CHAPTER III

STONE'S SPACE

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Introduction :

As distributive semilattice is a generalization of a distributive lattice . Interestingly we shall study some topological properties of the space of prime and maximal dual ideals in distributive semilatice.

Stone (11) has instroduced a topology for the set of all prime ideals of a distributive lattice. Many more attempts have been made for investigating the properties of the Stone's space for distributive lattice.

Balachandran [2] has made an extensive study of Stone's topology of the distributive lattice and has obtained results supplementing to those of Stone. In the same way Venkatanarasimhan [19] has studied indetail the space of prime dual ideals for a pseudo completed lattice.

In this chapter we have collected some properties of the Stone's topology for the set of prime dual ideals in bounded distributive $\wedge$-semilattice.

In 3.1 we have studied some properties of the Stone's topology on the set of prime dual ideals of a distributive semiattice. Mainly it is shown that $\wp$,the set of prime dual ideals in bounded distributive $\wedge$-semilattice is compact and T_0.

As every maximal dual idea is prime in bounded distributive $\wedge$ semilattice, Ω the set of all maximal dual ideals contains $\wp$,the set of all prime dual ideals. Hence we have to focus our attention on Ω together with the restricted Stone's topology on Ω. In 3.2

it is shown that every prime dual ideal in distributive semilattice is contained in a unique maximal dual ideal if $\mathcal{M}$ is retract of $\wp$.

By defining new topology T' on $\wp$ different from the Stone's topology on $\wp$. The new topological space $(\wp, T')$ is studied in 3.3.

In 3.4 mainly we have studied that V(a) is compact and $\{V(a)/a \in S\}$ is a subbase for the open sets in $(\wp, T')$

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3.1 The space of prime dual ideals:

Throughout S stands for a bounded distributive $\wedge$- semilattice. Denote by $\wp$, the set of all prime dual indeals in S. For any dual ideal A in S , let V(A) denote the set of all prime dual ideals in S, not constaining A. i.e. $V(A) = \{P \varepsilon \wp / A \not\subseteq P\}$

Some properties of V(A) are mentioned in the following Result which are used in defining the topology on the set of all prime dual ideal in S.

Result :3.1.1: For dual ideals Ai in S, i ε I (I is any indexing set) we have the following,

1) $V(\bigvee_i Ai) = \bigcup_i V(Ai)$

2) $V(A_1 \barwedge A_2 \barwedge \ldots \barwedge An) = V(A_1) \cap V(A_2) \cap \ldots \cap V(An)$

3) $V(S) = \wp$

4) $V([1)) = \Phi$

Proof : We have $V(A) = \{P \varepsilon \wp / A \not\subset P\}$

1) Let $P \varepsilon V(\bigvee_i Ai)$

Then $\bigvee_i Ai \not\subset P$ and hence $Ai \not\subset P$ for some i .

Hence $P \varepsilon \bigcup_i V(Ai)$, proving that $P \varepsilon V(Ai)$, for some i.

Thus, we have $V(\bigvee_i Ai) \subseteq \bigcup_i V(Ai)$ ...... (I)

For the reverse inclusion.

Let $p \varepsilon \bigcup_i V(Ai)$

B t then $A i \not\subseteq P$ for some i

i.e. $P \varepsilon V(Ai)$,for some i.

This proves that $\bigvee_i Ai \not\subseteq P$. Hence $P \varepsilon V(\bigvee_i Ai)$.

proving that $\bigcup_i V(Ai) \subseteq V(\bigvee_i Ai)$...... (II)

Therefore from (I) & (II) we get

$V(\underset{i}{\vee} Ai) = \underset{i}{\cup} V(Ai)$

2) If $P \in V(A_1 \wedge A_2 \wedge \ldots \wedge A_n)$ then

$A1 \wedge A_2 \wedge \ldots \wedge An \not\subset P$.

Hence $Ai \not\subseteq P$ for every i, $1 \leq i \leq n$.

i.e. $p \in \bigcap_{i=1}^{n} V(Ai)$

Thus $V(A_1 \wedge A_2 \wedge \ldots \wedge An) \subseteq \cap V(Ai)$.... (III)

Now if $P \in \bigcap_{i=1}^{n} V(Ai)$

This gives $P \in V(Ai)$, for every i

i.e. $Ai \not\subseteq P$, for every i.

As P is a prime dual ideal, $\bigwedge_{i=1}^{n} Ai \not\subset P$.

But then $P \in V(\bigwedge_{i=1}^{n} Ai)$

Therefore $\bigcap_{i=1}^{n} V(Ai) \subseteq V(\bigwedge_{i=1}^{n} Ai)$... (IV)

Combining (III) & (IV) we get

$\bigcap_{i=1}^{n} V(Ai) = V(\bigwedge_{i=1}^{n} Ai)$

i.e. $V(A_1 \wedge A_2 \wedge \ldots \wedge An) = V(Ai) \cap V(A_2) \cap V(An)$

3) As S is not contained in any member of $\wp$, we get $V(S) = \wp$

4) Since every prime dual ideal contains 1, it follows that $V([1)) = \Phi$

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Define $U(A) = \wp - V(A)$, the complement of V (A) in $\wp$. Then from the above

Result 3.1.1: we get

Result 3.1.2 : For any dual ideal Ai in S , we have

1) $U(\underline{\vee}_{i=1}^{n} A_i) = \cap_{i=1}^{n} U(A_i)$

2) $U(A_1 \barwedge A_2 \barwedge \ldots \barwedge A_n) = U(A_1) \cup U(A_2) \cup \ldots \cup U(A_n)$

3) $U(S) = \Phi$

4) $U([1]) = \wp$

***

Consider the topology T defined on $\wp$ for which V(A) is an open set. This topology is the Stone's topology and ($\wp$,T) is the Stone's space. At the out set we study some properties of Stone's space (($\wp$,T). Many results of Venkatanarasimhan [19] follow from our results.

Result : 3.1.3: Let X be any subset of $\wp$, then $Cl_{\wp} X = U(X0)$, X0 being the intersection of all members of X

Proof : Let B = U (Xo)

i.e. $B = \{P \in \wp \mid X_0 \subseteq P\}$

i.e. $B = \{P \in \wp \mid \cap X \subseteq P\}$

$= \{P \in \wp \mid \cap_{F \in X} F \subseteq P\}$

Let $F \in X$, then $\cap_{F \in X} F \subseteq F$ & $F \in \wp$, imply that

$F \in B$ Thus we get $X \subseteq B$.

Let $U(A) = \wp - V(A)$ be any closed set containing X But then $A \subseteq F$, for all $F \in X$

Hence $\{P \in \wp \mid \cap_{F \in X} F \subseteq P\} \subseteq \{P \in \wp \mid A \subseteq P\}$

This gives, $B \subseteq U(A)$

i.e. B is the smallest closed set containing X

Therefore $Cl._{\wp} X = B = U(X_0)$, Xo being the intersection of members of X.

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We define as usual for any subset B of S the hull of B: $h(B) = \{p \in \wp \mid B \subseteq P\}$ &

for any subset T of $\wp$, the Kernel of T in $\wp$, is defined as $k(T) = \cap\{ P \mid P \in T\}$.

Thus from the **Result 3.1.3**, we get T is closed if and only if $T = hk(T)$.

Hence we have ,

Result :3.1.4: T is the hull Kernel topology on $\wp$.

Proof : Let $T \subseteq \wp$

We shall prove that h(k(T)) is the smallest closed set containing T.

i.e. for any $T \subseteq \wp$, h (k(T)) is the closure of T in $\wp$.

i.e. $Cl._{\wp}\{T\} = \{P \varepsilon \wp / \bigcap_{Q \varepsilon T} Q \subseteq P\} = h(k(T))$.

Since $k(T) = \bigcap_{Q \varepsilon T} Q$

We get $k(T) \subseteq Q$ for all $Q \varepsilon T$ & hence $T \subseteq h(k(T))$

Also, since V(k(T)) is open and

$h(k(T)) = \wp - V(k(T))$

Thus we get h(k(T)) is closed.

Let C be any closed set in $\wp$ containing T.

Then $C = \wp - V(A)$, for some $A \subseteq S$.

Since $X \subseteq C$, $C \cap V(A) = \Phi$

i.e. $Q \notin V(A)$, for all $Q \in T$ & hence $A \subseteq Q$, for all $Q \in T$

i.e. $A \subseteq \bigcap_{Q \in T} Q$

i.e. $A \subseteq k(T)$ (as $k(T) = \bigcap_{Q \in T} Q$)

And hence $P \; h(k(T))$

This gives $k(T) \subseteq P$

i.e. $A \subseteq P$

This gives $P \in C$ (as $C = \wp - V(A)$)

Therefore $h(k(T)) \subseteq C$.

i.e. $h(k(T))$ is the smallest closed set containing T.

i.e. $h(k(T))$ is the closure of T.

i.e. $Cl_{\wp}\{T\} = \{P \in \wp \mid \bigcap_{Q \in T} Q \subseteq P\} = h(k(T))$

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In view of the above **Result3.1.4**. the topology on $\wp$ is known as the hull-kernel topology.

Next we prove

Result : 3.1.5.:$(\wp, T)$ is a To - space

Proof : Let $Q_1, Q_2 \in \wp$

Let $Cl_{\wp}\{Q_1\} = Cl_{\wp}\{Q_2\}$

By **Result : 3.1.3** $Cl_{\wp}\{Q_1\} = \{P \in \wp / Q_1 \subseteq P\}$

$Cl_{\wp}\{Q_2\} = \{P \in \wp | Q_2 \subseteq P\}$

But $Q_1 \in Cl_{\wp}\{Q_1\}$ implies $Q_1 \in Cl_{\wp}\{Q_2\}$

Hence $Q_2 \subseteq Q_1$

Similarlly, $Q_2 \in Cl_{\wp}\{Q_2\}$ implies that $Q_2 \in Cl_{\wp}\{Q_1\}$

Hence $Q_1 \subseteq Q_2$

Thus $Cl_{\wp}\{Q_1\} = Cl_{\wp}\{Q_2\}$ gives $Q_1 = Q_2$

which in turn proves that $(\wp, T)$ is a To - space [See **Def. 1.30**]

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S being distributive semilattice with 0.

We get,

Result : 3.1.6: $(\wp, T)$ is compact.

Proof: Let $\wp = \bigcup_{i \in I} V(Ai)$ (I is any indexing set)

Then [By Result 3.1 (1 & 3)]

$V(S) = \wp = \bigcup_{i \in I} V(Ai) = V(\underset{i \in I}{\vee} Ai)$

If $\vee Ai \neq S$ then there would exists a prime dual ideal containing $\vee Ai$ leading to $V(\vee Ai) \neq \wp$ **[By Results 2.4 & 2.6]**

But this contradicts our assumption.

Hence, $\underset{i \in I}{\vee} Ai = S$.

As $0 \in S$, we get $0 \in \underset{i \in I}{\vee} Ai$ (as $S = \underset{i \in I}{\vee} Ai$)

Therefore there exists a finite number of elements

$a_{i1}, a_{i2}, \ldots\ldots\ldots a_{in}$ ($a_{ij} \in A_{ij}$) such that

$$0 = a_{i1} \wedge a_{i2} \wedge \ldots\ldots \wedge a_{in} \in \underset{i \in I}{V} A_{ij}$$

Therefore $\underset{i \in I}{V} A_i \subseteq A_{i1} \vee A_{i2} \vee \ldots\ldots \vee A_{in}$

Consequently,

$$\wp = V(\underset{i}{V} A_i) \subseteq V(A_{i1} \vee A_{i2} \vee \ldots\ldots \vee A_{in}) = V(A_{i1}) \cup V(A_{i2}) \cup \ldots\ldots \cup V(A_{in})$$

Hence the result

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About the T_1 - points of $\wp$ we have,

Result: 3.1.7: P is a T_1 - point of $(\wp, T)$ if and only if P is a maximal dual ideal of S.

Proof :By Result 3.1.3 and Def. 1.24, it follows that $Cl_{\wp}\{M\} = \{M\}$, $M \subseteq \wp$

This proves that every maximal dual ideal is a T_1-point of $\wp$.

Now if $P \in \wp$ is a T_1-point of $\wp$ then [By Def. 1.33] it follows that P is a maximal dual ideal. I.e. the set of all T_1-points in $\wp$ is the set of all maximal dual ideals of S.

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If Π denotes the set of all maximal dual ideals in S. then we have,

Π = the set of all T_1-points of $\wp$.

Further we have,

Result : 3.1.8: The closure of set of T_1-points of $(\wp, T)$ is U(D) where D is the dual ideal of dense elements of S.

Proof: By Result 3.1.7.

The closure of the set of all T_1-points in $\wp$ = $Cl_{\wp}\,\mathcal{M} = \{P \in \wp \mid \cap \mathcal{M} \subseteq P\}$

But $\cap\mathcal{M} = D$ [By **Result 2.19**]

Hence $Cl_{\wp}\,\mathcal{M} = \{P \in \wp \mid D \subseteq P\}$

$= U(D)$

i.e. $Cl_{\wp}\,\mathcal{M} = U(D)$.

Hence the proof.

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A sufficient condition for the space $\wp$ to be Π_0 - space is given in the following

Result : 3.1.9: Let S be a bounded semilattice.

Then $(\wp, T)$ is Π_0 if $D = [1)$

Proof : Let V(A) be any non-empty open subset of $\wp$.

Let if possible $A \subseteq M$, for each $M \in \mathcal{M}$ then $A \subseteq \cap\mathcal{M}$ & hence $A \subseteq D$ [As $D = \cap\mathcal{M}$]

But by data, $D = [1)$

We get $A \subseteq D = [1)$

Thus $A = [1)$ proving that $V(A) = V([1)) = \phi$ [By **Result 3.1.1 (4)**]

This contradicts the fact that V(A) is non-empty.

Hence there exists at least one maximal dual ideal, say M such that $A \not\subseteq M$. But then $M \in V(A)$.

As $Cl_{\wp}\{M\} = \{M\}$

V(A) contains the closed set $\{M\}$

Hence [By **Def. 1. 36**] it follows that $(\wp, T)$ is Π_0-space.

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3.2 The space of maximal dual ideals.

L' us denote the set of all maximal dual ideals of S by Λ

As every maximal dual ideal in a distributive semilattice S is prime [By **Result 2.6**]

We get $\Lambda \subseteq \wp$, the set of prime dual ideals in S.

An interesting property of the subspace (Λ,T) is established in the following

Result : 3.2.1: The subspaces (Λ,T) is the smallest of the subspaces X of $(\wp,T)$ such that X is not weakly separable from any point out side it.

Proof: First we will prove that Λ is not weakly separable from any point out side it.

Let $P \in \wp$ such that $P \notin \Lambda$. Then as every proper dual ideal is contained in some maximal dual ideal [By **Result 2.4**], there exists $M \in \Lambda$ such that $P \subseteq M$.

But then $M \varepsilon \{M\} \cap Cl_{\wp}\{P\}$ proving that Λ is not weakly separable from any point out side it. [By **Def 1.40**]

To prove that Λ is the smallest subspace of $(\wp,T)$ satisfying the given condtition.

Let there exists $X \subseteq \wp$, such that $X \cap Cl_{\wp}\{P\} = \phi$, for any $P \notin X$.

Let if possible $\Lambda \not\subset X$. Hence there exists $M \in \Lambda$ such that $M \notin X$. As $M \in \wp$ by the property of X, we get $X \cap Cl_{\wp}\{M\} \neq \phi$ i.e. $X \cap \{\Lambda\} \neq \phi$ since $Cl_{\wp}\{M\} = \{M\}$[by **Result3.1.7**] But then $M \subseteq X$ which is a contradiction .Thus Λ is the smallest sub space of $\wp$ satisfying the given condition .

Sufficient condition for a subspace X of $\wp$ to be compact is given in the following.

Result : 3.2.2: If X is any subset of $\wp$ containing Λ then (X,T) is compact.

proof :Let $X \subseteq \bigcup_{i \in I} V(A)$ Then [By **Result 3.1.1(1)**]

$$X \subseteq V(\underset{i\in I}{\underline{\vee}} Ai)$$

Therefore no member of X contains $\underset{i\in I}{\underline{\vee}}Ai$ and as $\Gamma \subseteq X$ no member of Γ contains $\underset{i\in I}{\underline{\vee}}Ai$.

But this will imply $S = \underset{i\in I}{\underline{\vee}} Ai$. Hence $0 \in S$ implies that $0 \in \underset{i\in I}{\underline{\vee}} Ai$

Hence $o = a_{i1} \wedge a_{i2} \wedge \ldots \wedge a_{in}$ where $a_{ij} \in Aij$, for $1 \le j \le n$

But then,

$S = [a_{i1} \wedge a_{i2} \wedge \ldots \wedge A_{in})$

$= [a_{i1}) \ \underline{\vee} \ [a_{i2}) \ \underline{\vee} \ldots \underline{\vee} \ [a_{in})$

$\subseteq A_{i1} \ \underline{\vee} \ A_{i2} \ \underline{\vee} \ldots \underline{\vee} \ A_{in}$

Therefore, $X \subseteq V(A_{i1} \ \underline{\vee} \ A_{i2} \ \underline{\vee} \ \ldots \underline{\vee} A_{in})$

$= V(A_{i1}) U V(A_{i2}) U \ldots U V(A_{in})$

Thus every open cover of X contains a finite subcover proving that X is compact.

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A class of distributive lattices in which every prime ideal is contained in a unique maximal ideal is studied indetail in [8]. In the following theorem we give a topological condition under which every prime dual ideal in a distributive semilattice is contained in a unique maximal dual ideal.

Result : 3.2.3.:If Γ is retraction of $\wp$ then every prime dual ideal in S, is contained in a unique maximal dual ideal.

Proof :Let us assume that Γ is retract of $\wp$. Hence there exists a retraction say f of $\wp$ onto Γ.

Let $f(P)=M$, for some $P \subseteq M$. We will prove that, M is the unique maximal dual ideal containing P. As Π is T_1-space . $\{M\}$ is closed in Π. By continuity of f, $f^{-1}(\{M\})$ is closed in $\wp$. As $P \in f^{-1}(\{M\})$ and $Cl_{\wp}\{P\}$ is the smallest closed set containing P. We get $Cl_{\wp}\{P\} \subseteq f^{-1}\{M\}$. Now if $P \subseteq M_1$ and $M_1 \neq M$ in M then $M_1 \subseteq Cl_{\wp}\{P\}$.

Hence $M_1 \in f^{-1}(\{M\})$.i.e. $f(M_1)=M$. But $f(M_1)=M_1$, f being retraction.

Therefore $M=M_1$. This implies that, every prime dual ideal in S is contained in a unique maximal dual ideal.

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3.3 The space ($\wp$,T')

In this article we define a new topology T' on the set of all prime dual ideals $\wp$ of S.

Let us define $F(A) = \{P \in \wp / P \cap A \neq \Phi\}$. Where A is any ideal in S. the following result illustrates some properties of F(A).

Result :3.3.1.: 1. $F(\bigvee_{i \in I} A_i) = \bigcup_{i \in I} F(A_i)$ where I is any indexing set.

2. $F(A_1 \cap A_2 \cap \ldots \cap A_n) = F(A_1) \cap F(A_2) \cap \ldots \cap F(A_n)$

3. $F(S)= \wp$

4. $F((0])= \phi$

Proof : 1. Let $P \in F(\bigvee_{i \in I} A_i)$

Then we get $P \cap (\bigvee_{i \in I} A_i) \neq \phi$ implying that $P \cap A_i \neq \phi$ for some $i \in I$.

i.e. $P \in F(A_i)$ for some $i \in I$.

Hence $P \in \bigcup_{i \in I} F(A_i)$

Thus $F(\bigvee_{i \in I} A_i) \subseteq \bigcup_{i \in I} F(A_i)$ ....(I)

If $P \in \bigcup F(A_i)$ then $P \cap A_i \neq \phi$ for some $i \in I$

i.e. $P \cap V(A_i) \neq \phi$, proving that $P \in F(VA_i)$

Thus $\bigcup_{i \in I} F(A_i) \subseteq F(\bigvee_{i \in I} A_i)$ ....(II)

From (I) & (II) we get the proof of 1.

2. Let $P \in F\ (A_1 \wedge A_2 \wedge \ldots \wedge A_n)$

i.e. $P \cap A_i \neq \phi$, for every $i \in I$, $(1 \leq i \leq n)$

proving that $P \in \bigcap_{i=1}^{n} F(A_i)$

i.e. $F(\bigcap_{i=1}^{n} A_i) \subseteq \bigcap_{i=1}^{n} F(A_i)$ ......(III)

Now if $P \in \bigcap F(A_i)$, then $P \in F(A_i)$ for every i. $(1 \leq i \leq n.)$

i.e. $P \wedge A_i \neq \phi$ for all i, $(1 \leq i \leq n.)$

This implies that $a_i \in P \cap A_i$ for all i

But this in turn proves that, $a_1 \wedge a_2 \wedge \ldots \wedge a_n$ $P \cap (\bigcap_{i=1}^{n} A_i)$

i.e. $P \cap (\bigcap_{i=1}^{n} A_i) \neq \phi$

i.e. $P \in F(\bigcap_{i=1}^{n} A_i)$

But this implies that $\bigcap_{i=1}^{n} F(A_i) \subseteq F(\cap A_i)$ ....(IV)

Thus from (III) & (IV) we get

$\bigcap_{i=1}^{n} F(A_i) = F(\bigwedge_{i=1}^{n} A_i)$

3. $F(S) = \wp$

As every prime dual ideal of S is contained in S.

Therefore $F(S) = \wp$

4. $F((0]) = \phi$

As (o]=S is not contained in any prime dual ideal. We get $F((0])=\phi$

Define $F'(A) = \wp - F(A)$. Then from **Result (3.3.1)** we get following .

Result : 3.3.2:

1. $F'(\bigvee_{i\in I} Ai) = \bigwedge_{i\in I} F'(Ai)$

2. $F'(A_1 \wedge A_2 \wedge \ldots \wedge An) = F'(A_1) \cup F'(A_2) \cup \ldots \cup F(An)$

3. $F'(S) = \phi$

4. $F'((o]) = \wp$

The above **Result 3.3.2** shows that F' defines a closure operations in $\wp$, there by giving rise to a topology say T' on $\wp$

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3.4 Subbase

We begin with the following

Def.3.4.1: Let S be a bounded distributive $\wedge$ -semilattice and let a,b ε S. Define V(a) = $\{P \in \wp / a \notin P\}$

where $\wp$ is the set of all prime dual ideal in S.

We have a property of V(a) in the following.

Result :3.4.1: Let a,b ε S and let V(a) be open set in $\wp$ then $b \geq a$ gives $V(b) \subset V(a)$

Proof :Let $P \varepsilon V(b)$.

We get $b \notin P$.

If $a \leq b$ and $a \varepsilon P$, then $b \varepsilon P$, a contradiction.

Therefore $a \notin P$.

Then we get $P \varepsilon V(a)$.

Thus $b \geq a$ gives $V(b) \subset V(a)$

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Further we have

Result :3.4.2.:Let $a \varepsilon S$. Then $V(a)$ is a compact in $\wp$.

Proof: Let Δ be a class of subsets of S.

Let $\{V(A)/A \subseteq \Delta\}$ be an open cover of $V(a)$.

i.e. $V(a) \subseteq \bigcup_{A \varepsilon \Delta} V(A) = V(\underline{\vee}_{A \varepsilon \Delta} A)$ [By **Result 3.1.1(1)**]

$= V(B)$ where $B = \underline{\vee}_{A \varepsilon \Delta} A$

i.e. $V(a) \subseteq V(B)$, where $\underline{\vee}_{A \in \Delta} A = B$

Suppose $a \notin B$.Then $(a] \cap B = \Phi$ [By **Result 2.23**]

There exists a prime dual ideal P such that $B \subseteq P$ and $(a] \cap P = \Phi$

Therefore $a \notin P$.

Hence $P \varepsilon V(a)$

But $V(a) \subset V(B)$

Therefore $P \varepsilon V(B)$

This gives $B \not\subset P$, a contradiction to the choice of P.

Therefore $a \varepsilon B$.

i.e. $a \varepsilon \underline{V}_{A \varepsilon \Delta} A$

i.e. $a \geq a_1 \wedge a_2 \wedge \dots \wedge a_n$ such that, $a_1 \varepsilon A_1$, $a_2 \varepsilon A_2$, ... $a_n \varepsilon A_n$ i.e. $a \varepsilon \underline{V}_{i=1}^{n} Ai$

Thus we get $V(a) \subseteq V(\underline{V}_{i=1}^{n} Ai)$

This shows that the given open cover of V(a) has a finite open subcover.

Therefore V(a) is compact.

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Two properties of Stone space $\wp$ a bounded distributive $\wedge$ -semilattice are studied in the following.

Result. 3.4.3:The Stone space $\wp$ of a bounded distributive $\wedge$ -semilattice has sthe following two properties.

I) $\wp$ is a To - space in which the compact open sets form a base for the open sets.

II) If A is a closed set in $\wp$, {Uk /k ε K} is an updirected family of compact open sets of $\wp$ and $Uk \cap A \neq \Phi$ then

$\cap\{Uk/k \varepsilon K\} \cap A \neq \Phi$

Proof :To show sthat (I) holds.,we have to prove the following

1) $\wp$ is To - space.

2)V(a) is compact open set, and

3) V (a) form a base for the open sets of $\wp$

(1) & (2) are proved [see **Results 3.1.5 and 3.4.2**]

Now, we prove (3) i.e. V(a) form a base for the open sets of $\wp$.

In other wards, for a,b ε S, P ε V(a) $\cap$ V(b),

we have to find c ε S with P ε V(c) such that V(c) $\subseteq$ V(a) $\cap$ V(b) where P ε $\wp$

For a,b ε S, P ε V(a) $\cap$ V(b), we get

P ε V(a) and P ε V(b).

This gives a $\notin$ P and b $\notin$ P

i.e. [a) $\cap$ [b) $\not\subseteq$ P

As P is prime, there exists c ε S such that c ε [a) $\cap$ [b), which gives c $\notin$ P such that

c > a, c >, b.

Therefore P ε V(a) such that V(c) $\subseteq$ V(a) and V(c) $\subseteq$ V(b). [By **Result 3.4.1**]

Hence P ε V(c) such that V(c) $\subseteq$ V(a) $\cap$ V(b)

Thus for P ε V(a) $\cap$ V(b) there exists c ε S with P ε V(c) such that V(c) $\subseteq$ V(a) $\cap$ V(b).

Hence {V(a) /a ε S} form a base for the open sets of $\wp$.

To verify; (II) for $\wp$, let A be a closed set in $\wp$.

Therefore A = $\wp$-V(F) is closed in $\wp$ where V(F) is an open set in $\wp$. And {Uk/k ε K}

is an updirected family of compact open sets of $\wp$.

i.e. Uk = V(a_k), for some k ε K.

i.e. Uk = {P ε $\wp$ /a_k $\notin$ P}

Now consider I = {x/x $\leq$ a_k for some k ε K}

First we prove that I is an ideal

i) Since $o \leq a_k$ for every $k \in K$

Therefore $o \in I$. we get $I \neq \Phi$

ii) Let $x \leq y$ and $y \in I$

We get $y \leq a_k$ for some $k \in K$

As $x \leq y$ and $y \leq a_k$ we get $x \leq a_k$, for some $k \in K$

This gives $x \not\leq I$

iii) Let $x.y \in I$. Then $x \leq a_{k1}$ for some $k_1 \in K$.

and $y \leq a_{k2}$ for some $k_2 \in K$. There exists Uk_3 in updirected family such that $U(a_{k3}) \subseteq$

$U(a_{k1}) \cap U(a_{k2})$

Now since $Ua_{k3} \subseteq Ua_{k1}$ and $Ua_{k3} = V(a_{k3})$

This gives $V(a_{k3}) \subseteq V(a_{k1})$

Consider $a_{k1} \not\leq a_{k3}$

Take $Q \in \wp$ such that $a_{k1} \in Q$ and $a_{k3} \notin Q$.

This gives $Q \in V(a_{k3})$ .i.e. $Q \in V(a_{k1})$

Hence $a_{k1} \notin Q$, a contradiction Therefore $V(a_{k3}) \subseteq V(a_{k1})$ this gives $a_{k3} \geq a_{k1}$

Similarlly, since $U_{k3} \subseteq U_{k1}$ and $Ua_{k3} = V(a_{k3})$ Therefore $V(a_{k3}) \subseteq V(a_{k2})$.

This gives $a_{k3} \geq a_{k2}$. Now as $a_{k3} \leq a_{k}$ gives $a_{k3} \in I$, take $z = a_{k3} \in I$. Thus for $x, y \in I$ there exist $z \in I$ such that $z \geq x$ and $z \geq 1$. Therefore from (i),(ii) & (iii) we get I is an ideal.

Now since $U_k \cap A \neq \Phi$. Therefore $U_k \cap [\wp - V(F)] \neq \Phi$

i.e. $U_k \subseteq \wp$ and $U_k \not\subset V(F)$. Therefore $V(a_k) \subseteq \wp$ and $V(a_k) \not\subseteq V(F)$, where $U_k = V(a_k)$

Now if $a_k \in F$ then $a_k \in P$ for all $P \supseteq F$. This gives $a_k \in P$ for all $P \in \wp - V(F) = U_k$

i.e. $a_k \varepsilon P$ for all $P \varepsilon V(a_k)$ Therefore $a_k \varepsilon P$ for all P such that $a_k \notin P$, a contradiction.

Therefore $a_k \notin F$, for all k. Now we prove that $I \cap F = \Phi$

If $I \cap F \neq \Phi$ then there exist $x \varepsilon I \cap F$. i.e. $x \leq a_k$, for some k this gives $x \varepsilon F$ and $x \leq a_k$ i.e. $a_k \varepsilon F$ a contradiction. Therefore there exists a prime dual ideal P with $P \supseteq F$ such that $I \cap P = \Phi$.

Then $a_k \notin P$ and so $P \varepsilon V(a_k)$ for all $k \varepsilon K$. Also $P \supseteq F$

This gives $P \notin V(F)$ i.e. $P \varepsilon \wp - V(F)$

Therefore $P \varepsilon A$

And as $a_k \varepsilon I$ for all k and $P \cap I = \Phi$

Therefore $a_k \notin P$

This gives $P \varepsilon V(a_k)$ for all k

i.e. $P \varepsilon U_k$ for all k

Therefore $P \varepsilon [\cap\{U_k/k \varepsilon K\}]$

Thus $P \varepsilon A \cap [\cap\{U_k/k \varepsilon K\}]$ This shows that $A \cap [\cap\{U_k/k \varepsilon K\}] \neq \Phi$.

Hence the result.

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