C H A P T E R - I

NOTIONS AND CONCEPTS.

"..... the electromagnetic
field - even in the null case
leaves such a distinctive
imprint on the geometry in
which it lies that the
electromagnetic field follows
from the geometry"

- GERDCH (1965).

1. INTRODUCTION

The existance of intense magnetic field in inter-stellar objects justifies the study of electromagnetic phenomena in cosmic physics. The decisive importance of electromagnetic field in inter-stellar space and inter-galactic space necessiciates the development of magneto-hydrodynamics (MHD) to Relativistic magnetohydrodynamics (RMHD). This explains the effects of electromagnetic fields on the motion of charged particles in space (Alfv. 1963).

The rapid progress in the branch of RMHD can be visualised by the work of Lichnerowicz (1967), who gave an elegant account of the basic equations for RMHD, with reference to the existance and uniqueness of solutions. The other contributors in this field are Yodzis (1971), DeBray (1972), Date (1973), Shaha (1973). Maugin (1972) generalised the Lichnerowicz's formalism to the form of a self-consistent scheme incorporating electromagnetic field interacting with matter field. In various successive series of papers, Maugin and Eringen (1972 a), Maugin and Eringen (1972 b), Maugin and Eringen (1972 c) have invastigated various aspects of RMHD. The evolution of supermassive relativistic objects like pulsars, neutron stars at the end of thermo-nuclear reactions show ferro-magnetic properties. Newringer and Rosenweig (1964) gave synthesis of the ferro-fluid. A classical concept of steady state in RMHD has introduced by

Yodzis (1971) in order to obtain a relativistic analog of ferraro's theorem. The classical concept as considered by Yodzis is modified by Banerji (1974). Cissiko (1978) has established the general relativistic equations characterising the ferro fluid. G. Prasad (1980) has discussed the implication of an electric counterpart of ferraro's law of isorotation in the case of pulsars. Ray and Banerji (1982) studied the growth of magnetic energy density in the ferro fluid in collapsing state.

These works have prompted us to study the properties of relativistic perfect ferro fluid with infinite electrical conductivity and variable magnetic permeability (F-magnetofluid).

The weak conservation laws and law of isorotation pertaining to the space time of F-magnetofluid are the main streams of the dissertation work.

2. Stress-energy tensor for ferro magneto fluid :

To exploit the properties of the relativistic perfect fluid with infinite electrical conductivity and constant magnetic permeability, Lichnerowicz (1967) used the stress energy tensor

$$T^{ab} = (\varrho + p + \mu H^2) U^a U^b - (p + \tfrac{1}{2} \mu H^2) g^{ab} - \mu H^a H^b . \quad \ldots(2.1)$$

The modified form of the stress-energy tensor for the polarised magnetised perfect fluid with infinite electrical conductivity and constant magnetic permeability established through action principle is given by Maugin (1972) in the form,

$$T^{ab}_{(em)} = - H^a B^b + B_c H^c h^{ab} - \frac{1}{2} B_c B^c g^{ab} , \qquad \ldots (2.2)$$

where

B^a is the magnetic induction vector,

h^{ab} is the projection operator given as,

$$h^{ab} = g^{ab} - U^a U^b . \qquad \ldots (2.3)$$

H^a is the magnetic field vector,

U^a is the flow vector.

with

$$U^a U_a = 1, \qquad \ldots (2.4)$$

$$H^a H_a = - H^2 , \qquad \ldots (2.5)$$

$$U^a H_a = 0 \qquad \ldots (2.6)$$

If we consider the ferro fluid then the magnetic induction vector, magnetic field vector and polarization vector are related by (Cissoko, 1978).

$$B = H + M, \quad M = \chi B. \qquad \ldots (2.7)$$

The ferro's fluid is suppose to have isotropic properties, so we write,

$$B = \mu H. \qquad \ldots (2.8)$$

where $\mu = (1 - x)^{-1}$ is variable magnetic permeability.

So with the help of equation (2.8), we consider the relativistic perfect fluid with infinite electrical conductivity and variable magnetic permeability described by the stress energy tensor (2.2) as given by

$$T^{ab}_{(em)} = \mu \left[\left\{ \left(\frac{\mu}{2} - 1 \right) g^{ab} + U^a U^b \right\} H^2 - H^a H^b \right] . \quad \ldots (2.9)$$

This is the stress-energy tensor for electromagnetic field.

We consider the total stress-energy tensor in the form

$$T^{ab} = T^{ab}_{(pf)} + T^{ab}_{(em)} \quad \ldots (2.10)$$

where $T^{ab}_{(pf)}$ is the stress-energy tensor for perfect fluid given as,

$$T^{ab}_{(pf)} = (\rho + p) U^a U^b - p g^{ab}. \quad \ldots (2.11)$$

Here, ρ is the matter density and p is the isotropic pressure.

With the help of equations (2.9) and (2.11) we rewrite equation (2.10) as,

$$T^{ab} = (\rho + p + \mu H^2) U^a U^b - \left[p + \mu \left(1 - \frac{\mu}{2} \right) H^2 \right] g^{ab} - \\ - \mu H^a H^b. \quad \ldots (2.12)$$

If we set

$$\frac{1}{2}\,\mu H^2 = m, \qquad \ldots (2.13)$$

then we put equation (2.12) in the form

$$T^{ab} = (\rho + p + 2m)\, U^a U^b - \left[p + 2m - m\mu\right] g^{ab} - \mu H^a H^b. \qquad \ldots (2.14)$$

This form of stress energy tensor characterizes the properties of infinitely conducting ferro-magneto fluid with variable magnetic permeability. Hereafter throughout the work we write ferro-magnetofluid as F-magnetofluid denoted by stress energy tensor (2.14).

3. Contractions of stress-energy tensor T^{ab} :

We perform successive contractions of stress-energy tensor (2.14) characterising the F-magnetofluid as follows.

$$T^{ab}\, U_a = (\rho + m\mu)\, U^b. \qquad \ldots (3.1)$$

By using condition (2.4) we get,

$$T^{ab}\, U_a U_b = (\rho + m\,\mu). \qquad \ldots (3.2)$$

From equations (2.5) and (2.6) we obtain,

$$T^{ab}\, H_a = -\,(p + 2m - m\mu) H^b + \mu H^2 H^b\,. \qquad \ldots (3.3)$$

Also by contracting equation (3.3) by H_b we get,

$$T^{ab} H_a H_b = (P - m \mu) H^2 . \qquad \ldots (3.4)$$

It follows from equations (3.2) and (3.4), the time-like eigen-value is $(\varrho + m\mu)$ and the space-like eigen-value is $(p - m\mu)H^2$.

The contraction of equation (3.3) with U_b furnishes,

$$T^{ab} H_a U_b = 0. \qquad \ldots (3.5)$$

The trace of the stress-energy tensor T^{ab}(2.14) is,

$$T = T^{ab} g_{ab} = \left[\varrho - 3p - 4m (1 - \mu) \right] . \qquad \ldots (3.6)$$

We transvect the stress-energy tensor T^{ab} (2.14) with the projection operator h_{ab} and use equations (2.4) and (2.6) we get,

$$T^{ab} h_{ab} = - (3p + 4m - 3\mu m). \qquad \ldots (3.7)$$

4. Energy Conditions :

We discuss below the energy conditions to be satisfied by the F-magnetofluid.

(i) The WEAK ENERGY conditions state that the stress-energy tensor T^{ab} at each point obeys the inequality condition,

$$T^{ab}\, U_a U_b \geqslant 0 . \qquad \ldots (4.1)$$

In case of F-magnetofluid we have from equation (4.1)

$$(\rho + m\,\mu) \geqslant 0. \qquad \ldots (4.2)$$

From this, we conclude that the total internal energy density of the F-magnetofluid is non-negative.

(ii) The well-known ENERGY CONDITION (Hawking and Ellis, 1968) implies that,

$$T^{ab}\, U_a U_b - \frac{1}{2} T \geqslant 0 . \qquad \ldots (4.3)$$

It is called as stress-energy condition to be satisfied by the stress-energy tensor. For the given stress-energy tensor of the F-magnetofluid, we have

$$\frac{1}{2} (\rho + 3p - 2m\,\mu + 4m) \geqslant 0,$$

$$\text{i.e., } (\rho + 3p) \geqslant 2(m\,\mu - 2m). \qquad \ldots (4.4)$$

5. Kinematics of fluid flow :

The term "kinematical" quantities have definite dynamical implications for the time development of the gravitational field.

The kinematical parameters associated with the time-like congruence U^a of the fluid flow according to Greenberg (1970) are,

(i) The expansion parameter

$$\theta = U^a{}_{;a} , \qquad \ldots (5.1)$$

(ii) The symmetric shear tensor

$$\sigma_{ab} = U_{(a;b)} - \dot{U}_{(a}U_{b)} + \frac{1}{3}\,\theta\, h_{ab}. \qquad \ldots (5.2)$$

(iii) The anti-symmetric rotation tensor

$$\omega_{ab} = U_{[a;b]} - \dot{U}_{[a}\, U_{b]} . \qquad \ldots (5.3)$$

where the term,

$$\dot{U}_a = U_{a;b}\, U^b . \qquad \ldots (5.4)$$

is known as the acceleration.

We have projection operator

$$h_{ab} = g_{ab} - U_a U_b \, . \qquad \ldots (5.5)$$

with the properties,

$$h^a_a = 3 \, , \; h_{ab} \, U^b = 0. \qquad \ldots (5.6)$$

This shows that h_{ab} is 3-space projection operator which is orthogonal to fluid flow. The shear tensor and the rotation tensor are trace free.

i.e., $$\sigma^a_a = 0 = \omega^a_a \, . \qquad \ldots (5.7)$$

Also by expressions (5.2) and (5.3) we have

$$\sigma_{ab} \, U^b = \omega_{ab} \, U^b = 0. \qquad \ldots (5.8)$$

On the same line, it follows from the unitary character of the flow vector that,

$$\dot{U}_a \, U^a = 0 \, . \qquad \ldots (5.9)$$

The invariants of these tensors are defined as,

$$\sigma_{ab} \, \sigma^{ab} = 2 \, \sigma^2 \, , \qquad \ldots (5.10)$$

$$\omega_{ab} \, \omega^{ab} = 2 \, \omega^2 \, . \qquad \ldots (5.11)$$

6. Field equations for F-magnetofluid :

(i) Equations of motion :

The fundamental equations in General Relativity are the Einstein's field equations

$$R_{ab} - \frac{1}{2} R\, g_{ab} = - K\, T_{ab}, \qquad \ldots (6.1)$$

where

R_{ab} is the Ricci tensor,

T_{ab} is the energy-momentum tensor,

K is the gravitational constant.

Ernstein's field equations describe how the gravitational field raps up the space-time surrounding the matter but do not give any information about the motion of objects in the space-time.

We use the Einstein's field equations (6.1) to write the Ricci tensor in the form of the stress-energy tensor as given by,

$$R_{ab} = - K \left(T_{ab} - \frac{1}{2K} R g_{ab} \right). \qquad \ldots (6.2)$$

In order to get Ricci scalar we inner multiply equation (6.2) by g^{ab},

$$R = kT. \qquad \ldots (6.3)$$

Finally by substituting the value of stress-energy tensor T^{ab} from equation (2.14) we write the expression for Ricci tensor as,

$$R_{ab} = -K \left[(\varrho + p + 2m)\, U_a U_b - \frac{1}{2} (\varrho - p + 2m\mu) g_{ab} - \right.$$

$$\left. - \mu H_a H_b \right]. \qquad \ldots (6.4)$$

(ii) Maxwell equations :

In Relativistic magnetohydrodynamics (RMHD) the 4-current J^a is not determineable, hence we have the only one applicable set of Maxwell equations (Lichnerowicz, 1967),

$$\left[\mu \,(H^a U^b - U^a H^b)\right]_{;b} = 0, \qquad \ldots (6.5a)$$

i.e. $$\mu\,(H^a U^b_{;b} + H^a_{;b}\, U^b - U^a_{;b}\, H^b - U^a H^b_{;b} +$$

$$+ \mu_{;b}\,(H^a U^b - U^a H^b) = 0. \qquad \ldots (6.5b)$$

Contractions of equation (6.5b) with U_a and H_a and making use of equations (2.4), (2.5), (2.6) we get,

$$\mu\,(\dot{U}_a H^a + H^b{}_{;b}) + \mu_{;b}\, H^b = 0. \qquad \ldots (6.6)$$

$$\mu (H^2\, \theta + \frac{1}{2} \dot{H}^2 + U^a_{;b}\, H_a H^b) + H^2\, \hat{\mu} = 0,$$

i.e. $$\mu (H^2\, \theta + \frac{1}{2} \dot{H}^2 + U_{a;b}\, H^a H^b) + H^2\, \dot{\mu} = 0. \qquad \ldots (6.7)$$

(iii) Heat equations :

The interrelation connecting the thermodynamical variables for polarised and magnetised fluid due to Maugin (1972) as

$$T_o \, dS = d\epsilon + p d \left(1/ \rho_o \right) - \frac{1}{2 \rho_o} \mu \, (1-\mu) dH^2 \quad \ldots(6.8)$$

where,

ρ_o is the proper material density,

ϵ is the internal energy density,

S is the specific entropy,

T_o is the rest temperature,

p is the isotropic pressure.

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