PREFACE

So many times we come across the situation where we define certain things vaguely. For example, Instead of describing weather today in terms of the exact percentage of cloud cover, we could just say that it is sunny, which is more uncertain and less precise but more useful. In fact, it is important to realize that the imprecision or vagueness that is characteristic of natural language does not necessarily imply a loss of accuracy or meaningless. For example, it is more accurate to say that it is usually warm in summer than to say that it is 45°c.

In order for a term such as 'SUNNY' to accomplish the desired introduction of vagueness. However, we cannot use it to mean precisely 0% cloud cover. It's meaning is not totally arbitrary. Definitely cloud cover of 100% is not sunny. Not only that if there is 80% cloud cover it is not sunny. We accept certain intermediate states such as 10% or 20% cloud cover is Sunny.

Fuzziness is not a priori an obvious concept and demands some explanation. "Fuzziness" is what Mr. Black calls "vagueness" when he distinguishes it from "generality" and from "ambiguity".

Similarly, When we deal with some situation such as "Much taller" or "Less rain" or "High quality" or "Smaller in size", etc., we come across a term known as Fuzzy Relation or more precisely a Fuzzy Lattice.

Since Lofti A. Zadeh published his paper on fuzzy set theory in 1965, it has received more and more attention from researchers in a wide range of scientific areas, especially in the past few years. This theory is attractive because it is based on a very intuitive, although somewhat subtle, idea capable of generating many intellectually appealing results that provide new insights to old, often-debated questions.

A great deal work has been done in Fuzzy set theory. The concept of a fuzzy binary relation was introduced in Zadeh's very first paper on fuzzy sets and further developed in his 1971 paper. Today after 30 years of intensive development, the theory of fuzzy binary relations is probably one of the most important and influential branches of fuzzy set theory. Here our aim is to fuzzify the lattice theoretical concepts. We will study fuzzy lattices in detail, as fuzzy lattices play an important role in the theory of topological modular lattices and L-Fuzzy topology.

Also our aim is to study fuzzy lattice ideals. For instance, we will obtain the extension of Stone's prime ideal theorem of distributive lattices to the fuzzy prime ideals.

The dissertation is being divided into six chapters. First chapter contains the preliminary definitions required to study further topics. Mainly, We will discuss the definitions of Fuzzy set, Fuzzy relation and binary relation, Fuzzy reflexive, perfectly antisymmetric, Max-min transitive, Symmetric relations, Fuzzy Union and Intersection. Second chapter contains

a detail study of Fuzzy partial ordered set. We will discuss the definition of Fuzzy partial order relation, partial order set, Dominating and Dominated class, Fuzzy Upper and Lower bound. Efforts are taken to prove some theorems of Fuzzy partial order sets.

Third chapter contains the detail study of Fuzzy lattices with some examples. A discussion of a theorem in this chapter enables us to convert Fuzzy lattice into Fuzzy lattice Algebra. A list of some properties of fuzzy lattice is given. As diagram plays an important part in lattice theory, emphasize is given to the diagram for discussing the Fuzzy lattices. (Heeding Alice's advice: "and what is the use of a book without pictures or conversations", L. Carroll [1865]). Fourth chapter contains definition and examples of Fuzzy sublattice.

Fifth chapter contains a detail discussion of fuzzy ideals. It contains definition of Fuzzy Ideals, Dual Ideals, Ideals generated by crisp subset, Dual ideals generated by crisp subset, Principal ideal generated by an element, Prime ideals and Distributive lattices. Successfully, a Stone type theorem for fuzzy distributive lattice is proved. The last Sixth chapter is about Fuzzy Lattice Homomorphism. Also in this chapter we deal with definition of Fuzzy congruence relation and Quotient lattice. Mainly Fundamental theorem of Fuzzy lattice homomorphism is proved.

(Sachin H. Dhanani)