PREFACE

The dissertation entitled "A STUDY OF SPACE-TIME GEOMETRY OF A METRIC IN EINSTEIN-CARTAN THEORY OF GRAVITATION", is devoted to study the influence of the Spin tensor on the space-time geometry of a non-static spherically dust distribution in the Einstein-Cartan theory of gravitation.

Einstein introduced the General Theory of Relativity in 1915 to study gravitation. He observed that gravitation is an interaction which can not be switched on and off at will. It is omnipresent and everlasting and hence it is universal. Therefore, it must act all particles (massive as well as mass less) in the same manner. In an attempt to achieve such force, he identified gravitation with the intrinsic property of space-time region through the equation

Gravitation = Space-time geometry.

Thus the general theory of relativity enables us to understand the mysterious gravitational force through the geometry of the space-time structure. The theory is supported by experimental tests but still it has defects in the sense that it does not incorporate the intrinsic spin of matter and singularities can not be prevented.

The Einstein-Cartan theory of gravitation is a generalization of Einstein's theory of gravitation. The influence of the intrinsic spin of matter on the space-time structure is considered in the theory. The theory was originated by Cartan (1922) by introducing torsion in to gravitational theory. The theory is popularly known as U_4 theory of gravitation. Obviously the geometry of theory is non-Riemannian.

The mathematical tools that we have used in the dissertation are then tetrad formalism of Newman and Penrose (1962) and its extension to space-time with torsion by Jogia and Griffiths (1980).

These formalisms plays very important role in simplifying and reducing the number of relativistic field equations. For the last more than four decades the progress made in understanding of Einstein field equations, in finding exact solutions of Einstein field equations, in the study of black holes as well as in the study of electromagnetic fields, both in Einstein's theory of gravitation and Einstein-Cartan theory of gravitation may be attributed to these formalisms.

The first Chapter is introductory. A brief exposition of exterior differential forms is presented in the Section 2. The basic notions are described in the Section 3. The prime mathematical artifact Newman-Penrose tetrad formalism and its extension to space-times with torsion Jogia and Griffiths is summarized in Section 4 and 5 respectively. Originality is not claimed in this chapter. The results of the next two chapters contain original results. The original results reported in the Chapter 2 are published in the journal of Indian Academy of Mathematics Vol. 31, No. 2, 2009 and that of Chapter 3 are prepared in the form of two research papers and is communicated respectively to the Indian journal of pure and applied mathematics and the Astrophysics space science journal for publication.

In the general theory of relativity Matte (1953) was a first to introduce the decomposition of electric part and magnetic parts of the Weyl tensor with respect to some time-like vector field. Several author's [Misra et. al (1968), McIntosh et. al (1994), Hall (1973), Barnes (1973), Ahsan (1999), Hasmani et. al (2008)] have invested some examples of space-times which are purely electric and purely magnetic. By noting important role of purely electric and purely magnetic space-times in the general relativity, a study of electric and magnetic parts of Weyl tensor in Kerr-Newman space-time is the subject matter of Chapter 2.

The Kerr-Newman space-time is considered in this chapter. The tetrad components of Connection 1-forms, Curvature 2-forms and Riemannian curvature tensor with respect to the Kerr-Newman space-time obtained by Katkar (2008) are verified and the results are exploited to construct the electric and magnetic parts of Weyl tensor. They are respectively derived in the form:

$$E_{ij}=\frac{1}{2}R^{-6}\{[2mr(r^2-3a^2\cos^2\theta)-e^2(3r^2-a^2\cos^2\theta)][l_il_j-2l_{(i}n_{j)}+n_in_j]-$$
$$-4[mr\,(r^2-3a^2\cos^2\theta)-e^2(r^2-a^2\cos^2\theta)]\,m_{(i}\overline{m}_{j)}\}$$

and

$$H_{ij}=(-a\cos\theta)R^{-6}[mr(3r^2-a^2\cos^2\theta)-2re^2][l_il_j-2l_{(i}n_{j)}+n_in_j+2m_{(i}\overline{m}_{j)}]$$

It has been observed that both the angular momentum per unit mass and the electric charge of the gravitating body are the sources of the electric part and magnetic part of the Weyl tensor. We see that if angular momentum per unit mass of a gravitating body is zero, the magnetic part of Weyl tensor ceases to be zero in the Kerr-Newman space-time while electric part E_{ij} still exists. At very far distance from the gravitating object ($r \to \infty$) one can see that the electric and magnetic parts of the Weyl tensor vanish.

A study of non-static spherically symmetric space-time in Einstein-Cratan theory of gravitation and the construction of electric and magnetic parts of Weyl tensor with respect to this metric is the subject matter of Chapter 3.

Extension of the 'amazingly useful' Newman-Penrose formalism to space-time with torsion by Jogia and Griffiths (1980) is exploited to study the role of torsion on the geometry of the space-time structure. In the Second Section, one finds a brief review of notions and tensors that are pertinent to the study of the metric in the U_4 theory of gravitation.

The structure equations in the U_4 theory of gravitation are given by Katkar (2008). We utilize it to find the tetrad components of Connection 1-forms and Curvature 2-forms in Einstein-Cartan theory of gravitation in the Section 3. In the Section 4, the tetrad components of Curvature tensor, Ricci tensor and Weyl tensor are obtained with reference to the non-static spherically symmetric metric in the U_4 theory of gravitation. The roll of Spin components on the geometry of the space-time is observed. It has been observed that the spin tensor influences the space-time geometry of the Einstein-Cartan theory of gravitation. If the Spin tensor component $s_\circ = 0$ and $s_1 \neq 0$, the space-time of U_4 theory of gravitation is shown to be

Petro-type I. If however, the tetrad components of the Spin tensor are functions of r and t alone then we have $\psi_\circ = \psi_1 = \psi_3 = \psi_4 = 0$ proving the space-time of U_4 theory of gravitation is Petro-type D. However, if $s_\circ \neq 0$ and $s_1 = 0$, then none of the Weyl tensor vanishes, showing that $s_\circ$ has predominant effect on the space-time geometry of U_4. In the absence of the Spin tensor the space-time reduces to the space-time D of Einstein theory of gravitation. In Section 5, we utilize the results of Section 4 to find the expression for the electric part and the magnetic part of Weyl tensor in U_4 theory of gravitation. It has been noticed that the Spin influences the electric and magnetic parts of Weyl tensor. In the absence of the Spin tensor we notice that the magnetic part of the Weyl tensor vanishes.