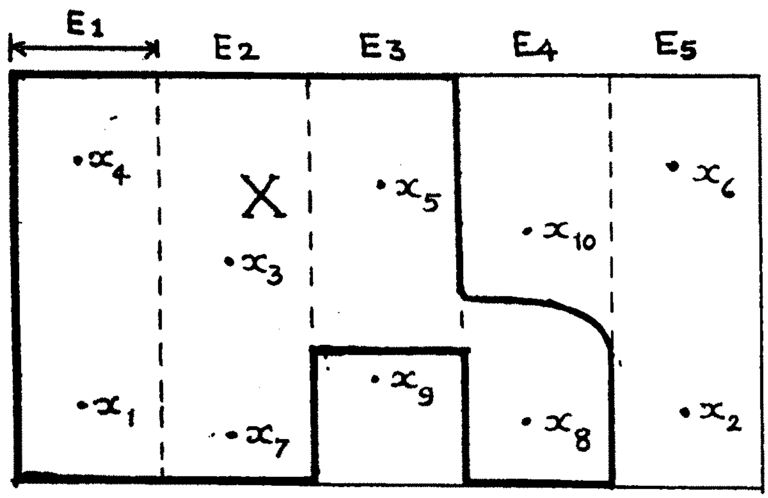
$X = \{ X_1, X_3, X_4, X_5, X_7, X_8 \} \subseteq U,$

Then

$\underline{R}X = E_1 \cup E_2 = A_1.$

$\underline{R}X = E_1 \cup E_2 \cup E_3 \cup E_4 = A_2$

Therefore $X \subseteq U$ is a generater for rough set (A_1, A_2).



DIG (I:4:2)

Remarks (I:4:8)

(i) If $X \subseteq U$ is a generator for rough set (A_1, A_2) in an approximation space $K = (U, \varepsilon)$, then $A_1 \subseteq X \subseteq A_2$.

(ii) For a rough set (A_1, A_2) in the space $K = (U, \varepsilon)$, may have many generater.

The following example illustrates the above remarks.

Example (I:4:9)

Let $K = (U, \mathcal{E})$ Where $U = \{X_1, X_2 ---, X_{12}\}$,

$\mathcal{E} = \{ E_1, E_2, ---, E_7 \}$ and

$E_1 = \{ X_1, X_4 \}$ $E_2 = \{ X_6, X_9 \}$

$E_3 = \{ X_2, X_{10} \}$ $E_4 = \{ X_5 \}$

$E_5 = \{ X_8, X_{11} \}$ $E_6 = \{ X_3, X_{12} \}$

$E_7 = \{ X_7 \}$

Consider,

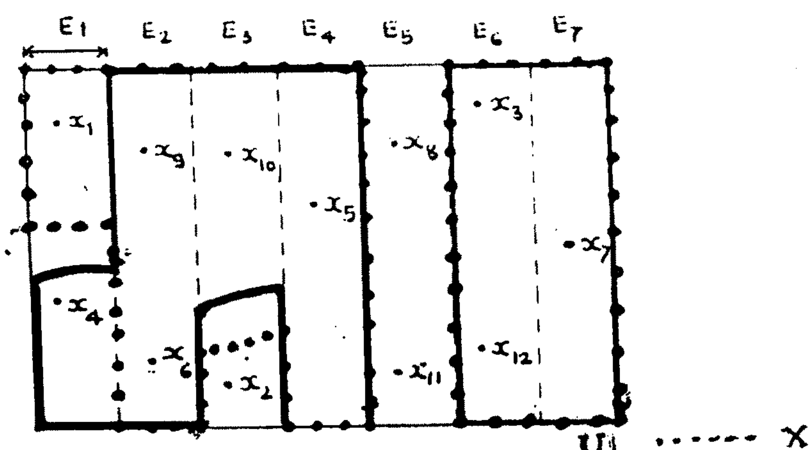
$A_1 = \{ X_3, X_5, X_6, X_7, X_9, X_{12} \}$.

$A_2 = \{ X_1, X_2, X_3, X_4, X_5, X_6, X_7, X_9, X_{10}, X_{12} \}$

Then (A_1, A_2) is a rough set in the K.

Let $X = \{ X_1, X_3, X_5, X_6, X_7, X_9, X_{10}, X_{12} \} \subseteq U$

$Y = \{ X_3, X_4, X_5, X_6, X_7, X_9, X_{10}, X_{12} \} \subseteq U$



DIG (I:4:3)

Clearly X and Y are generater for the rough set (A_1, A_2) in an approximation space K.

And $A_1 \subseteq X$; $Y \subseteq A_2$.

Therefore given rough set (A_1, A_2) has two generator.

Definition (I:4:10)[C]

Let (A_1, A_2) and (B_1, B_2) be two rough sets in the approximation space $K = (U, \mathcal{E})$.

The <u>union</u> and <u>intersection</u> of rough sets are defined as follows

$(A_1, A_2) \cup (B_1, B_2) = (A_1 \cup B_1, A_2 \cup B_2)$ and

$(A_1, A_2) \cap (B_1, B_2) = (A_1 \cap B_1, A_2 \cap B_2)$

Definition (I:4:11)

Let (A_1, A_2) be a rough set in the approximation space $K = (U, \mathcal{E})$. The <u>complement</u> of a rough set (A_1, A_2) is a rough set $(-A_2, -A_1)$, where $-A_2$, $-A_1$ are usual complements of A_1, A_2 with respect to U.

Definition (I:4:12)

Let (A_1, A_2) and (B_1, B_2) be rough sets in the space $K = (U, \mathcal{E})$.

(i) A rough set (A_1, A_2) is <u>included</u> in rough set (B_1, B_2) if and only if $A_1 \subseteq B_1$ and $A_2 \subseteq B_2$.

(ii) A rough set (A_1, A_2) is <u>properly included</u> in a rough set (B_1, B_2) if and only if $A_1 \subset B_1$ or $A_2 \subset B_2$.

(iii) A rough set (A_1, A_2) is <u>equal to</u> rough set (B_1, B_2) if and only if

$A_1 = B_1$ and $A_2 = B_2$.

Remarks (I:4:13)

Let X, Y be the generators of rough sets (A_1, A_2) and (B_1, B_2) respectively.

(i) $(A_1, A_2) \subseteq (B_1, B_2)$ iff $\underline{R}\,X \subseteq \underline{R}\,Y$ and $\bar{R}\,X \subseteq \bar{R}\,Y$

(ii) $(A_1, A_2) \subset (B_1, B_2)$ iff $\underline{R}\,X \subset \underline{R}\,Y$ or $\bar{R}\,X \subset \bar{R}\,Y$

(iii) $(A_1, A_2) = (B_1, B_2)$ iff $\underline{R}\,X = \underline{R}\,Y$ and $\bar{R}\,X = \bar{R}\,Y$

I:5 PROPERTIES OF ROUGH SETS

Let $K = (U, \mathcal{E})$ an approximation space and $X, Y \subseteq U$ be generators of rough sets (A_1, A_2) and (B_1, B_2) respectively. The followings are properties of these rough sets.

(i) $A_1 \subseteq X \subseteq A_2$; $B_1 \subseteq Y \subseteq B_2$

(ii) $\underline{R}\,\Phi = \Phi = \bar{R}\Phi$; $\underline{R}\,U = U = \bar{R}U$

(iii) $\bar{R}(X \cup Y) = \bar{R}X \cup \bar{R}Y = A_2 \cup B_2$

(iv) $\underline{R}(X \cup Y) \supseteq \underline{R}X \cup \underline{R}Y = A_1 \cup B_1$

(v) $\underline{R}(X \cap Y) = \underline{R}X \cap \underline{R}Y = A_1 \cap B_1$

(vi) $\bar{R}(X \cap Y) \subseteq \bar{R}X \cap \bar{R}Y = A_2 \cap B_2$

(vii) If $X \subseteq Y$ then

a) $A_1 \subseteq B_1$

b) $A_2 \subseteq B_2$

i.e. If $X \subseteq Y$, then $(A_1, A_2) \subseteq (B_1, B_2)$

The following proposition is the direct consequence of the Def. (I:4:5) and Note (I:4:6)

Proposition (I:5:1)

Let $K = (U, \mathcal{E})$. $X \subseteq U$ is a generator for a rough set (A_1, A_2) in the space K, then

(i) $\underline{R}\underline{R}X = A_1 = \bar{R}\underline{R}X$,   (ii) $\bar{R}\bar{R}X = A_2 = \underline{R}\bar{R}X$.

Proposition (I:5:2)

Let $K = (U, \mathcal{E})$. If $X \subseteq U$ is a generator for rough set (A_1, A_2), then

(i) $\underline{R}(-X) = -\bar{R}(X)$

(ii) $\bar{R}(-X) = -\underline{R}(X)$

Proof

(i) Let $x \in \underline{R}(-X)$ iff $[x]_R \subseteq -X$

iff $[x]_R \cap X = \Phi$

iff $x \notin \bar{R}(X)$

iff $x \in -\bar{R}(X)$

(ii) Let $x \in \bar{R}(-X)$ iff $[x]_R \cap -X \neq \Phi$

iff $[x]_R \not\subseteq X$

iff $x \notin \underline{R}X$

iff $x \in -\underline{R}X$

Corollary (I:5:3)

Let $K = (U, \mathcal{E})$. If $X \subseteq U$ is a generator for rough set (A_1, A_2) in the space K, then $-X$ is a generator for rough set $(-A_2, -A_1)$.

The following example shows that the strict containtment holds in the properties (iv) and (vi).

Example (I:5:4)

In the approximation space $K = (U, \mathcal{E})$,

$U = \{ x_1, x_2, \ldots, x_8 \}$, $\mathcal{E} = \{ E_1, E_2, E_3, E_4 \}$ where

$E_1 = \{ x_1, x_5 \}$

$E_2 = \{ x_2, x_8 \}$

$E_3 = \{ x_3, x_7 \}$

$E_4 = \{ x_4, x_6 \}$

Consider,

$A_1 = \{ x_1, x_5 \}$

$A_2 = \{ x_1, x_2, x_4, x_5, x_6, x_8 \}$

Clearly (A_1, A_2) is a rough set in the space K.

Let $X = \{ x_1, x_2, x_4, x_5 \}$, then X is a generator for (A_1, A_2)

Next consider,

$B_1 = \Phi$, $B_2 = \{ x_1, x_2, \ldots, x_8 \} = U$

and $Y = \{ x_5, x_6, x_7, x_8 \}$

Y is a generator for rough set (B_1, B_2) in the space K.

Since $X \cup Y = \{ x_1, x_2, x_4, x_5, x_6, x_7, x_8 \}$,

$\underline{R}(X \cup Y) = E_1 \cup E_2 \cup E_4 = \{ x_1, x_2, x_4, x_5, x_6, x_8 \}$

and $\underline{R}X \cup \underline{R}Y = A_1 \cup B_1 = E_1 \cup \Phi = \{ x_1, x_5 \}$

Therefore, $\underline{R}X \cup \underline{R}Y \subseteq R(X \cup Y)$. But $\underline{R}X \cup \underline{R}Y \neq \underline{R}(X \cup Y)$.

Again since $X \cap Y = \{ x_5 \}$;

$\bar{R}(X \cap Y) = E_1 = \{ x_1, x_5 \}$

and $\bar{R}X \cap \bar{R}Y = A_2 \cap B_2 = E_1 \cup E_2 \cup E_4$

Hence $\bar{R}(X \cap Y) \subseteq \bar{R}X \cap \bar{R}Y$. But $\bar{R}(X \cap Y) \neq \bar{R}X \cap \bar{R}Y$.

Remark (I:5:5)

In example (I:5:4) we observe that,

(i) $\underline{R}(X \cup Y) = \{ x_1, x_2, x_4, x_5, x_6, x_8 \} \neq \{ x_1, x_5 \} = A_1 \cup B_1$

Hence $X \cup Y$ is not a generator for rough set

$(A_1, A_2) \cup (B_1, B_2) = (A_1 \cup B_1, A_2 \cup B_2)$.

(ii) $\bar{R}(X \cap Y) = \{ x_1, x_5 \} \neq \{ x_1, x_2, x_4, x_5, x_6, x_8 \}$

$= A_2 \cap B_2$

Hence $X \cap Y$ is not a generator for rough set

$(A_1, A_2) \cap (B_1, B_2) = (A_1 \cap B_1, A_2 \cap B_2)$.

I:6 GENERATOR FOR ROUGH SETS

According to Pawlak [P_1], every rough set has a generator, but according to Chanas and Kuchta [C], a rough set may or may not have a generator. If exist then may or may not be unique.

Let (A_1, A_2) be a rough set in an approximation space $K = (U, \hat{E})$.

Case (i)

If $A_2 = \Phi$, then obviousely $A_1 = \Phi$ and $X = \Phi$ is a generator for (A_1, A_2).

Case (ii)

If $A_1 = A_2$, then $X = A_1$ is a generator for (A_1, A_2).

Case (iii)

If $A_1 = \Phi$ and $A_2 \neq \Phi$, let $A_2 = \cup \{ E_\lambda \varepsilon \acute{E} \mid \lambda \varepsilon I \}$,
$I \subseteq \Lambda$

This has following subcases :

a) If some E_λ is singltone set, then (A_1, A_2) has no generator. For otherwise A_1 will not be empty.

b) If no E_λ is singleton set, then by Axiom of choice construct $X = \{ x_\lambda \mid x_\lambda \varepsilon E_\lambda, \lambda \varepsilon I \}$ where precisely one x_λ is chosen from E_λ for each $\lambda \varepsilon I$. Clearly X is a generator for (A_1, A_2).

Case (iv)

If $A_1 \neq \Phi$ and $A_2 \neq \Phi$, then suppose that,
$A_1 = \cup\{ E_\lambda \mid E_\lambda \varepsilon \acute{E}, \lambda \varepsilon I \}$, $I \subseteq \Lambda$.
Let $A_2 = A_1 \cup \{ F_s \mid F_s \varepsilon \acute{E}, s \varepsilon J \}$, $J \subseteq \Lambda$.
and $F_s \nsubseteq A_1$ for any $s \varepsilon J$.
This has the following subcases,

a) If some F_s is singleton set then (A_1, A_2) has no generator.

b) If no F_s is singleton set, construct the set Y consisting of precisely one element from each F_s, $s \varepsilon J$.

Now chose $X = A_1 \cup Y$. Clearly X is a generator for (A_1, A_2).

The above discussion leads us to the following proposition.

Proposition (I:6:1)

Let (A_1,A_2) be a non-trivial rough set (i.e. $A_1 \neq \Phi \neq A_2$) in an approximation space K = (U, £), then a rough set (A_1,A_2) has generator if and only if $A_2 - A_1$ contains no singleton set.

Note (I:6:2)

The generator constructed in the above discussion (I:6) is a minimal generator for rough set (A_1,A_2).

1:7 SUMMERY

We have shown in this chapter that the first approach to rough set presumes the knowledge of the subset of the universal set U, while in the second approach it does not.