CHAPTER I

PRELIMINARIES

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In this chapter we give some basic definitions which will be used in subsequent chapters.

1. DEFINITIONS

Def.1.1 : Partially Ordered Set or Poset [5] :Let P be a nonvoid set. Define a relation '$\leq$ 'on P which has following properties for all a,b,c$\in$ P.

i)$a \leq a$ (reflexivity)

ii)$a \leq b$ and $b\leq a \Rightarrow a = b$ (antisymmetry)

iii)$a \leq b$ and $b\leq c \Rightarrow a \leq c$ (transitivity)

The relation satisfying above three conditions is called partial ordering relation and the set P equipped with such relation '$\leq$'is called partially ordered set or poset denoted by $(p,\leq)$.

A Poset $(P,\leq)$is called as chain (or totally ordered set or linearly ordered set) if it satisfies the following condition for all a,b $\in$P

iv)$a \leq b$ or $b \leq a$ (linearity)

Def.1.2 : Upper bound [5] : Let $H \subseteq P$, $a \in p$. Then a is an upper bound of H, if $h \leq a$ for all $h \in H$.

Def.1.3 : Updirected poset [15] : A partially ordered set $(P, \leq)$such that for any a,b$\in$ P, the set of upper bounds of {a,b} is nonvoid is said to be up directed poset.

Def.1.4 :Zero element and Unit element of poset [5] :A zero of a poset $(P, \leq)$ is an element o$\in$ P with $o\leq x$ for all x$\in$P.

A Unit element of a poset $(P, \leq)$ is an element $1 \in P$ with $x \leq 1$ for all $x \in P$.

Def 1.5: Bounded Poset [5] : A bounded Poset is one that has both 0 and 1

Def 1.6: Covering Property [5] : In a Poset $(P, \leq)$, a covers b (in notation $a \succ b$) if $a > b$ and for no x in P $a > x > b$.

Def 1.7 : Atom [5] : An element a of poset $(P, \leq)$ is an atom if $a \succ 0$ for $0 \varepsilon P$

Def 1.8: Meet semilattice (Λ - semilattice) as a Poset [5] :

A poset S is a Λ -semilattice if inf {a,b} [or $a \Lambda b$] exists in S for all a,b ε S

Def 1.9 : Meet -semilattice as an algebra [5] : An algebra $< S, \wedge >$ is called a semilattice if S is a non-void set with binary operation Λ satisfying following properties for all a,b,c ε S

i) $a \Lambda a = a$ (idempotency)

ii) $a \Lambda b = b \Lambda a$ (commutativity)

iii) $(a \Lambda b) \Lambda c = a \Lambda (b \Lambda c)$ (associativity)

Def 1.10 : Join exists (V exists) [5] : Join exists (V-exists) means if sup {a,b} exists for any two elements.

Def 1.11 Relative annihilator of a in b [16] : Let S be a Λ semilattice Relative annihilator of a in b (a,b ε S) is denoted by $<a,b>$ and is defined as follows, $<a,b> = \{x \varepsilon S / x \Lambda a \leq b\}$

Def 1.12 Annihilator of an element [16] : Let S be a semilattice with 0 . Annihilator of an element a ε S is denoted by $(a)^*$ and is defined as follows, $(a)^* = \{x \varepsilon S / a \Lambda x = 0\}$

Def 1.13 : Dense element [5] : Let S be semilattice with 0 and $x \varepsilon S$. An element x is said to be dense in S if $(x)^* = \{0\}$

Def .1.14 : Distributive Λ - semilattice [15]

A Λ- semilattice S is distributive if $c > a \wedge b$ $(a,b,c \varepsilon S)$ implies the existance of a_1, b_1, ε such that $a_1 \geq a$, $b_1 \geq b$ and $a_1 \wedge b_1 = c$.

Def 1.15 : Semi - ideal [15] : A semi- ideal I of a Λ- semilattice S is a non-empty subset of S such that $y \leq x$ and $x \varepsilon I$ imply $y \varepsilon I$.

Def 1.16 : Ideal [15] : An ideal I of a Λ- semilattice S is a non - empty subset of S such that (I_1) $y \leq x$ and $x \varepsilon I$ imply $y \varepsilon I$,

(I2) for any $x, y \varepsilon I$ there exists $z \varepsilon I$ such that $z \geq x$ and $z \geq y$.

Def 1.17 : Principal ideal [5] : The ideal generated by a subset H of the semilattice S, provided that $H \neq \Phi$ will be denoted by (H] , and if $H = \{a\}$ we write (a] for ({a})], we shall call (a] a principal ideal. I.e. the set $\{x/x \varepsilon S, x \leq a\}$

Def 1.18 : Prime ideal [5] : A proper ideal I of semilattice S (i.e. $I \neq S$) is prime if $a,b \varepsilon S$ and $a \wedge b \varepsilon I$ imply that $a \varepsilon I$ or $b \varepsilon I$. In other words an ideal I of S is prime if whenever two ideals I_1 and I_2 are such that $\Phi \neq I_1 \cap I_2 \subseteq I$ then I_1 or I_2 belongs to I.

Def 1.19 : Maximal ideal [15] : Let S be a semilattice. A proper ideal I of S (i.e. $I \neq S$) is maximal if the only ideal strictly containing 1 in S.

Def 1.20 : Minimal prime ideal [5] : A prime ideal P of S is called minimal if there is no prime ideal Q with $Q \subset P$.

Def 1.21 : Dual ideal [15] : A dual ideal of a semilattice S is a non-emply subset D of S such that $a \wedge b \in D$ if and only if $a \in D$ and $b \in D$

Def 1.22 : Principal dnal ideal [5] : The dual ideal generated by a subset H of semilattice S provided that $H \neq \Phi$ will be denoted by [H) and if H = {a} we write [a) for [{a}), we shall call [a) a principal dual ideal i.e. the set $\{x / x \in S, x \geq a\}$

Def 1.23 : Prime dual ideal [15] : A dual ideal D of semilattice S, is prime if whenever two dual ideals D_1 and D_2 are such that $\Phi \neq D_1 \cap D_2 \subseteq D$ then D_1 or D_2 belongs to D.

Def 1.24 : Maximal dual ideal [15] : A proper dual ideal D of semilattice S (i.e. $D \neq S$) is maximal if the only dual ideal strictly containing D in S.

Def 1.25 : Complement of an element [5] : Let S be a semilattice with 0 and 1 and a be any element of S. An element $b \in S$ is called a cemplement of a if $a \wedge b = 0$ and $a \vee b$ exist and is 1.

We denote the complement of a by a'.

Def : 1.26 : Topological space [10] By a topology on a set X, we mean a class T of subsets of X satisfying

i) $\Phi, X \in T$

ii) $\{G_i / i \in \Delta\} \subseteq T \Rightarrow \bigcup_{i \in \Delta} G_i \in T$

iii) $G_1, G_2 \in T \Rightarrow G_1 \cap G_2 \in T$

The pair (X,T) is called a topological space. The members of X are called points and members of T are called open sets.

A subset C of X is called closed if X-C is open.

When there is no ambiguity, we simply say that X is a topological space with out specifically mentioning the topology on it.

Def 1.27 : Base for open sets [10] :A family of non void sets $B \subseteq T$ is a base for open sets if every open sets is a union of members of B.

Def : 1.28 : Subbase for open sets [10] : A family of non-void sets $C \subseteq T$ is a subbase for open sets if the finite intersection of members of C form a base for open sets.

Def : 1.29 : Closure of a set [10] : Let X be a topological space and let $A \subseteq X$. Then there exists a smallest closed set $\overline{A}$ containing A , $\overline{A}$ is the closure of A.

De 1.30 : To space [10] : A topological space X is said to be To space if $\overline{\{x\}} = \overline{\{y\}} \Rightarrow x = y$, for $x,y \in X$.

Def 1.31 : T_1 - space [10]:A topological space X is called T_1 - space if for given any pair of distinct points each has a neighborhood nor containing the other.

Def 1.32 : Housdorff's space or T_2 -space [10] :A Housdorff's space or T_2 -space is a topological space in which each pair of distinct points can be separated by open sets.

Def 1.33 : T_1 - point [10] : Let X be a topological space. A point p of X is said to be T_1 if f does not belong to closure of p for any point f of X other than p.

Def 1.34 : Anti T_1 - point [10] : A point of X is said to be anti T_1 point if p does not belong to closure of f for any point f of X other than p.

Def 1.35 : Compact space [10] :A compact space is a topological space in which every open cover has a finite subcover .

Def 1.36 : Π_0 - space [10] : A topological space X is said to be Π_0 if every non empty open set of X contains a non empty closed set

Def . 1.37 : Normal space [10] : A topological space is normal space if it is T_1 - space and if every pair of disjoint closed sets is separated by disjoint open sets.

.. .1.38 : Continuous map [10] : Let X and Y be any two topological spaces and f: X → Y then f is said be continiuous if f^{-1} (O) is open in X, for every open set O in Y.

Def .1.39 : Homeomorphism [10] :Let X and Y be any two topological spaces. A function f : X → Y is said to be homeomerphism if both f and f^{-1} are continuous.

Def 1.40 : Weakly sparable [10] :Let X be any topological space and A, B be any two disjoints subsets of X. A is said to be weakly sparable from B if there exist an open set in X containing A and disjoint with B.

Def.1.41 : Retraction [10] :A subspace A of a topological space B is said to be retraction of B, if there exists a continuous map f:B → A such that f (a) =a for all a ε A.

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