CHAPTER - FOUR

FUZZY SUBLATTICES

Chapter 4

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§4.1 Definition:

"Let (X, Λ, V) be a fuzzy lattice. Let S be non-empty subset of X. If (S, Λ, V) is a fuzzy lattice then we call S as a fuzzy sublattice of fuzzy lattice X."

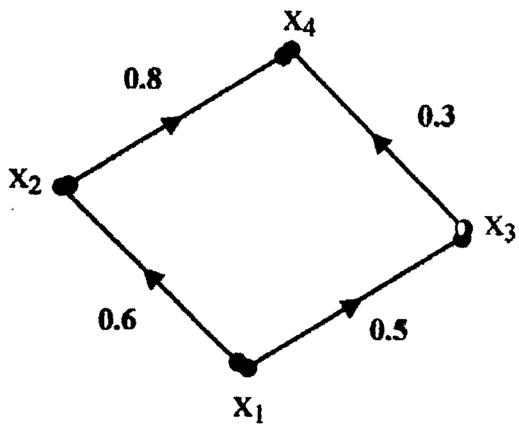
Example:

$X = \{ x_1, x_2, x_3, x_4\}$

R(X,X) is given by membership matrix as follows:

	x_1	x_2	x_3	x_4
x_1	1	0.6	0.5	0.6
x_2	0	1	0	0.8
x_3	0	0	1	0.3
x_4	0	0	0	1

The Hasse diagram is,



Here the ordered pair (X, R) is a fuzzy lattice.

Consider $S_1 = \{x_1, x_2\}$

$x_1 \wedge x_2 = x_1 \in S_1$ and $x_1 \vee x_2 = x_2 \in S_1$

Hence S_1 is a fuzzy sublattice of X.

Consider, $S_2 = \{x_1, x_2, x_3\}$

$x_2 \vee x_3 = x_4 \notin S_2$

Hence S_2 is not fuzzy sublattice of X.

Remark: Every non-empty non-fuzzy subset of fuzzy lattice need not be a fuzzy sublattice.

Definition: Let $(X, \wedge, \vee)$ be a fuzzy lattice. For any $x, y \in X$,

Let R be fuzzy partial order define on X

Define, $[x, y] = \{z \in X / R(x, z) > 0 \text{ and } R(z, y) > 0\}$

$[x, y) = \{z \in X / R(x, z) > 0, R(z, y) > 0 \text{ and } y \neq z\}$

$(x, y] = \{z \in X / R(x, z) > 0, R(z, y) > 0 \text{ and } x \neq z\}$

$(x, y) = \{z \in X / R(x, z) > 0, R(z, y) > 0, y \neq z \text{ and } x \neq z\}$

- **Theorem 4.1**

Let $(X, \wedge, \vee)$ be a fuzzy lattice and R be the fuzzy partial order define on X. Let $x, y \in X$. Let $R(x, y) > 0$. Show that $[x, y]$, $[x, y)$, $(x, y]$, (x, y) are fuzzy sublattices of X.

Proof:

Let $z_1, z_2 \in [x, y]$. To prove: $z_1 \wedge z_2 \in [x, y]$ and $z_1 \vee z_2 \in [x, y]$

Now, $z_1 \in [x, y] \Rightarrow R(x, z_1) > 0$ and $R(z_1, y) > 0$

$z_2 \in [x, y] \Rightarrow R(x, z_2) > 0$ and $R(z_2, y) > 0$

$\therefore$ $R(x, z_1 \wedge z_2) > 0$ and $R(z_1 \wedge z_2, y) > 0$ (by theorem 2.1)

$\therefore$ $z_1 \wedge z_2 \in [x, y]$. (by definition of $[x, y]$)

Also, $R(x, z_1 \vee z_2) > 0$ and $x, y \in X$ (by theorem 2.2)

$\therefore$ $z_1 \vee z_2 \in [x, y]$. (by definition of $[x, y]$)

Hence $[x, y]$ is fuzzy sublattice of X.

Similarly, $[x, y)$, $(x, y]$, (x, y) are fuzzy sublattices of X. □

§4.2 Fuzzy Convex Sublattices:

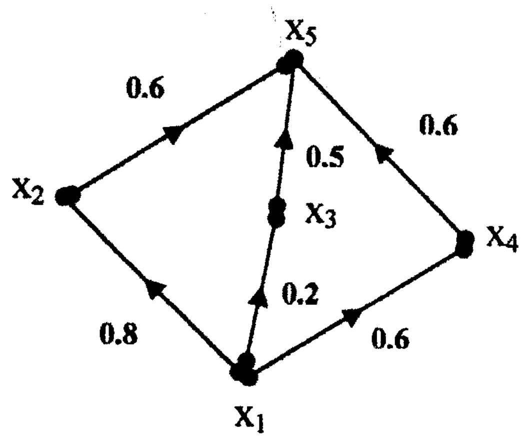
"A fuzzy sublattice S of a fuzzy lattice (X, R) is said to be fuzzy convex sublattice if for x, y ∈ X there exists t ∈ X such that R(x, t) > 0, R(t, y) > 0, x ≠ t and y ≠ t then t ∈ S."

Example: $X=\{x_1, x_2, x_3, x_4, x_5\}$ and

R(X, X) is given by grade membership matrix as follows:

	x_1	x_2	x_3	x_4	x_5
x_1	1	0.8	0.2	0.6	0.6
x_2	0	1	0	0	0.6
x_3	0	0	1	0	0.5
x_4	0	0	0	1	0.6
x_5	0	0	0	0	1

The Hasse diagram is,



Here the ordered pair (X, R) is a fuzzy lattice.

Consider, $S_1 = \{x_1, x_2, x_5\} \subseteq X$.

Here S_1 is a fuzzy convex sublattice.

Consider, $S_2 = \{x_1, x_5\} \subseteq X$.

Now $R(x_1, x_3) > 0$, $R(x_3, x_5) > 0$ and $x_3 \notin S_2$

Hence S_2 is not a fuzzy convex sublattice.

Remark: Every fuzzy sublattice need not be fuzzy convex sublattice.

- **Theorem 4.2**

Let (X, R) be a fuzzy lattice. Let $x, y \in X$. Let $R(x, y) > 0$.

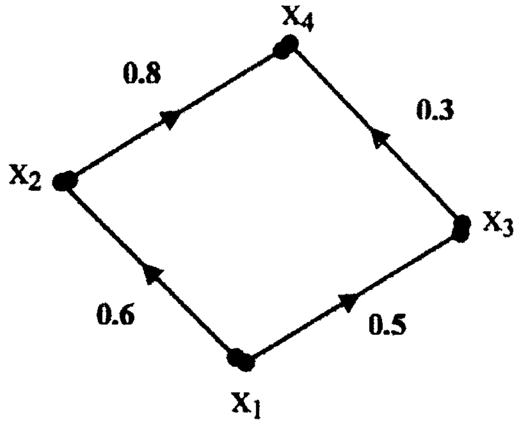
Show that [x, y], [x, y), (x, y], (x, y) are fuzzy convex sublattices.

Remark:

1) Crisp intersection of any number of fuzzy sublattices of a fuzzy lattice is a fuzzy sublattice.
2) Crisp union of any two fuzzy sublattices need not be a fuzzy sublattice.

Counter example: $X = \{ x_1, x_2, x_3, x_4\}$

The Hasse diagram is,



Here X is a fuzzy lattice.

Here $S_1 = \{x_1, x_2\}$, $S_2 = \{x_1, x_3\}$ are fuzzy sublattices of X.

Consider $S_1 \cup S_2 = \{x_1, x_2, x_3\} \subseteq X$.

Now, $x_2 \vee x_3 = x_4 \notin S_1 \cup S_2$.

- Thus, $S_1 \cup S_2$ is not a fuzzy sublattice of X.