PREFACE

This thesis entitled " THE CURVATURE, TORSION, BITORSION OF THE WORLD LINE OF THE TIME LIKE PARTICLE IN SPACE-TIME METRICES" is devoted to the study of the geometry of world line of the time-like particle in space-times. Investigation of the geometry of world line of the particle is imperative to understand the universe and relate the General Relativity to the rest of the geometry. The intrinsic scalars viz. the curvature field K, the torsion field T and the bitorsion field B of the curve determine the geometry of the curve. The invention of rheotetrad by Radhakrishna (1993) prompted us to study the geometry of the world line of the time-like particle for the given metric as the background space-time. The thesis consists of four chapters.

In the first chapter, one finds a brief review of the Newman-Penrose spin coefficient formalism, which is amazingly useful and most powerful mathematical technique in the General theory of relativity, highlighting its silent features. As we have pointed out that rheotetrad of Radhakrishna provides clue to study ~~the~~ ~~of~~ the geometry of world line in the given space-time. This rheotetrad is constructed with the help of a single time-like vector field and its intrinsic derivatives together with intrinsic scalars viz. the curvature field K, the torsion field T and the bitorsion field B. Recently, Katkar and Khairmode by exploiting the Newman-Penrose spin coefficient formalism, have obtained the expressions for the intrinsic scalars in terms of Newman-Penrose spin coefficient. A brief exposition of the rheotetrad and Newman-Penrose concomitants of intrinsic scalars are presented in the same chapter. Accordingly no originality is claimed in the contents of the first chapter. The rest of the tree chapters contain the original contribution of the author.

Three space-time metrics viz.

- i) Reissener-Nordstorm space-time ,
- ii) Bertotti-Robertson space-time and
- iii) Robertson-Walker space-time

are chosen and the of the geometry of world line of the time-like particle in the above space-times is described respectively in the next three chapters. The investigation are given chapterwise and sectionwise as follows.

Chapter-II is concerned with the geometry of the time like particle in Reissener-Nordstorm space-time. The concept of tetrad and notation are cited in section-1. By using the mathematical tool of differential forms, the components of tetrad vector fields relative to Reissener-Nordstorm space-time are obtained in Section-2. Section-3 deals with the description of Reissener-Nordstorm space-time in the Newman-Penrose formalism. In section-4, we present all Christoffel symbols with respect to the Reissener-Nordstorm space-time. With respect to the basis vectors, cited in section-3,the 12 complex spin coefficients are worked out in section-5. It is shown that the spin coefficients $\kappa, \sigma, \tau, \lambda, \nu$ vanish for the Reissener-Nordstorm space-time. Consequently on the basis of Goldberg-Sachs theorem it is observed that the Reissener-Nordstorm space-time is of Petrov-type D. In order to find the curvature at a point of the Reissener-Nordstorm space-time for the orientation determined by two real or complex null vector fields, twenty independent components of Riemann Curvature tensor are derived in Section-6. Riemann Curvature at a point of a given space-time for the orientation determined by two unit vectors is given by Weatherburn (1963). Katkar and Khairmode have obtained the expression for the Riemann Curvature at a point of a space-time for the orientation determined by either two real or two complex null vector fields. Using this formula we have obtained in section-7 that $K=-[\frac{2M}{r^3}-\frac{3}{2}(\frac{Q^{*2}}{r^2})]$ as the curvature at a point of the Reissener-Nordstorm space-time for the orientation determined by two real vector fields; while $K=\frac{-2M}{r^3}+\frac{Q^{*2}}{r^4}$ for the orientation determined by two complex vector fields. It is noted that the Riemann Curvature at a point of a space-time for the orientation determined by one real and one complex null vector fields is indeterminate. The ultimate goal of the thesis is to study the geometry of the world line of the time like particle.Section-8 deals with the geometry of the world-line of the time like particle in the Reissener-Nordstorm space-time. It is observed that

$$K=-\left(\frac{Mr-Q^{*2}}{r^2\sqrt{\Delta}}\right),$$

$$T=0$$

and

$$B=0 .$$

It is shown that the world line in the space-time is a torsion free plane curve.

Chapter-III is devoted to the study of the geometry of the world line of the time like particle in Bertotti-Robertson space-time. Accordingly components of the tetrad vectors relative to the Bertotti-Robertson space-time are discussed in Section-2.In section-3, the Bertotti-Robertson space-time is described in Newman-Penrose formalism. In Section-4 we cite the forty Christffel symbols with respect to the given space-time. In the next Section-5, the 12 complex spin coefficients are determined. It is shown that the Bertotti-Robertson space-time is of Petrov-type D. Out of twenty independent components of Riemann curvature tensor only two are non-vanishing with respect to Bertotti-Robertson space-time and are presented in Section-6. In section-7,we have derived $\mathrm{K}=\frac{5}{e^2}$ and $\mathrm{K}=\frac{-1}{e^2}$ as the expressions for the Riemann curvature at a point of the Bertotti-Robertson space-time for the orientation determined by two real or two complex null vector fields respectively. In the last section-8, the expressions for the curvature field K, the torsion field T and the bitorsion field B of the world line of the time-like particle in the Bertotti-Robertson space-time are determined. It is observed that

$$\mathrm{K}=\frac{-1}{\sqrt{2}}\left(\frac{e}{\sqrt{2}r^2}+\frac{3}{\sqrt{2}e}\right),$$

$$\mathrm{T}=0,$$

$$\mathrm{B}=0.$$

Consequently, the world line Bertotti-Robertson space-time is a torsion free plane curve.

In chapter-IV, investigation of the geometry of the world line of the time-like particle in the Robertson-Walker space-time is made in the same line of the previous chapters. Accordingly, tetrad components relative to the Robertson-Walker space-time are described in the section-2. Robertson-Walker space-time in the Newman-Penrose formalism is presented in the Section-3. Eleven non-vanishing Christoffel symbols and twenty nine vanishing Christoffel symbols with respect to the space-time are determined in Section-4. Petrov-type D characteristic of the Robertson-Walker space-time is investigated by determining the 12 complex spin coefficients in section-5. Section-6 deals with the investigation of the twenty independent components of Riemann Curvature tensor, out of which only seven are non-vanishing and remaining eleven vanish. Using these components of Riemann curvature tensor, the expressions

$$\mathrm{K} = \frac{-s''(t)}{4s(t)} \text{ and}$$

$$\mathrm{K} = \frac{\left[f'^2(r) - f^2(r)s'^2(t) - 1\right]}{\left(s^2(t)f^2(r)\right)}$$

for the Riemann curvature at a point of the Robertson-Walker space-time for the orientation determined by two real or two complex null vector fields are evaluated in Section-7. in the section-8, it is shown that the world line of the time-like particle in the Robertson-Walker space-time is just a straight line.