PREFACE

The dissertation entitled "differential forms in Einstein-Cartan theory of gravitation" is devoted to study of Cartan's equations of structure in Einstein- Cartan theory of gravitation and its concomitants in Newman-Penrose-Jogia-Griffiths formalism.

Einstein put forward his General Theory of Relativity in 1915 when it was realized that gravitational phenomenon is incompatible with the concept of inertial frame, which is so basic to Special Theory of Relativity. Since gravitation is universal and hence it must interact with everything including light. This universal characteristic of gravitation prompted Einstein to identify, it with the curvature of space-time and he proposed surprising result.

Gravitation = space-time geometry,

Through the field equations of gravitation

$$R_{ij} + \frac{1}{2} R g_{ij} = K T_{ij} \ ,$$

where T_{ij} is the stress energy momentum tensor that represents the source of gravitation. The theory is considered as greatest height of imagination, but still it does not explain the phenomenon of intrinsic spin of matter.

Cartan (1923, 24) considered the intrinsic spin of matter on the space time and generalized Einstein theory of gravitation to Einstein- Cartar theory of gravitation. The theory is considered as a possible route to a unified theory of gravity and electromagnetism.

Unlike the Einstein gravitation theory, the Einstein-Cartar theory of gravitation has non-Riemannian geometry.

The mathematical tools that we have been exploiting are differential forms and Newman-Penrose-Jogia-Griffiths formalism. Differential form is a powerful mathematical tool for the use of Scientists and Engineers. Its use can reduce the complexity of the computation. For example there are 40 christoffel symbols to compute in tensor formulation, where as if we use Cartan's method of differential forms there are only six connection forms to determine.

Differential forms and exterior differential play a control role in the Einstein –Cartan theory of gravitation and even in the modern literature. We have introduced the techniques of differential forms and exterior calculus briefly in the first chapter. The material of this chapter can be traced from the following books and no originality is claimed in this chapter. Schutz (1980), Choquet Bruhat (1982), F. De'Felice and Clarke C. J. S. (1990)

In the chapter 2, we addressed ourselves to find out the equation of structure in Einstein- Cartan theory of gravitation. Cartan's equations of structure in this theory take the form

$$d\theta^{\alpha} = -\omega^{0\alpha}_{\beta} \wedge \theta^{\beta} + T^{\alpha},$$

where $$T^{\alpha} = K_{\beta\gamma}{}^{\alpha}\theta^{\beta} \wedge \theta^{\gamma}$$

and $$\Omega^{\alpha}{}_{\beta} = \Omega^{0\alpha}{}_{\beta} - dK_{\gamma\beta}{}^{\alpha}\theta^{\gamma} + \begin{bmatrix} K_{\sigma\beta}{}^{\alpha}\omega^{0\sigma}_{\varepsilon\gamma} - K_{\rho\beta}{}^{\alpha}K_{\varepsilon\gamma}{}^{\rho} - \\ -K_{\gamma\sigma}{}^{\alpha}K_{\varepsilon\beta}{}^{\sigma} + K_{\gamma\sigma}{}^{\alpha}\omega^{0\sigma}_{\varepsilon\beta} \end{bmatrix}\theta^{\varepsilon}\wedge\theta^{\gamma},$$

where $K_{\gamma\beta}{}^{\alpha}$ is the contortion tensor. It can be seen that in the limiting case when contortion tensor vanishes the Einstein-Cartan theory of gravitation reduces to Einstein theory of gravitation.

The result $\Omega^{0\alpha}_{\beta}\wedge\theta^{\beta}=0$ of Einstein gravitation is shown to take the form $$-\Omega^{0\alpha}_{\beta}\wedge\theta^{\beta}+\omega^{0\alpha}_{\beta}\wedge T^{\beta}+dT^{\alpha}=0$$

in Einstein-Cartan theory of gravitations.

Analogous to the Newman-Penrose tetrad formalism Jogia and Griffiths (1980) have developed a singular type of null formalism which is suitably adopted for studying the Einstein-Cartan theory of gravitation. Newman-Penrose-Jogia-Griffiths tetrad formalism is exploited to express the tensor components of the contortion tensor in terms of 12 complex tetrad components of contortion tensor. Some prosperities of the contortion tensor are cited in the same section. The necessary and sufficient conditions for vanishing of the contortion tensor are also obtained in terms of its 12 complex tetrad components. In the following section utilizing the Newman-Penrose-Jogia-Griffiths formalism the Cartan's tensor equations of structure are transcribed in to scalar equations.

Newman-Penrose (1962) spin coefficient formalism is "amazingly useful" tetrad formalism in many applications. In the chapter 3, an attempt has been made to develop formalism similar to that of Newman and Penrose which can be used for 2-dimensional Riemannian space. With the help of only two complex null vector fields m_i and $\overline{m_i}$ the formalism is developed. The relation between the metric tensor of 2-dimensional Riemannian space and two complex null vector fields is given by

$$g_{ik} = m_i \overline{m_k} + \overline{m_i} m_k,$$

where $$m_i m^i = \overline{m_i}\, \overline{m}^i = 0 \text{ and } m_i \overline{m}^i = 1.$$

The detail exposition of this formalism is portrayed in this chapter. As an application of this formalism, it is applied to study the geometry of 2 dimensional Riemannian space V_2. It is interesting to see that it works beautifully. The components of connection 1-forms and curvature 2- form are expressed in this formalism. It is show that the curvature 2-form of 2-dimensional space V_2 is exact (Frankel 1997) satisfying $\Omega^{\alpha}_{\beta} = d\omega^{\alpha}_{\beta}$.

In this case $$\Omega_{12} = d\omega_{12} = K\, \theta^1 \wedge \theta^2,$$

where K is the curvature of the space V_2. Using this new formalism the well-known result that the Riemannian space V_2 has constant curvature has been proved. The commutator relation and the field equations for V_2 are also derived in the form

$$\left(\overline{\delta}\delta - \delta\overline{\delta}\right)\phi = \kappa\overline{\delta}\phi + \overline{\kappa}\delta\phi$$

and $$\overline{\delta}\kappa + \delta\overline{\kappa} - 2\kappa\overline{\kappa} + \psi + \phi_{12} - \frac{R}{6} = 0 \text{ respectively.}$$