PREFACE

This dissertation entitled "**Principal Element Lattices**" is based on abstract commutative ideal theory and is an expository work. M. Ward and R. P. Dilworth began study in abstract form of ideal theory of commutative rings, around 1938. It was intended that, results of commutative ring theory are to be abstracted. For such generalization, the system to be chosen was 'lattice structure' due to its simplicity. It was then obvious to introduce new binary operation called multiplication on lattices. Consequently, such lattices are known as multiplicative lattices.

For such multiplicative lattices, theorems analogues of the Noether decomposition theorems of commutative rings were formulated and proved in abstract version successfully. However the results corresponding to the deeper results on the ideal structure of rings were not obtained quite satisfactorily, because of the lack of a proper abstraction of *principal ideals*. In [21], M. Ward and R. P. Dilworth introduced a weak concept of 'principal element' as follows:

" An element a of a multiplicative lattice L is *principal*, if $x \leq a$ implies $\exists$ an element $y \in L$ such that $x = ay$."

Though this concept was sufficient for the proofs of the decomposition theorem into primaries and some other results, it was latter on realised that it had serious defects and hence this concept of principal element is not adequate for the further development.

It was R. P. Dilworth [9] who was able to give a stronger formulation for the notion of principal elements in two identities, after a very long era near about of 23 years, as follows :

" In a multiplicative lattice L, an element $a \in L$ is said to be *join principal*, if $x \vee y{:}a = (xa \vee y){:}a$ and *meet principal*, if $x \wedge ay = (x{:}a \wedge y)a$, for all $x, y \in L$. An element is *principal*, if it is both join and meet principal."

This concept of principal elements is thus the cornerstones on which the theory of multiplicative lattices and abstract ideal theory now largely rests. And then R. P. Dilworth demonstrated the richness of principally generated multiplicative lattices by giving a purely lattice theoretic development of the most basic constructions and results of classical ideal theory with a series of research work.

Till now the classical beauty of the theory has been enrich by many mathematicians especially to mention the names are D. D. Anderson, C. Jayaram, E. W. Johnson, J. A. Johnson, Francisco Alarcon and others. The key richness of the setting lies, of course, in the definition of a principal element which is expressed into two identities. Almost all basic concepts are studied in the chapter one.

The concept of weak Noether lattices is introduced by D. D. Anderson and C. Jayaram in [3] in 1993, of course, as a generalization of the concept of Noether lattices. Using this concept we study some equivalent conditions for a weak Noether lattice to be a principal element lattice. In [1], the concept of r-lattice is strongly introduced and studied well. The results established therein are mainly concerned with r-lattices. Note that, the difference between r-lattices and weak r-lattices is nothing but "modularity". In fact, many of the results proved in [1] don't require the condition of modularity and thus they can be carried over without any change to weak r-lattice. This fact is pointed out by D. D. Anderson himself with C. Jayaram in [1]. Further, in this chapter, we study an equivalent condition for a weak r-lattices to be a principal element lattices.

In the chapter 3, we study π-lattices, UFD lattices and Dedekind domains. Also we study their characterization. Throughout this chapter, L denotes a multiplicative lattice with 1 compact unless otherwise stated.

While the chapter 4 is concerned with invertible elements of multiplicative lattices. The concept of invertible elements is described in terms of regular elements. We then study invertible prime elements and using them, we study some equivalent conditions for a weak r-lattice L to be a finite direct product of Dedekind domains.

The work done in this dissertation is purely expository. We have just studied principal element lattices with the main help of the research paper [3].

The numbers in the square brackets indicates the number of references listed in "References", which is enclosed at the end of this dissertation.