CHAPTER-II
FUZZY ROUGH SETS

CHAPTER II

FUZZY ROUGH SETS

II:1 INTRODUCTION

Fuzzy set theory deals with vagueness, while rough set theory deals with indiscernibility. Thus they deal with two different aspects of uncertainty. Therefore, some attempts have been made to combine these two aspects. In this chapter and next we shall discuss these approaches.

II:2 FUZZY SETS

Definition (II:2:1)[Z_2]

A <u>fuzzy set</u> A of U is a function A : U —> [0,1].

Note (II:2:2)

Let A be a fuzzy set of U.

(i) If we replace the closed interval [0,1] by the set {0,1}, then fuzzy set A of U is a characteristic function of subset $\{x \in U \mid A(x) = 1\}$. Hence fuzzy set is a generalization of a crisp set.

(ii) For any $x \in U$, A(x) is the grade of membership of x in A.

Definition (II:2:3) [M,Z_2]

Let A and B be two fuzzy sets of U.

(i) A fuzzy set A is a subset of fuzzy set B if,

$A(x) \leq B(x), \forall x \in U$

and denoted as $A \subseteq B$.

(ii) A fuzzy set A of U is said to be proper subset of fuzzy set B if,

$A(x) \leq B(x), \forall x \in U$; but $B(x) \nleq A(x)$.

and denoted as $A \subset B$.

(iii)A fuzzy set A is equal to fuzzy set B if,

$A(x) = B(x), \forall x \in U$.

Remarks (II:2:4)

The inclusion relation '$\subseteq$' defined in the above Def.(II:2:3) is a partial order relation on the set of fuzzy sets of U.

Definition (II:2:5) [M,D_2,D_3]

Let A,B be two fuzzy sets of U.

The union and intersection of fuzzy sets A,B are fuzzy sets of U, defined are as follows :

$(A \cup B)(x) = \max \{ A(x), B(x) \}$

$(A \cap B)(x) = \min \{ A(x), B(x) \}$

Definition (II:2:6) [M, D_2,D_3]

Let A be a fuzzy set of U. The complement of A is a fuzzy set $\bar{A}$ of U defined as,

$\bar{A}(x) = 1-A(x), \quad \forall x \in U$.

II.3 FUZZY ROUGH SETS

The concept of rough set is fuzzified in various ways, by Pawlak [P_1], Dubois and Prade [D_1,D_2] and others [W_1,C,K,N_1,N_2] etc. Here we discuss the approach by Pawlak; Chanas and Kuchta; Wygralak.

II:4 PAWLAK'S, WYGRALAK'S APPROACH TO FUZZY ROUGH SETS [P_3,W_2]

First we consider Pawlak's [P_3] and Wygralak's [W_1] approach to fuzzy rough sets.

Definition (II:4:1) [P_3]

Let K = (U,R) be the approximation space, and $X \subseteq U$ be a rough set in the space K.

The <u>fuzzy rough set</u> x_r of U is a function,

$x_r : U \longrightarrow \{0, \frac{1}{2}, 1\}$ defined by,

$$x_r(x) = \begin{cases} 1 \text{ iff } x \in POS_R(X) \\ \frac{1}{2} \text{ iff } x \in BON_R(X) \\ 0 \text{ iff } x \in NEG_R(X) \end{cases}$$

where $POS_R(X)$, $BON_R(X)$, $NEG_R(X)$ are defined as in the previous chapter.

We shall see in the following examples, in general that if X, $Y \subseteq U$ rough sets then,

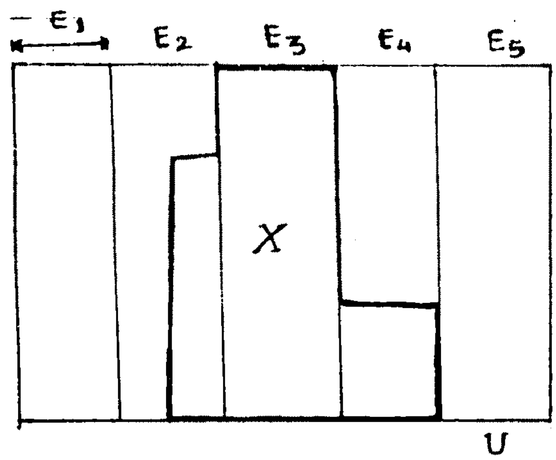
$$(X \cup Y)_r \neq X_r \cup Y_r \quad \text{and}$$
$$(X \cap Y)_r \neq X_r \cap Y_r \text{ .}$$

Example (II:4:2)

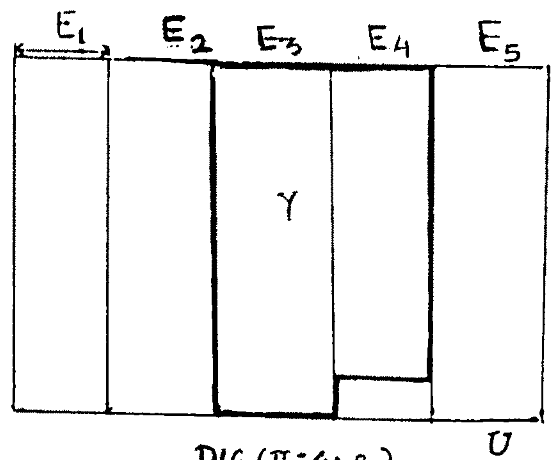
Let K = (U,R). The quotent set,

U/R = $\{E_1, E_2, E_3, E_4, E_5\}$ of equivalence classes of U induced by R.

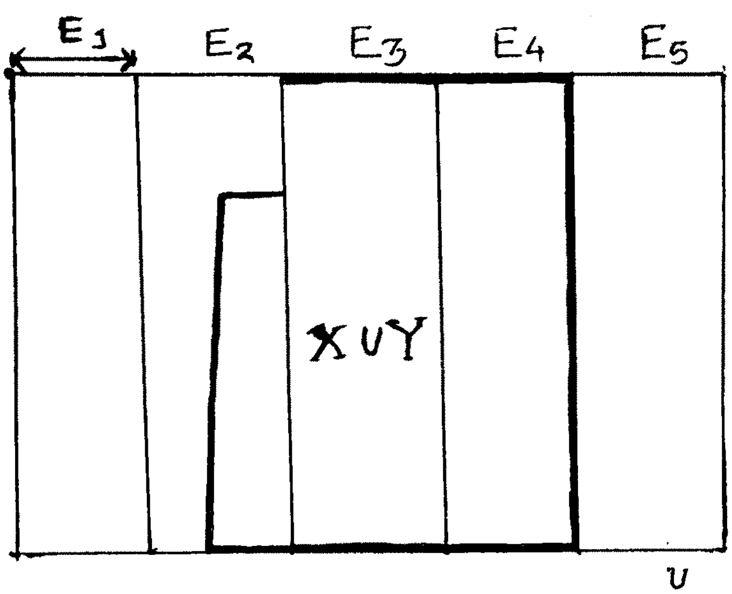
Consider rough sets X,Y and X U Y as follows :



DIG(II:4:1)



DIG(II:4:2)



DIG(II:4:3)

Using Def.(II:4:1) and Diag.(II:4:3) we get,

$$(X \cup Y)_r(x) = \begin{cases} 1 \text{ iff } x \in E_3 \cup E_4 \\ \frac{1}{2} \text{ iff } x \in E_2 \\ 0 \text{ iff } x \in E_1 \cup E_5 \end{cases}$$

Next, we find max $\{X_r(x), Y_r(x)\}$, using Diag.(II:4:1) as well as (II:4:2).

If $x \in E_1$, then $X_r(x) = 0$; $Y_r(x) = 0$

Therefore, max $\{ X_r(x), Y_r(x) \} = 0$, iff $x \in E_1$ (i)

If $x \in E_2$, then $X_r(x) = \frac{1}{2}$; $Y_r(x) = 0$

Therefore, max $\{ X_r(x), Y_r(x) \} = \frac{1}{2}$, iff $x \in E_2$ (ii)

If $x \in E_3$, then $X_r(x) = 1$; $Y_r(x) = 1$

Therefore, max $\{ X_r(x), Y_r(x) \} = 1$, iff $x \in E_3$ (iii)

If $x \in E_4$, then $X_r(x) = \frac{1}{2}$; $Y_r(x) = \frac{1}{2}$

Therefore, max $\{ X_r(x), Y_r(x) \} = \frac{1}{2}$, iff $x \in E_4$ (iv)

If $x \in E_5$, then $X_r(x) = 0$; $Y_r(x) = 0$

Therefore, max $\{ X_r(x), Y_r(x) \} = 0$, iff $x \in E_5$ (v)

From (i), (ii), (iii), (iv) and (v)

$$\max \{ X_r(x), Y_r(x) \} = \begin{cases} 1 \text{ iff } x \in E_3 \\ \frac{1}{2} \text{ iff } x \in E_2 \cup E_4 \\ 0 \text{ iff } x \in E_1 \cup E_5 \end{cases}$$

Hence,

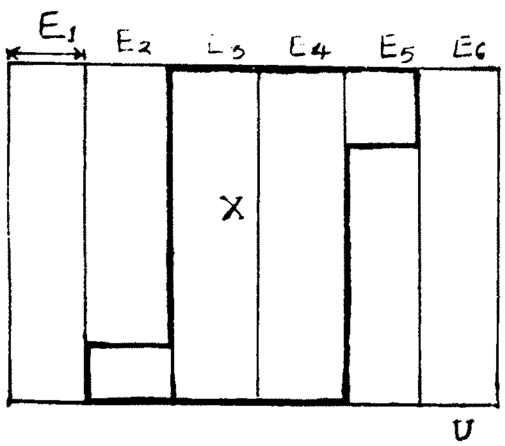
$$(X \cup Y)_r (x) \neq \max \{ X_r(x), Y_r(x) \}.$$

Example (II:4:3)

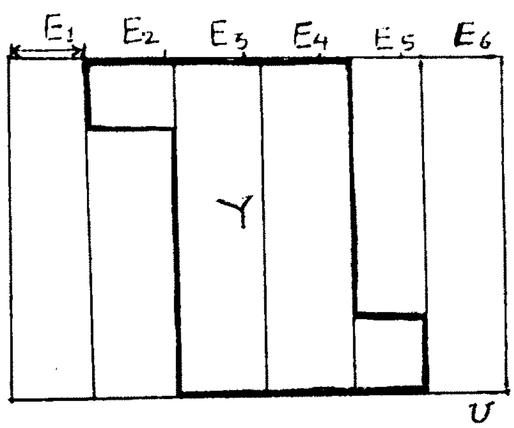
Let K = (U,R). The quotient set

$U/R = \{ E_1, E_2, E_3, E_4, E_5, E_6 \}$ of equivalence classes of U, induced by R.

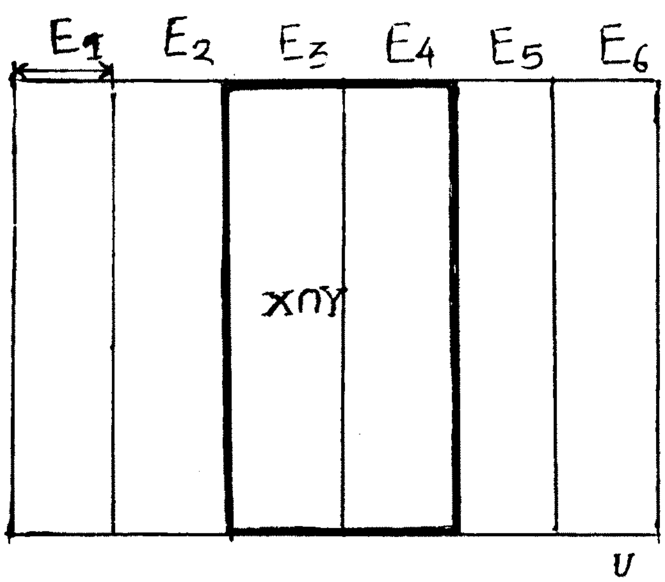
Consider the rough sets X,Y and X ∩ Y in the space K as follows :



DIG (II:4:4)



DIG (II:4:5)



DIG (II:4:6)

Using Def.(II:4:1) and Diag.(II:4:6)

$$(X \cap Y)_r(x) = \begin{cases} 1 \text{ iff } x \in E_3 \cup E_4 \\ 0 \text{ iff } x \in E_1 \cup E_2 \cup E_5 \cup E_6 \end{cases}$$

Next, we find $\min\{ X_r(x), Y_r(x) \}$ using Diag.(II:4:4) as well as Diag.(II:4:5) as follows :

If $x \in E_1$, then $X_r(x) = 0$; $Y_r(x) = 0$

Therefore, $\min\{ X_r(x), Y_r(x) \} = 0$, iff $x \in E_1$ (i)

If $x \in E_2$, then $X_r(x) = \frac{1}{2}$; $Y_r(x) = \frac{1}{2}$

Therefore, $\min\{ X_r(x), Y_r(x) \} = \frac{1}{2}$, iff $x \in E_2$ (ii)

If $x \in E_3$, then $X_r(x) = 1$; $Y_r(x) = 1$

Therefore, $\min\{ X_r(x), Y_r(x) \} = 1$, iff $x \in E_3$ (iii)

If $x \in E_4$, then $X_r(x) = 1$; $Y_r(x) = 1$

Therefore, $\min\{X_r(x), Y_r(x) \} = 1$, iff $x \in E_4$ (iv)

If $x \in E_5$, then $X_r(x) = \frac{1}{2}$; $Y_r(x) = \frac{1}{2}$

Therefore, $\min\{ X_r(x), Y_r(x) \} = \frac{1}{2}$, iff $x \in E_5$ (v)

If $x \in E_6$, then $X_r(x) = 0$; $Y_r(x) = 0$

Therefore, $\min\{ X_r(x), Y_r(x) \} = 0$, iff $x \in E_6$ (vi)

From (i), (ii), (iii), (iv), (v) and (vi)

$$\min\{ X_r(x), Y_r(x) \} = \begin{cases} 1 \text{ iff } x \in E_3 \cup E_4 \\ \frac{1}{2} \text{ iff } x \in E_2 \cup E_5 \\ 0 \text{ iff } x \in E_1 \cup E_6 \end{cases}$$

Hence

$$(X \cap Y)_r(x) \neq \min \{ X_r(x), Y_r(x) \}.$$

The reasons, why we can not get

$(X \cup Y)_r = X_r \cup Y_r$ and $(X \cap Y)_r = X_r \cap Y_r$ are that the equalities

$\underline{R}(X \cup Y) = \underline{R}X \cup \underline{R}Y$ and $\bar{R}(X \cap Y) = \bar{R}X \cap \bar{R}Y$

do not holds in general as seen in example (I:3:1) of the previous chapter. However, a way out is suggested by Wygralak [W_1] by defining new operators $\sqcup$ and $\sqcap$ called union and intersection as follows :

Definition (II:4:4) [W_1W_2]

Let X, Y be two subsets of U. The <u>union</u> of fuzzy rough sets X_r and Y_r of U corresponding to X and Y is,

$$(X_r \sqcup Y_r)(x) = \begin{cases} 1, \text{ if } X_r(x)=Y_r(x)=\tfrac{1}{2} \text{ and } [x]_R \subseteq X \cup Y. \\ \\ \max\{ X_r(x), Y_r(x) \}, \text{ Otherwise.} \end{cases}$$

Similarly the <u>intersection</u> of fuzzy rough sets X_r, Y_r of U corresponding to X and Y is,

$$(X_r \sqcap Y_r)(x) = \begin{cases} 0, \text{if } X_r(x)=Y_r(x)=\tfrac{1}{2} \text{ and } [x]_R \cap (X \cap Y) = \Phi \\ \\ \min\{ X_r(x), Y_r(x) \}, \text{ Otherwise.} \end{cases}$$

Proposition (II:4:5) [W_1]

Let $X, Y \subseteq U$ and X_r, Y_r be fuzzy rough sets of U. Then,

$$X_r \sqcup Y_r = (X \cup Y)_r \quad \text{and}$$
$$X_r \sqcap Y_r = (X \cap Y)_r.$$

Proof

We have,

$$(X_r \sqcup Y_r)(x) = \begin{cases} 1, \text{if } X_r(x) = Y_r(x) = \tfrac{1}{2} \text{ and } [x]_R \subseteq X \cup Y \\ \\ \max\{ X_r(x), Y_r(x) \}, \text{ Otherwise.} \end{cases}$$

and

$$(X \cup Y)_r(x) = \begin{cases} 1 \text{ iff } x \in POS_R(X \cup Y) \\ \tfrac{1}{2} \text{ iff } x \in BON_R(X \cup Y) \\ 0 \text{ iff } x \in NEG_R(X \cup Y) \end{cases}$$

Consider the following cases :

Case (i)

Let $x \in \underline{R}(X \cup Y)$. i.e. $[x]_R \subseteq X \cup Y$.

This has following subcases.

(a) Let $[x]_R \subseteq X$. i.e. $X_r(x) = 1$

Therefore, $(X_r \sqcup Y_r)(x) = \max\{ X_r(x), Y_r(x) \} = 1$.

(b) Let $[x]_R \subseteq Y$. i.e. $Y_r(x) = 1$

Therefore, $(X_r \sqcup Y_r)(x) = \max\{ X_r(x), Y_r(x) \} = 1$.

(c) If $[x]_R \subseteq X \cup Y$, $[x]_R \not\subseteq X$ and $[x]_R \not\subseteq Y$,

then $[x]_R \cap X \neq \Phi$ and $[x]_R \cap Y \neq \Phi$

Therefore, $X_r(x) = \frac{1}{2} = Y_r(x)$.

Hence, $(X_r \sqcup Y_r)(x) = 1$.

Thus, when $x \in \underline{R}(X \cup Y)$, then

$(X_r \sqcup Y_r)(x) = (X \cup Y)_r(x) = 1$.

Case (ii)

Let $x \in \bar{R}(X \cup Y) - \underline{R}(X \cup Y)$ i.e. $x \in \bar{R}(X \cup Y)$

$= \bar{R}X \cup \bar{R}Y$ and $x \notin \underline{R}(X \cup Y)$.

This has following subcases.

(a) If $x \in \bar{R}X$ and $x \notin \underline{R}(X \cup Y)$, then

$x \in \bar{R}X$ and $[x]_R \not\subseteq X$ and $[x]_R \not\subseteq Y$.

i.e. $x \in \bar{R}X - \underline{R}X$ and $x \notin \underline{R}Y$

Therefore, $X_r(x) = \frac{1}{2}$ and $Y_r(x) \neq 1$

Hence,

$(X_r \sqcup Y_r)(x) = \max\{X_r(x), Y_r(x)\} = \frac{1}{2}$

(b) If $x \in \bar{R}Y$ and $x \notin \underline{R}(X \cup Y)$, then

$x \in \bar{R}Y$ and $[x]_R \not\subseteq X$, $[x]_R \not\subseteq Y$

i.e. $x \in \bar{R}Y - \underline{R}Y$ and $x \notin \underline{R}X$.

Therefore, $Y_r(x) = \frac{1}{2}$ and $X_r(x) \neq 1$

Hence, $(X_r \sqcup Y_r)(x) = \max\{X_r(x), Y_r(x)\} = \frac{1}{2}$

Thus, when $x \in \bar{R}(X \cup Y) - \underline{R}(X \cup Y)$

$(X_r \sqcup Y_r)(x) = \frac{1}{2} = (X \cup Y)_r(x)$

Case (iii)

Let $x \notin \bar{R}(X \cup Y) = \bar{R}X \cup \bar{R}Y$

i.e. $x \notin \bar{R}X$ and $x \notin \bar{R}Y$.

Therefore, $X_r(x) = 0$ and $Y_r(x) = 0$

Hence $(X_r \sqcup Y_r) = \max\{ X_r(x), Y_r(x) \} = 0$

Thus, when $x \notin \bar{R}(X \cup Y)$ then

$(X_r \sqcup Y_r)(x) = 0 = (X \cup Y)_r(x)$

Similarly

$(X_r \sqcap Y_r)(x) = (X \cap Y)_r(x)$

and hence the proposition.

The following examples illustrate the situation :

Example (II:4:6) :

Consider the approximation space K and the quotient set U/R as in Example (II:4:2). Using Diagram (II:4:1); Diag(II:4:2); Diag.(II:4:3) and Def.(II:4:1), we get,

$$(X \cup Y)_r(x) = \begin{cases} 1 \text{ iff } x \in E_3 \cup E_4 \\ \frac{1}{2} \text{ iff } x \in E_2 \\ 0 \text{ iff } x \in E_1 \cup E_5 \end{cases}$$

By using proposition (II:4:5) and Diag. (II:4:1), Diag. (II:4:2), Diag. (II:4:3) we find $X_r \sqcup Y_r$ as follows

If $x \in E_1$, then $X_r(x) = 0$; $Y_r(x) = 0$, and

$(X_r \sqcup Y_r)(x) = \max\{ X_r(x), Y_r(x) \} = 0$.

Thus $(X_r \sqcup Y_r)(x) = 0$, when $x \in E_1$ (i)

If $x \in E_2$, then $X_r(x) = \frac{1}{2}$; $Y_r(x) = 0$, and

$(X_r \sqcup Y_r)(x) = \max\{ X_r(x), Y_r(x) \} = \frac{1}{2}$.

Thus $(X_r \sqcup Y_r)(x) = \frac{1}{2}$, when $x \in E_2$ (ii)

If $x \in E_3$, then $X_r(x) = 1$; $Y_r(x) = 1$, and

$(X_r \sqcup Y_r)(x) = \max\{ X_r(x), Y_r(x) \} = 1$.

Thus $(X_r \sqcup Y_r)(x) = 1$, when $x \in E_3$ (iii)

If $x \in E_4$, then $X_r(x) = \frac{1}{2}$; $Y_r(x) = \frac{1}{2}$, and

$[x]_R \subseteq X \cup Y$.

Hence $(x_r \sqcup Y_r)\ (x) = 1$

Thus $(X_r \sqcup Y_r)\ (x) = 1$, when $x \in E_4$ (iv)

Finally, if $x \in E_5$, then $X_r(x) = 0$; $Y_r(x) = 0$ and

$(X_r \sqcup Y_r) = \max\{ X_r(x), Y_r(x) \} = 0$

Thus, $(X_r \sqcup Y_r)\ (x) = 0$, when $x \in E_5$ (v)

Therefore, by (i), (ii), (iii), (iv) and (v) we have

$$(X_r \sqcup Y_r)\ (x) = \begin{cases} 1, & \text{iff } x \in E_3 \cup E_4 \\ \frac{1}{2} & \text{iff } x \in E_2 \\ 0 & \text{iff } x \in E_1 \cup E_5 \end{cases}$$

Thus, $X_r \sqcup Y_r$ produces the same fuzzy rough set $(X \cup Y)_r$, which was computed using Diag.(II:4:3).

Example (II:4:7)

Consider approximation space K and the quotient set U/R as in previous example (II:4:3).

Using Def.(II:4:1) and Diag.(II:4:6), we get,

$$(X \cap Y)_r(x) = \begin{cases} 1 \text{ iff } x \in E_3 \cup E_4 \\ 0 \text{ iff } x \in E_1 \cup E_2 \cup E_5 \cup E_6 \end{cases}$$

By using proposition (II:4:5) and Diag. (II:4:4), Diag. (II:4:5) and Diag.(II:4:6) we find $X_r \sqcap Y_r$ as follows :

If $x \in E_1$, then $X_r(x) = 0$; $Y_r(x) = 0$ and

$(X_r \sqcap Y_r)(x) = \min\{X_r(x), Y_r(x)\} = 0$.

Thus, $(X_r \sqcap Y_r)(x) = 0$, when $x \in E_1$ (i)

If $x \in E_2$, then $X_r(x) = \frac{1}{2}$; $Y_r(x) = \frac{1}{2}$ and

$[x]_R \cap (X \cap Y) = \Phi$,

Hence, $(X_r \sqcap Y_r)(x) = 0$.

Thus $(X_r \sqcap Y_r)(x) = 0$, when $x \in E_2$ (ii)

If $x \in E_3$, then $X_r(x) = 1$; $Y_r(x) = 1$ and

$(X_r \sqcap Y_r)(x) = \min\{X_r(x), Y_r(x)\} = 1$.

Thus, $(X_r \sqcap Y_r)(x) = 1$, when $x \in E_3$ (iii)

If $x \in E_4$, then $X_r(x) = 1$; $Y_r(x) = 1$ and

$(X_r \sqcap Y_r)(x) = \min\{X_r(x), Y_r(x)\} = 1$.

Thus, $(X_r \sqcap Y_r)(x) = 1$, when $x \in E_4$ (iv)

If $x \in E_5$, then $X_r(x) = \frac{1}{2}$; $Y_r(x) = \frac{1}{2}$ and

$[x]_R \cap (X \cap Y) = \Phi$,

Hence, $(X_r \sqcap Y_r)(x) = 0$.

Thus $(X_r \sqcap Y_r)(x) = 0$, when $x \in E_5$ (v)

Finally, if $x \in E_6$, then $X_r(x) = 0$; $Y_r(x) = 0$ and

$(X_r \sqcap Y_r)(x) = \min \{X_r(x), Y_r(x)\} = 0$.

Thus, $(X_r \sqcap Y_r)(x) = 0$, when $x \in E_6$ (vi)

From equations (i), (ii), (iii), (iv), (v) and (vi)

$$(X_r \sqcap Y_r)(x) = \begin{cases} 1 \text{ iff } x \in E_3 \cup E_4 \\ 0 \text{ iff } x \in E_1 \cup E_2 \cup E_5 \cup E_6 \end{cases}$$

Thus $X_r \sqcap Y_r$ produces the same fuzzy rough set $(X \cap Y)_r$ which was computed using Diag.(II:4:6).

Definition (II:4:8) [P_3, W_2]

Let $K = (U,R)$ and X be a rough set in the approximation space K.

Let X_r is a fuzzy rough set of U corresponding to X. The <u>complement</u> of a fuzzy-rough set X_r is a fuzzy rough set $-X_r$ of U, defined by

$$-X_r(x) = \begin{cases} 1 \text{ iff } x \in POS_R(-X) \\ \frac{1}{2} \text{ iff } x \in BON_R(X) \\ 0 \text{ iff } x \in NEG_R(-X) \end{cases}$$

Proposition (II:4:9) [P_3]

Let X_r be a fuzzy rough set of U corresponding to rough set X in the space K. Then for any $x \in U$

$$-X_r(x) = 1 - X_r(x)$$

Proof : We have

$$\begin{aligned}
-X_r(x) = 1 \quad & \text{iff } x \in POS_R(-X) \\
& \text{iff } x \in \underline{R}(-X) \\
& \text{iff } x \in -\bar{R}(x) \\
& \text{iff } x \notin \bar{R}(x) \\
& \text{iff } X_r(x) = 0 \\
& \text{iff } 1-X_r(x) = 1.
\end{aligned}$$

Thus $-X_r(x) = 1$ iff $1 - X_r(x) = 1$

$$\begin{aligned}
-X_r(x) = 0 \quad & \text{iff } x \in NEG_R(-X) \\
& \text{iff } x \in -\bar{R}(-X) \\
& \text{iff } x \notin \bar{R}(-X) \\
& \text{iff } x \notin -\underline{R}(X) \\
& \text{iff } x \in \underline{R}(X) \\
& \text{iff } X_r(x) = 1 \\
& \text{iff } 1-X_r(x) = 0
\end{aligned}$$

Thus $-X_r(x) = 0$ iff $1 - X_r(x) = 0$

Next, $-X_r(x) = \frac{1}{2}$ iff $x \in BON_R(X)$

$$\begin{aligned}
& \text{iff } X_r(x) = \tfrac{1}{2} \\
& \text{iff } 1-X_r(x) = \tfrac{1}{2}
\end{aligned}$$

Thus, $-X_r(x) = \frac{1}{2}$ iff $1-X_r(x) = \frac{1}{2}$

Therefore, $-X_r(x) = 1-X_r(x)$.

II:5 CHANAS AND KUCHTA'S APPROACH TO FUZZY ROUGH SETS [C]

Now we will discuss the fizzification of rough sets defined by Chanas and Kuchta.

Definition (I:5:1) [C]

Let (A_1,A_2) be a rough set in an approximation space $K = (U,\mathcal{E})$. A fuzzy rough set is a function.

$(A_1,A_2)_r : U \longrightarrow \{0, \frac{1}{2}, 1\}$ defined by

$$(A_1,A_2)_r(x) = \begin{cases} 1 \text{ iff } x \in A_1 \\ \frac{1}{2} \text{ iff } x \in A_2 - A_1 \\ 0 \text{ iff } x \notin A_2. \end{cases}$$

Using above membership function following proposition holds

Proposition (II:5:2) [C]

Let (A_1,A_2), (B_1,B_2) be two rough sets in the approximation space $K = (U,\mathcal{E})$. Then

a) $[(A_1,A_2) \cup (B_1,B_2)]_r = (A_1,A_2)_r \cup (B_1,B_2)_r$ and

b) $[(A_1,A_2) \cap (B_1,B_2)]_r = (A_1,A_2)_r \cap (B_1,B_2)_r$

Proof (a)

For any $x \in U$, we are to prove that,

$(A_1 \cup B_1,\ A_2 \cup B_2)_r(x) = \max\{\ (A_1,A_2)_r(x),\ (B_1,B_2)_r(x)\ \}$

i.e. to prove that,

$$\max\{(A_1,A_2)_r(x),(B_1,B_2)_r(x)\} = \begin{cases} 1 \text{ if } x \in A_1 \cup B_1 \\ \frac{1}{2} \text{ if } x \in [(A_2 \cup B_2)-(A_1 \cup B_1)] \\ 0 \text{ if } x \notin A_2 \cup B_2 \end{cases}$$

Consider the following cases

Case (1)

Let $x \in A_1 \cup B_1$

This has following subcases

(i) If $x \in A_1$, then $(A_1,A_2)_r(x) = 1$ and

$\max\{ (A_1,A_2)_r(x), (B_1,B_2)_r(x) \} = 1$.

(ii) Similarly, if $x \in B_1$

$\max\{ (A_1,A_2)_r(x), (B_1,B_2)_r(x) \} = 1$.

Case (2)

Let $x \in A_2 \cup B_2 - A_1 \cup B_1$

i.e. $x \in A_2 \cup B_2$ and $x \notin A_1 \cup B_1$

i.e. $x \in A_2$ or $x \in B_2$ and $x \notin (A_1 \cup B_1)$

This has following subcases.

(i) Let $x \in A_2$ and $x \notin A_1 \cup B_1$

Then $x \in A_2$ and $x \notin A_1$ and $x \notin B_1$

i.e. $x \in A_2-A_1$ and $x \notin B_1$

Therefore $(A_1,A_2)_r(x) = \frac{1}{2}$ and $(B_1,B_2)_r(x) \neq 1$.

Hence, $\max\{ (A_1,A_2)_r(x), (B_1,B_2)_r(x) \} = \frac{1}{2}$.

(ii) Similarly if $x \in B_2$ and $x \notin A_1 \cup B_1$,

$\max\{ (A_1,A_2)_r(x), (B_1,B_2)_r(x) \} = \frac{1}{2}$.

Case (3)

Let $x \notin A_2 \cup B_2$

Then $x \notin A_2$ and $x \notin B_2$

Therefore, $(A_1,A_2)_r(x) = 0$ and $(B_1,B_2)_r(x) = 0$

Hence, $\max\{ (A_1,A_2)_r(x), (B_1,B_2)_r(x) \} = 0$.

From cases (1) (2) and (3), we have

$$\max\{(A_1,A_2)_r(x), (B_1,B_2)_r(x)\} = \begin{cases} 1 & \text{if } x \in A_1 \cup B_1 \\ \frac{1}{2} & \text{if } x \in A_2 \cup B_2 - A_1 \cup B_1 \\ 0 & \text{if } x \notin A_2 \cup B_2 \end{cases}$$

This proves the first part of the theorem.

(b) Similarly by using basic properties of intersection we can prove that,

$$\min\{(A_1,A_2)_r(x), (B_1,B_2)_r(x)\} = \begin{cases} 1 & \text{if } x \in A_1 \cap B_1 \\ \frac{1}{2} & \text{if } x \in A_2 \cap B_2 - A_1 \cap B_1 \\ 0 & \text{if } x \notin A_2 \cap B_2 \end{cases}$$

Definition (II:5:3)

Let $(A_1,A_2)_r$ be a fuzzy rough set of U. Then the <u>complement</u> of $(A_1,A_2)_r$ is a fuzzy rough set $(-A_2,-A_1)_r$ of U defined as

$$(-A_2,-A_1)_r(x) = \begin{cases} 1 & \text{if } x \in -A_2 \\ \frac{1}{2} & \text{if } x \in A_2 - A_1 \\ 0 & \text{if } x \in A_1. \end{cases}$$

Proposition (II:5:4)

Let (A_1,A_2) be a fuzzy rough set of U. Then for any x ε U.

$$(-A_2,-A_1)_r(x) = 1-(A_1,A_2)_r(x)$$

Proof Now

$$\begin{aligned}(-A_2,-A_1)_r(x) = 1 \text{ iff } & x \,\varepsilon\, -A_2 \\ \text{iff } & x \notin A_2 \\ \text{iff } & (A_1,A_2)_r(x) = 0 \\ \text{iff } & 1-(A_1,A_2)_r(x) = 1.\end{aligned}$$

$$\begin{aligned}(-A_2,-A_1)_r(x) = \tfrac{1}{2} \text{ iff } & x \,\varepsilon\, A_2 - A_1 \\ \text{iff } & (A_1,A_2)_r(x) = \tfrac{1}{2} \\ \text{iff } & 1-(A_1,A_2)_r(x) = \tfrac{1}{2}.\end{aligned}$$

Finally, $$\begin{aligned}(-A_2,-A_1)_r(x) = 0 \text{ iff } & x \,\varepsilon\, A_1 \\ \text{iff } & (A_1,A_2)_r(x) = 1 \\ \text{iff } & 1-(A_1,A_2)_r(x) = 0.\end{aligned}$$

Hence, $(-A_2,-A_1) = 1 - (A_1,A_2)_r$.

II:6 SUMMARY

In the previous chapter we discuss rough sets in two ways. In this chapter we define the fuzzy rough sets and discuss some set-theoretic operations like union, intersection and complementation of fuzzy rough sets in two ways with the same technique as in the previous chapter.

In the first approach Wygralak [W_1] define the special types of union ($\sqcup$) and intersection ($\sqcap$) of fuzzy rough sets to overcome the difficulties arising in the usual definitions of union and intersection. But in the second approach such difficulties do not arrive. This is one more point which goes in favor of the more general definition of rough set given by Wygralak [W_2].