CHAPTER - O

Notations and Definitions

CHAPTER - 0

NOTATIONS AND DEFINITIONS

Notations which are used in this dissertation.

Sr.No.	Notations	Meaning of Notation
1	$=$	Equal to
2	$<$	Strictly less than
3	$\leq$	Less than Equal to
4	$>$	Strictly greater than
5	$\geq$	Greater than equal to
6	$\therefore$	Therefore
7	$\because$	Since
8	$\rightarrow$	Tends to
9	$\Rightarrow$	Implies
10	∞	Infinity

11	C	Set of complex numbers
12	L	Operator of the Lapace transform
13	S	Operator of the Stieltjes transforms
14	m	Operator of the mixed Stieltjes transform
15	M	Operator of the Mellin transform
16	Γ	Gamma Function
17	$\phi(z)$	Laplace transform of function f(x)
18	S(t)	Stieltjes transform of function f(x)
19	m(t, z)	Mixed Stieltjes transform of function f(x)
20	M(p)	Mellin transform of function f(x).

0.2 Definitions :

I) Integral transform

Let k(z, x) be a function of two variables z and x, where z is parameter (may be real or complex) independent of t, the function F(z) defined by the integral,

$$F(z) = \int_{-\infty}^{\infty} k(z, x).f(x).dx \quad \ldots\ldots\ldots\ldots\ldots\ldots (0.2.1)$$

is called the integral transform of the function f(x) and is denoted by T[f(x)]. The function k(z, x) is called the kernel of the transformation.

II) Laplace Transform:

If f(x) is piecewise continuous and is of exponential order in the interval $o < x < \infty$, then Laplace transform of f(x) is denoted as $\phi(z)$ and defined by the equation,

$$L[f(x)] = \int_{0}^{\infty} e^{-zx} f(x)dx \quad \ldots\ldots\ldots\ldots\ldots\ldots (0.2.2)$$

where z is complex number.

III) Stieltjes Transform:

If f(x) piecewise continuous and locally integrable function in the interval $0 < x < \infty$, then Stieltjes Transform is denoted as S(t) and defined by the equation,

$$S(t) = \int_0^\infty \frac{f(x)}{t+x}.dx \quad \ldots\ldots (0.2.3)$$

$0 < t < \infty$

is called the Stieltjes transform of function f(x). It is denoted by S(t) or S[f(x)].

IV) Mellin Transform

The Mellin transform of function f(x), over the interval $0 < x < \infty$ is defined by the equation as

$$M[f(x)] = \int_0^\infty x^{p-1}.f(x)dx \quad \ldots\ldots (0.2.4)$$

It is denoted by M(p) or M[f(x)].

V) Piecewise continuous function.

A function f(x) is said to be piecewise continuous on a closed interval $a \le t \le b$, if it is defined on that interval and is such that the interval can be subdivided into finite number of intervals, in each of which f(x) is continuous and has finite right and left hand limits.

VI) Absolutely integrable function.

A function f(x) is said to be absolutely integrable over the interval I if the integral

$$\int_I |f(x)|\,dx \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (0.2.5)$$

is finite.

VII) Localization lemma.

If f(x) is piecewise continuous function over $0 \le x \le a$ where 'a' is finite then,

$$\int_0^a f(x)\frac{\sin \lambda x}{x}\,dx \to \frac{1}{2}\pi f(0^+) \qquad \ldots\ldots\ldots (0.2.6)$$

as $\lambda \longrightarrow \infty$

VIII) Dirichlet's integral formula

If f(x) is piecewise continuous function and x^{-1} f(x) is absolutely integrable over interval $0 < x < \infty$, then

$$\int_0^\infty f(x)\frac{\sin \lambda x}{x}dx \to \frac{1}{2}\pi f(0^+) \ldots (0.2.7)$$

as $\lambda \to \infty$

IX) Lemma

If f(x) is piecewise continuous and absolutely integrable over the whole real line R then,

$$\int_{-\infty}^\infty f(x+u)\frac{\sin \lambda u}{u}du \to \frac{1}{2}\pi[f(x^+)+f(x^-)] .. x \in R ..(0.2.8)$$

as $\lambda \to \infty$

X) Lebnitz's rule:

If $f(x, \alpha)$ is a continuous function of x and α in the interval $a \leq x \leq b$, where a and b are functions of parameter α and

$$I(\alpha) = \int_{a(\alpha)}^{b(\alpha)} f(x,\alpha)dx$$

then,

$$\frac{dI}{d\alpha} = \frac{d}{d\alpha}\int_a^b f(x,\alpha)\,dx$$

$$= \int_a^b \frac{\partial x}{\partial \alpha} f(x,\alpha)dx + f(b,\alpha)\frac{db}{d\alpha} - f(a,\alpha)\frac{da}{d\alpha} \quad \ldots(0.2.9)$$

& if a and b are not the functions of parameter , then

$$\frac{dI}{d\alpha} = \frac{d}{d\alpha}\int_a^b f(x,\alpha)\,d\alpha$$

$$= \int_a^b \frac{\partial}{\partial \alpha} f(x,\alpha)d\alpha \quad \ldots\ldots\ldots.(0.2.10)$$

XI) Change of order of integration.

If function f(x, y) is integrated with two variables x and y, and if limits of integration are function of variables x or y , also $y_1(x)$, $y_2(x)$ are functions of x and $x_1(y)$, $x_2(y)$ are functions of y then,

$$\int_a^b dx \int_{y_1(x)}^{y_2(x)} f(x,y)dy = \int_c^d dy \int_{x_1(y)}^{x_2(y)} f(x,y)dx \quad \ldots.(0.2.11)$$

& if limits of integration are not the functions of x and y, then

$$\int_a^b dx \int_c^d f(x, y)dy = \int_c^d dy \int_a^b f(x, y)dx \ldots (0.2.12)$$

Where a, b, c, d are constants numbers.

XII) Gamma Functions:

The gamma function of 't' is defined by

$$\Gamma t = \int_0^\infty e^{-x} x^{t-1} dx \ldots\ldots (0.2.13)$$

It 't' non-integral i.e. real number, then we have

$$\Gamma t \cdot \Gamma(1-t) = \frac{\pi}{\sin \pi t} \ldots\ldots (0.2.14)$$

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