C H A P T E R - I

TETRAD FORMALISM IN NULL VECTOR FIELDS

Introduction :

In the 4-dimensional space time continuum of the general Theory of Relativity various types of tetrads are available in literature. The most prominent among these formalisms is one which is proposed by Newman and Penrose in 1962, which consists of four 'invisible vectors'- in the sense that, their magnitudes vanish.

The Einstein field equations can also be formulated in different forms from the standard tensorial one (Carmeli 1982). One of these forms, which proved to be very powerful and useful in seeking exact solutions is the null tetrad formulation of Newman and Penrose. Number of articles based on this formulism are presented in the '*Abstracts of Contributed paper*' - the 9th International Conference on General Relativity and Gravitation held in July 1980 at Jena, (Germany) (Ed. Schmutzer 1980) in the four recent books, Cosmology and Gravitation (Ed. Bergmann and Sabbata 1980), General Relativity and Gravitation (Ed. Held 1980), General Relativity - An Einstein Centenary Survey, from Cambridge (Ed. Hawking and Israel 1979) and *Gravitation Quanta and the Universes* - An Einstein Centenary Symposium held at Ahmedabad (Ed. Prasanna, Narlikar, Vishveshwara 1980) bears testimony to its incredible influence. This powerful technique is used in the present dissertation to exploit the various geometrical and dynamical aspects of the space time corresponding to the charged perfect fluid in null electromagnetic field.

2. An Exposition of the N.P. formalism :

The spectacular success of the Newman Penrose formalism is due to

(i) Its suitability for computational work,

(ii) Its efficiency in making the Einstein equations transparent.

(iii) Its thorough utilisation of the Bianchi identities.

Computational Advantages :

In 1962 Newman and Penrose have introduced a complex null tetrad

$$Z^a_{(i)} = (l^a, n^a, m^a, \bar{m}^a), \; i = 1, 2, 3, 4.$$

where l^a, n^a are two real null vector fields and m^a, $\bar{m}^a$ can overhead bar indicates the complex conjugate) are complex null vectors. They satisfy the pseudo-normal relations

$$l_a n^a = 1,$$
$$m_a \bar{m}^a = -1. \qquad \ldots (2.1)$$

and the orthogonal relations

$$l_a m^a = l_a \bar{m}^a = m_a n^a = \bar{m}_a n^a = 0. \qquad \ldots (2.2)$$

together with self orthogonality relations

$$l_a l^a = n_a n^a = m_a m^a = \bar{m}_a \bar{m}^a = 0. \qquad \ldots (2.3)$$

defining their null character.

Thus out of ten inner products of the four vector fields, as many as eight vanish in N.P.formalism. This establishes the computational advantages of this formalism over the others.

The relationship between this tetrad and the geometry of space time is given by the 'Completeness relation'

$$g_{ab} = l_a n_b + n_a l_b - m_a \bar{m}_b - \bar{m}_a m_b. \qquad \ldots (2.4)$$

3. The Adaptability of the N.P. formalism :

The accessibility of this null formalism to non-null vector fields is easy; for example, the unit time like vector field u^a can be written either as

$$u^a = (2)^{-\frac{1}{2}} (l^a + n^a) \qquad \ldots (3.1)$$

or as

$$u^a = (2)^{-1/2} (m^a - \bar{m}^a) \qquad \ldots (3.2)$$

where $u^a u_a = 1$

The unit space like vector field u^a can be expressed as

$$h^a = (2)^{-1/2} (m^a + \bar{m}^a) \qquad \dots (3.3)$$

or as

$$h^a = (2)^{-1/2} (l^a - n^a) \qquad \dots (3.4)$$

or as

$$h^a = - i (2)^{-1/2} (m^a - \bar{m}^a) \qquad \dots (3.5)$$

because $h^a h_a = -1$.

4. Simplified version of Einstein field equations

Instead of the tensor quantities, their physical components with respect to the null tetrad are utilised and this makes the Einstein's field equations more transparent. A prominent exemplar of this efficiency is in the representation of the Riemann-Christofell curveture tensor in terms of five Weyl scalar functions, the six Ricci scalar functions and one curveture scalar. This formalism provides an algorithm for computing the curveture tensor of space time in a form suitable for a variety of applications, while at the same time providing additional information pertaining to the geometry of space time (Katkar, 1981).

Transparency Of The Curveture Tensor And The Five Weyl Scalars:

Let C_{abcd} is the Weyl conformal curveture tensor. We have the defining expression for this tensor

$$R_{abcd} = C_{abcd} - \frac{1}{2}(g_{ad} R_{bc} - g_{ac} R_{bd} + g_{bc} R_{ad} - g_{bd} R_{ac}) +$$

$$+ \frac{R}{6}(g_{ad} g_{bc} - g_{ac} g_{bd}) \qquad \ldots (4.1)$$

where $R_{ab} = R^{c}_{abc}$ is the Ricci tensor and $R = R^{a}_{a}$ is the scalar curvature. The free gravitational part of the curvature tensor R_{abcd} is Weyl tensor C_{abcd}.

The Weyl- tensor C_{abcd} in N.P. formalism is expressed by (Campbell and Wainwrite, 1977)

$$C_{abcd} = R_e \cdot \Big[- 2\psi_0 U_{ab} U_{cd} + 4\psi_1 (U_{ab} M_{cd} + M_{ab} U_{cd}) -$$

$$- 2\psi_2 (u_{ab} V_{cd} + 4M_{ab} M_{cd} + V_{ab} U_{cd}) +$$

$$+ 4\psi_3 (V_{ab} M_{cd} + M_{ab} V_{cd}) - 2\psi_4 V_{ab} V_{cd} \Big] , \ldots (4.2)$$

where $U_{ab} = 2\bar{m}_a n_b$,

$V_{ab} = 2 l_a m_b$,

and $M_{ab} = l_a n_b - m_a \bar{m}_b$.

Then the tetrad components of C_{abcd} are

$$\psi_0 = - C_{abcd}\, l^a m^b l^c m^d,$$

$$\psi_1 = - C_{abcd}\, l^a n^b l^c m^d,$$

$$\psi_2 = - C_{abcd}\, \bar{m}^a n^b l^c m^d,$$

$$\Psi_3 = - C_{abcd}\, \bar{m}^a n^b l^c n^d ,$$

$$\Psi_4 = - C_{abcd}\, \bar{m}^a n^b \bar{m}^c n^d \qquad \ldots (4.3)$$

Intrinsic Derivative Operators :

We have the four operators D, Δ, δ, $\bar{\delta}$ defined by

$$D\phi = \phi_{;a} l^a , \qquad \Delta\phi = \phi_{;a} n^a ,$$

$$\delta\phi = \phi_{;a} m^a , \qquad \bar{\delta}\phi = \phi_{;a} \bar{m}^a . \qquad \ldots (4.4)$$

Spin Coefficients : This formalism combines 24 Ricci rotation coefficients in to 12 complex spin coefficients. The Ricci rotation coefficients are defined through

$$\gamma_{ijk} = z_{ib;a}\, z_j^b\, z_k^a . \qquad \ldots (4.5)$$

which are antisymmetric in first two indices. In N.P. formalism these are known as spin coefficients which are defined as follows

$$k = l_{a;b}\, m^a l^b , \quad \rho = l_{a;b}\, m^a \bar{m}^b ,$$

$$\sigma = l_{a;b}\, m^a m^b , \quad \tau = l_{a;b}\, m^a n^b ,$$

$$\nu = - n_{a;b}\, \bar{m}^a n^b , \quad \mu = - n_{a;b}\, \bar{m}^a m^b ,$$

$$\lambda = - n_{a;b}\, \bar{m}^a \bar{m}^b , \quad \pi = - n_{a;b}\, \bar{m}^a l^b ,$$

$$\alpha = 1/2\, (l_{a;b}\, n^a \bar{m}^b - m_{a;b}\, \bar{m}^a \bar{m}^b),$$

$$\beta = 1/2\, (l_{a;b}\, n^a m^b - m_{a;b}\, \bar{m}^a m^b),$$

$$\gamma = 1/2\,(l_{a;b}\, n^a n^b - m_{a;b}\, \bar{m}^a\, n^b\,),$$

$$\epsilon = 1/2\,(l_{a;b}\, n^a l^b - m_{a;b}\, \bar{m}^a\, l^b\,)\,. \qquad \ldots (4.6)$$

<u>Ricci Scalars</u> :

The enumeration of the ten scalars which are just the tetrad components of Ricci tensor R_{ab} and R are given by

$$\phi_{00} = -\,1/2\; R_{ab}\; l^a l^b,$$

$$\phi_{01} = -\,1/2\; R_{ab}\; l^a m^b,$$

$$\phi_{02} = -\,1/2\; R_{ab}\; m^a m^b,$$

$$\phi_{10} = -\,1/2\; R_{ab}\; l^a\, \bar{m}^b,$$

$$\phi_{11} = -\,1/4\; R_{ab}\; (l^a n^b + m^a \bar{m}^b\,),$$

$$\phi_{12} = -\,1/2\; R_{ab}\; n^a m^b,$$

$$\phi_{20} = -\,1/2\; R_{ab}\; \bar{m}^a\, \bar{m}^b,$$

$$\phi_{21} = -\,1/2\; R_{ab}\; n^a\, \bar{m}^b\,,$$

$$\phi_{22} = -\,1/2\; R_{ab}\; n^a\, n^b\,,$$

$$\Lambda = 1/24\;\; R. \qquad \ldots (4.7)$$

<u>Commutator Relations</u> :

The commutator ratios formed out of intrinsic derivatives are :

$$[\Delta, D]\,\phi = (\gamma + \bar{\gamma})\,D\phi - (\bar{\tau} + \pi)\,\delta\,\phi - (\tau + \bar{\pi})\,\bar{\delta}\,\phi \; +$$

$$+ (\epsilon + \bar{\epsilon})\,\Delta\,\phi,$$

$$[\delta, \Delta]\,\phi = -\,\nu\,D\,\phi + (\mu - \gamma + \bar{\gamma})\,\delta\phi + \bar{\lambda}\,\bar{\delta}\phi + (\tau - \alpha - \beta)\,\Delta\,\phi,$$

$$[\delta, D]\,\phi = (\bar{\alpha} + \beta - \bar{\pi})\,D\phi - (\bar{\rho} + \epsilon - \bar{\epsilon})\,\delta\phi - \sigma\,\bar{\delta}\phi + k\,\Delta\phi,$$

$$[\bar{\delta}, \delta]\,\phi = (\bar{\mu} - \mu)\,D\phi - (\bar{\beta} - \alpha)\,\delta\phi - (\bar{\alpha} - \beta)\,\bar{\delta}\,\phi + (\bar{\rho} - \rho)\,\Delta\,\phi. \qquad \ldots (4.8)$$

where $[\delta\,,\,D] = \delta\,D - D\,\delta$.

By leballing the values like (Carmeli 1977, p.332)

$$Dx^a = x^a{}_{;b}\,l^b = l^a,$$

$$\delta\,x^a = x^a{}_{;b}\,m^b = m^a,$$

$$\bar{\delta}\,x^a = x^a{}_{;b}\,\bar{m}^b = \bar{m}^a,$$

$$\Delta\,x^a = x^a{}_{;b}\,n^b = n^a\,. \qquad \ldots (4.9)$$

We derive the metric equation

$$\Delta l^a - Dn^a = (\gamma + \bar{\gamma})\,l^a - (\bar{\tau} + \pi)\,m^a - (\tau + \bar{\pi})\,\bar{m}^a \; +$$

$$+ (\epsilon + \bar{\epsilon})\,n^a,$$

$$\delta\,l^a - Dm^a = (\bar{\alpha} + \beta - \bar{\pi})\,l^a - (\bar{\rho} + \epsilon - \bar{\epsilon})\,m^a - \sigma\,\bar{m}^a + kn^a,$$

$$\delta\,n^a - \Delta m^a = -\,\bar{\nu}\,l^a + (\mu - \gamma + \bar{\gamma})\,m^a + \lambda\,\bar{m}^a + (\tau - \bar{\alpha} - \beta)\,n^a,$$

$$\bar{\delta}\,m^a - \delta\,\bar{m}^a = (\bar{\mu} - \mu)\,l^a - (\bar{\beta} - \alpha)\,m^a - (\bar{\alpha} - \beta)\,\bar{m}^a + (\bar{\rho} - \rho)\,n^a. \qquad \ldots (4.10)$$

<u>The Ricci Identities</u> :

$$Y^{i}_{a;bc} - Y^{i}_{a;cb} = Y^{i}_{d}\, R_{abc}{}^{d} ,$$

can be written in terms of rotation coefficients

$$R_{hijk} = \gamma_{hij;k} - \gamma_{hik;j} - \gamma_{hmj}\, \gamma_{i}{}^{m}{}_{k} + \gamma_{hmk}\, \gamma_{i}{}^{m}{}_{j} + \gamma_{hi}{}^{m} \left(\gamma_{mjk} - \gamma_{mkj} \right). \qquad \ldots (4.11)$$

These generate the following eighteen N.P. equations

$$(NP_1) : D\rho - \bar\delta \kappa = (\rho^2 + \sigma\bar\sigma) + (\epsilon + \bar\epsilon)\rho - \bar\kappa\tau + \kappa (3\alpha + \bar\beta - \pi) + \Phi_{00}.$$

$$(NP_2) : D\sigma - \delta \kappa = (\rho + \bar\rho)\sigma + (3\epsilon - \bar\epsilon)\sigma + (\tau - \bar\pi + \bar\alpha + 3\beta)\kappa + \Psi_0.$$

$$(NP_3) : D\tau - \Delta \kappa = (\tau + \bar\pi)\rho + (\bar\tau + \pi)\sigma + (\epsilon - \bar\epsilon) - \kappa (3\gamma + \bar\gamma) + \Psi_1 + \Phi_{01}.$$

$$(NP_4) : D\alpha - \bar\delta \epsilon = (\rho + \bar\epsilon - 2\epsilon)\alpha + \beta\bar\sigma - \bar\beta\epsilon - \kappa\lambda - \bar\kappa\gamma + (\epsilon + \rho)\pi + \Phi_{10}.$$

$$(NP_5) : D\beta - \delta \epsilon = (\alpha + \pi)\sigma + (\bar\rho - \bar\epsilon)\beta - (\mu + \gamma)\kappa - (\bar\alpha - \bar\pi) + \Psi_1 .$$

$$(NP_6) : D\gamma - \Delta \epsilon = (\tau + \bar\pi)\alpha + (\bar\tau + \pi)\beta - (\epsilon + \bar\epsilon)\gamma - (\gamma + \bar\gamma)\epsilon + \tau\pi -)\kappa + \Psi_2 - \Lambda + \Phi_{11}.$$

$$(NP_7) : D\lambda - \bar{\delta}\pi = (\rho\lambda + \bar{\sigma}\mu) + \pi^2 + (\alpha - \bar{\beta})\pi - \nu\bar{k} -$$

$$- (3\epsilon - \bar{\epsilon})\lambda + \Phi_{20}.$$

$$(NP_8) : D\mu - \delta\pi = (\bar{\rho}\mu + \sigma\lambda) + \pi\bar{\pi} - (\epsilon + \bar{\epsilon})\mu -$$

$$- \pi(\bar{\alpha} - \beta) - \nu k + \Psi_2 + 2\Lambda .$$

$$(NP_9) : D\nu - \Delta\pi = (\pi + \bar{\tau})\mu + (\bar{\pi} + \tau)\lambda + (\gamma - \bar{\gamma})\pi -$$

$$- (3\epsilon + \bar{\epsilon})\nu + \Psi_3 + \Phi_{21}.$$

$$(NP_{10}) : \Delta\lambda - \bar{\delta}\nu = - (\mu + \bar{\mu})\lambda - (3\gamma - \bar{\gamma})\lambda +$$

$$+ (3\alpha + \bar{\beta} + \pi - \bar{\tau})\nu - \Psi_4.$$

$$(NP_{11}) : \delta\rho - \bar{\delta}\sigma = \rho(\bar{\alpha} + \beta) - \sigma(3\alpha - \bar{\beta}) + (\rho - \bar{\rho}) +$$

$$+ (\mu - \bar{\mu})k - \Psi_1 + \Phi_{01}.$$

$$(NP_{12}) : \delta\alpha - \bar{\delta}\beta = (\mu\rho - \lambda\sigma) + \alpha\bar{\alpha} + \beta\bar{\beta} - 2\alpha\beta + \gamma(\rho - \bar{\rho}) +$$

$$+ \epsilon(\mu - \bar{\mu}) - \Psi_2 + \Lambda + \Phi_{11}.$$

$$(NP_{13}) : \delta\lambda - \bar{\delta}\mu = (\rho - \bar{\rho})\nu + (\mu - \bar{\mu})\pi + (\bar{\alpha} + \beta)\mu +$$

$$+ \lambda(\bar{\alpha} - 3\beta) - \Psi_3 + \Phi_{21}.$$

$$(NP_{14}) : \delta\nu - \Delta\mu = (\mu^2 + \lambda\bar{\lambda}) + (\gamma + \bar{\gamma})\mu - \bar{\nu}\pi +$$

$$+ (\tau - 3\beta - \bar{\alpha})\nu + \Phi_{22}.$$

$$(NP_{15}) : \delta\gamma - \Delta\beta = (\tau - \bar{\alpha} - \beta)\gamma + \mu\tau - \sigma\nu - \epsilon\bar{\nu} + \alpha\bar{\lambda} -$$

$$- \beta(\gamma - \bar{\gamma} - \mu) + \Phi_{12} .$$

$$(NP_{16}) : \delta\tau - \Delta\sigma = (\mu\sigma + \bar{\lambda}\rho) + (\tau + \beta - \bar{\alpha})\tau - $$

$$- (3\gamma - \bar{\gamma})\sigma - k\bar{\nu} + \phi_{02},$$

$$(NP_{17}) : \Delta\rho - \bar{\delta}\tau = -(\rho\bar{\mu} + \sigma\lambda) + (\bar{\beta} - \alpha - \bar{\tau})\tau +$$

$$+ (\gamma + \gamma)\rho + \nu k - \Psi_2 - 2\Lambda,$$

$$(NP_{18}) : \Delta\alpha - \bar{\delta}\gamma = (\rho + \epsilon)\nu - (\tau + \beta)\lambda + (\bar{\gamma} - \mu)\alpha +$$

$$+ (\bar{\beta} - \bar{\tau})\gamma - \Psi_3 . \qquad \ldots (4.12)$$

<u>The Bianchi Identities</u> : The Bianchi identities

$$R_{abcd;e} + R_{abde;c} + R_{abec;d} = 0, \qquad \ldots (4.13)$$

in terms of Ricci rotation coefficients will give rise to the following relations in N.P. version.

$$(B_1) : \delta\Psi_0 - D\Psi_1 + D\phi_{01} - \delta\phi_{00} = (4\alpha - \pi)\Psi_0 -$$

$$- 2\Psi_1(2\rho + \epsilon) + 3k\Psi_2 + (\bar{\pi} - 2\bar{\alpha} - 2\beta)\phi_{00} +$$

$$+ 2(\epsilon + \bar{\rho})\phi_{01} + 2\sigma\phi_{10} - 2k\phi_{11} - \bar{k}\phi_{02},$$

$$(B_2) : \Delta\Psi_0 - \delta\Psi_1 + D\phi_{02} - \delta\phi_{01} = (4\gamma - \mu)\Psi_0 -$$

$$- 2(2\tau + \beta)\Psi_1 + 2\sigma\Psi_2 - \bar{\lambda}\phi_{00} + 2(\bar{\pi} - \beta)\phi_{01} +$$

$$+ 2\sigma\phi_{11} + (2\epsilon - 2\bar{\epsilon} + \bar{\rho})\phi_{02} - 2k\phi_{12},$$

$$(B_3) : 3(\bar{\delta}\Psi_1 - D\Psi_2) + 2(D\phi_{11} - \delta\phi_{10}) + \delta\phi_{01} - \Delta\phi_{00} =$$

$$= 3\lambda\Psi_0 - 9\rho\Psi_2 + 6(\alpha - \pi)\Psi_1 + 6k\Psi$$

$$+(\bar{\mu} - 2\mu - 2\gamma - 2\bar{\gamma})\phi_{00} + (2\alpha + 2\pi + 2\bar{\tau})\phi_{01} +$$

$$+ 2(\tau - 2\bar{\alpha} + \bar{\pi})\phi_{10} + 2(2\bar{\rho} - \rho)\phi_{11} +$$

$$+ 2\sigma\phi_{20} - \bar{\sigma}\phi_{02} - 2\bar{k}\phi_{12} - 2k\phi_{21}.$$

$$(B_4) : 3(\Delta\psi_1 - \delta\psi_2) + (D\phi_{12} - \delta\phi_{11}) + (\bar{\delta}\phi_{02} - \Delta\phi_{01}) =$$

$$= 3\nu\psi_0 + 6(\gamma - \mu)\psi_1 - 9\tau\psi_2 + 6\sigma\psi_3 -$$

$$- \nu\phi_{00} + 2(\mu - \mu - \gamma)\phi_{01} - 2\bar{\lambda}\phi_{10} + 2(\tau + 2\bar{\pi})\phi_{11} +$$

$$+ (2\alpha + 2\pi + \bar{\tau} - 2\bar{\beta})\phi_{02} + (2\bar{\rho} - 2\rho - 4\bar{\epsilon})\phi_{12} +$$

$$+ 2\sigma\phi_{21} - 2k\phi_{22}.$$

$$(B_5) : 3(\bar{\delta}\psi_2 - D\psi_3) + D\phi_{21} - \delta\phi_{20} + 2(\bar{\delta}\phi_{11} - \Delta\phi_{10}) =$$

$$= 6\lambda\psi_1 - 9\pi\psi_2 + 6(\epsilon - \rho)\psi_3 + 3k\psi_4 -$$

$$- 2\nu\phi_{00} + 2\lambda\phi_{01} + 2(\bar{\mu} - \mu - 2\tau)\phi_{10} +$$

$$+ (2\pi + 4\bar{\tau})\phi_{11} + (2\beta + 2\tau + \bar{\pi} - 2\bar{\alpha})\phi_{20} -$$

$$- 2\bar{\sigma}\phi_{12} - \bar{k}\cdot\phi_{22} + 2(\bar{\rho} - \rho - \epsilon)\phi_{21}.$$

$$(B_6) : 3(\Delta\psi_2 - \delta\psi_3) + D\phi_{22} - \delta\phi_{21} + 2(\bar{\delta}\phi_{12} - \Delta\phi_{11}) =$$

$$= 6\nu\psi_1 - 9\mu\psi_2 + 6(\beta - \tau)\psi_3 + 3\sigma\psi_4 -$$

$$- 2\nu\phi_{01} - 2\bar{\nu}\phi_{10} + 2(2\bar{\mu} - \mu)\phi_{11} + 2\lambda\phi_{02} -$$

$$- \bar{\lambda}\phi_{20} + 2(\pi + \bar{\tau} - 2\bar{\beta})\phi_{12} + 2(\beta + \tau + \pi)\phi_{21} +$$

$$+ (\bar{\rho} - 2\epsilon - 2\bar{\epsilon} - 2\rho)\phi_{22}.$$

$$(B_7) : \bar{\delta}\Psi_3 - D\Psi_4 + \bar{\delta}\phi_{21} - \Delta\phi_{20} = 3\lambda\Psi_2 - 2(\alpha+2\pi)\Psi_3 + (4\epsilon - \rho)\Psi_4 - 2\nu\phi_{10} + 2\lambda\phi_{11} + (2\gamma - 2\bar{\gamma} + \bar{\mu})\phi_{20} + 2(\bar{\tau} - \alpha)\phi_{21} - \bar{\sigma}\phi_{22} ,$$

$$(B_8) : \Delta\Psi_3 - \delta\Psi_4 + \bar{\delta}\phi_{22} - \Delta\phi_{21} = 3\nu\Psi_2 - 2(\gamma+2\mu)\Psi_3 + (4\beta - \tau)\Psi_4 - 2\nu\phi_{11} - \bar{\nu}\phi_{20} + 2\lambda\phi_{12} + 2(\gamma + \bar{\mu})\phi_{21} + (\bar{\tau} - 2\bar{\beta} - 2\alpha)\phi_{22}.$$

$$(B_9) : D\phi_{11} - \delta\phi_{10} - \bar{\delta}\phi_{01} + \Delta\phi_{00} + 3D\Lambda = (2\gamma - \mu + 2\bar{\gamma} - \bar{\mu})\phi_{00} + (\pi - 2\alpha - 2\bar{\tau})\phi_{01} + (\bar{\pi} - 2\bar{\alpha} - 2\tau)\phi_{10} + 2(\rho + \bar{\rho})\phi_{11} + \bar{\sigma}\phi_{02} + \sigma\phi_{20} - \bar{\kappa}\phi_{12} - \kappa\phi_{21}.$$

$$(B_{10}) : D\phi_{12} - \delta\phi_{11} - \bar{\delta}\phi_{02} + \Delta\phi_{01} + 3\delta\Lambda = (2\gamma - \mu - 2\mu)\phi_{01} + \bar{\nu}\phi_{00} - \bar{\lambda}\phi_{10} + 2(\bar{\pi} - \tau)\phi_{11} + (\pi + 2\bar{\beta} - 2\alpha - \bar{\tau})\phi_{02} + (2\rho + \bar{\rho} - 2\bar{\epsilon})\phi_{12} + \sigma\phi_{21} - \kappa\phi_{22}.$$

$$(B_{11}) : D\phi_{22} - \delta\phi_{21} - \bar{\delta}\phi_{12} + \Delta\phi_{11} + 3\Delta\Lambda = \nu\phi_{01} + \bar{\nu}\phi_{10} - 2(\mu + \bar{\mu})\phi_{11} - \lambda\phi_{02} - \bar{\lambda}\phi_{20} + (2\pi - \bar{\tau} + 2\beta)\phi_{12} + (2\beta - \tau + 2\bar{\pi})\phi_{21} + (\rho + \bar{\rho} - 2\epsilon - 2\bar{\epsilon})\phi_{22}.$$

... (4.13)

<u>Maxwell's Scalars</u> :

The tetrad components of electromagnetic field tensor F_{ab} in N.P. formalism are given by the three complex scalars:

$$\phi_0 = F_{ab}\, l^a m^b ,$$

$$\phi_1 = \frac{1}{2} F_{ab}\, (l^a n^b + \bar{m}^a m^b) ,$$

$$\phi_2 = F_{ab}\, \bar{m}^a n^b . \qquad \ldots (4.14)$$

By making use of these scalars the N.P. version of the electromagnetic bivector takes the form (Debney and Zund, 1971)

$$F_{ab} = -2\, R_e\, \phi_1 \; l_{[a} n_{b]} + 2i\, I_m\, \phi_1\, m_{[a} \bar{m}_{b]} + \phi_2\, l_{[a} m_{b]} +$$

$$+ \bar{\phi}_2\, l_{[a} \bar{m}_{b]} - \bar{\phi}_0\, n_{[a} m_{b]} - \phi_0\, n_{[a} \bar{m}_{b]} . \qquad \ldots (4.15)$$

<u>Electromagnetic Field Equations</u> :

The Maxwell's equations in the presence of source for the electromagnetic field

$$F^{ab}{}_{;b} = J^a ,$$

$$F_{[ab;c]} = 0 . \qquad \ldots (4.16)$$

We rewrite these equations in N.P. formalism as follows :

$$D\, \phi_1 - \bar{\delta}\, \phi_0 = (\pi - 2\alpha)\phi_0 + 2\, \rho\, \phi_1 - k\, \phi_2 + \frac{1}{2}\, I_0 ,$$

$$\delta\, \phi_2 - \Delta\, \phi_1 = -\, \nu\, \phi_0 + 2\mu\, \phi_1 + (\tau - 2\beta)\, \phi_2 + \frac{1}{2}\, I_2 ,$$

$$\delta\, \phi_1 - \Delta\, \phi_0 = (\mu - 2\gamma)\phi_0 + 2\,\tau\, \phi_1 - \sigma\, \phi_2 + \frac{1}{2}\, I_1 .$$

$$D\phi_2 - \bar{\delta}\,\phi_1 = -\lambda\,\phi_0 + 2\pi\,\phi_1 + (\rho - 2\epsilon)\,\phi_2 + \frac{1}{2} I_1 \quad \ldots (4.17)$$

$$DI_2 + \Delta I_0 - \delta\bar{I}_1 - \bar{\delta} I_1 = (\gamma + \bar{\gamma} - \mu - \bar{\mu})\, I_0 + (\bar{\beta} + \pi - \alpha - \bar{\tau}) I_1 + c.c. + (\rho + \bar{\rho} - \epsilon - \bar{\epsilon})\, I_2 \quad \ldots (4.18)$$

<u>The Stress Energy Momentum Tensor For Electromagnetic Field</u> :

The energy momentum tensor for a source free electromagnetic field is

$$T_{ab} = \frac{1}{4}\, g_{ab}\, F_{cd}\, F^{cd} - F_{ac}\, F_b{}^c \quad \ldots (4.19)$$

This in terms of complex null tetrad Z^a, and Maxwell scalars ϕ_A, the above expression takes the form

$$T_{ab} = \frac{1}{2}\Big[\phi_2\bar{\phi}_2\, l_a l_b + \phi_0\bar{\phi}_0\, n_a n_b + \bar{\phi}_0\phi_2\, m_a m_b + \phi_0\bar{\phi}_2\, \bar{m}_a\bar{m}_b\Big] + \phi_1\phi_1\Big[l_{(a}n_{b)} + m_{(a}\bar{m}_{b)}\Big] - \bar{\phi}_1\phi_2\, l_{(a}m_{b)} - \phi_1\bar{\phi}_2\, l_{(a}\bar{m}_{b)} - \bar{\phi}_0\phi_1\, n_{(a}m_{b)} - \phi_0\bar{\phi}_1\, n_{(a}\bar{m}_{b)} \quad \ldots (4.20)$$

<u>The Stress Energy Tensor For Null Electromagnetic Field</u> :

For null electromagnetic field described through mathematical conditions

$$F_{ab}\, F^{ab} = 0 = F_{ab}\, F^{*ab} \quad \ldots (4.21)$$

We can reduce the form of the stress energy tensor for electromagnetic field (Gumaste 1984) as

$$E_{ab} = \frac{1}{2} |\phi|^2 \; l_a l_b , \qquad \ldots (4.22)$$

where $\phi_{22} = \phi \bar{\phi} = |\phi|^2 .$ $\qquad \ldots (4.23)$

5. <u>Special Choice of Flows</u> :

Some special types of flows characterized by through restrictions on the kinematical parameters associated with the time like flow vector are listed below (the restrictions are written in N.P. version).

(A) <u>Geodesic Flow</u> : This flow is governed by the condition

$$G_1 = G_4 - G_3 = 0 \qquad \ldots (5.1)$$

In N.P. version it can be written as

$$\epsilon + \bar{\epsilon} + \gamma + \bar{\gamma} = 0,$$

$$\pi + \nu - \bar{\kappa} - \bar{\tau} = 0. \qquad \ldots (5.2)$$

(B) <u>The Time like Killing Flow</u> :

The time like killing flow is characterised by

$$G_{11} = G_{12} = 0,$$

$$G_{31} + G_{32} = 0,$$

$$G_{32} + G_{41} = 0,$$

$$G_{21} + G_{21} = 0,$$

$$G_5 = 0.$$

In N.P. version we can write

$$\epsilon + \bar{\epsilon} = \gamma - \bar{\gamma} = 0,$$

$$\bar{\tau} - \nu + \alpha + \bar{\beta} = 0,$$

$$\alpha + \bar{\beta} + \pi - \bar{\kappa} = 0,$$

$$\rho + \bar{\rho} - \mu - \mu = 0,$$

$$\bar{\sigma} - \lambda = 0. \qquad \ldots (5.4)$$

(C) <u>The Rigid Flow</u> : The Rigid flow follows the necessary conditions

$$G_{11} - G_{12} = 0,$$

$$G_{21} + \bar{G}_{21} = 0,$$

$$G_{31} + G_{41} + 2\bar{G}_{32} = 0,$$

$$G_5 = 0. \qquad \ldots (5.5)$$

In N.P. formalism it can be written as

$$\epsilon + \bar{\epsilon} - \gamma - \bar{\gamma} = 0,$$

$$\rho + \bar{\rho} - \mu - \bar{\mu} = 0,$$

$$\tau - \nu + \pi - \bar{\kappa} + 2\alpha + 2\bar{\beta} = 0,$$

$$\bar{\sigma} - \lambda = 0. \qquad \ldots (5.6)$$

APPENDIX

(A_1) N.P. Concomitants of Covariant Derivatives of Null Congruences :

Covariant derivatives of null congruences are expressed in terms of spin coefficients

$$
\begin{aligned}
l_{a;b} = {} & (\gamma + \bar{\gamma}) l_a l_b + (\epsilon + \bar{\epsilon}) l_a n_b + \Big[-(\alpha + \bar{\beta}) l_a m_b - \\
& - \bar{\tau} m_a l_b + \bar{\sigma} m_a m_b + \bar{\rho} m_a \bar{m}_b - \bar{k} m_a n_b \Big] + \\
& + \text{C.C.,}
\end{aligned} \qquad \ldots (A_{1.1})
$$

$$
\begin{aligned}
n_{a;b} = {} & \Big[\nu m_a l_b - \lambda m_a m_b - \mu m_a \bar{m}_b + \pi m_a n_b \Big] + \text{C.C.} \\
& - (\gamma + \bar{\gamma}) n_a l_b - (\epsilon + \bar{\epsilon}) n_a n_b + (\alpha + \bar{\beta}) n_a m_b + \text{C.C.}
\end{aligned} \qquad \ldots (A_{1.2})
$$

$$
\begin{aligned}
m_{a;b} = {} & \bar{\nu} l_a l_b - \bar{\mu} l_a m_b - \bar{\lambda} l_a \bar{m}_b + \bar{\pi} l_a n_b + (\gamma - \bar{\gamma}) m_a l_b + \\
& + (\bar{\beta} - \alpha) m_a m_b + (\bar{\alpha} - \beta) m_a \bar{m}_b + (\epsilon - \bar{\epsilon}) m_a n_b - \\
& - \tau n_a l_b + \rho n_a m_b + \sigma n_a \bar{m}_b - k n_a n_b ,
\end{aligned} \qquad \ldots (A_{1.3})
$$

$$
\begin{aligned}
\bar{m}_{a;b} = {} & \nu l_a l_b - \lambda l_a m_b - \mu l_a \bar{m}_b + \pi l_a n_b + (\bar{\gamma} - \gamma) \bar{m}_a l_b + \\
& + (\alpha - \bar{\beta}) \bar{m}_a m_b + (\beta - \bar{\alpha}) \bar{m}_a \bar{m}_b + (\bar{\epsilon} - \epsilon) \bar{m}_a n_b - \\
& - \tau n_a l_b + \bar{\sigma} n_a m_b + \bar{\rho} n_a \bar{m}_b - \bar{k} n_a n_b .
\end{aligned} \qquad \ldots (A_{1.4})
$$

(A_2) The Divergence of the Null Congruences :

$$l^{a}{}_{;a} = \epsilon + \bar{\epsilon} - \rho - \bar{\rho} , \qquad \ldots (A_{2.1})$$

$$n^{a}{}_{;a} = \mu + \bar{\mu} - \gamma - \bar{\gamma} , \qquad \ldots (A_{2.2})$$

$$m^{a}{}_{;a} = \bar{\pi} - \bar{\alpha} + \beta - \tau \qquad \ldots (A_{2.3})$$

$$\bar{m}^{a}{}_{;a} = \pi - \alpha + \bar{\beta} - \bar{\tau} \qquad \ldots (A_{2.4})$$

(A_3) Intrinsic Derivatives of the Congruences of the N.P. tetrad :

(i) The intrinsic derivatives of the Congruence l_a :

$$Dl_a = l_{a;b}\, l^b = (\epsilon + \bar{\epsilon})l_a - \bar{\kappa} m_a - \kappa \bar{m}_a , \qquad \ldots (A_{3.1})$$

$$\delta l_a = l_{a;b} m^b = (\bar{\alpha} + \beta)l_a - \bar{\rho} m_a - \sigma \bar{m}_a , \qquad \ldots (A_{3.2})$$

$$\bar{\delta}\, l_a = l_{a;b}\, \bar{m}^b = (\alpha + \bar{\beta})l_a - \rho\, \bar{m}_a - \bar{\sigma}\, m_a , \qquad \ldots (A_{3.3})$$

$$\Delta l_a = l_{a;b}\, n^b = (\gamma + \bar{\gamma})l_a - \bar{\tau}\, m_a - \tau\, \bar{m}_a . \qquad \ldots (A_{3.4})$$

(ii) The intrinsic derivatives of the congruence m_a :

$$Dm_a = m_{a;b}\, l^b = \bar{\pi}\, l_a + (\epsilon - \bar{\epsilon})m_a - \kappa n_a , \qquad \ldots (A_{3.5})$$

$$\delta m_a = m_{a;b}\, m^b = \bar{\lambda}\, l_a - (\bar{\alpha} - \beta)m_a - \sigma n_a , \qquad \ldots (A_{3.6})$$

$$\bar{\delta} m_a = m_{a;b}\, \bar{m}^b = \bar{\mu}\, l_a - (\bar{\beta} - \alpha)m_a - \rho\, n_a , \qquad \ldots (A_{3.7})$$

$$\Delta\, m_a = m_{a;b}\, n^b = \bar{\nu}\, l_a + (\gamma - \bar{\gamma})m_a - \tau\, n_a \qquad \ldots (A_{3.8})$$

(iii) The intrinsic derivatives of the Congruence $\bar{m}_a$:

$$D\bar{m}_a = \bar{m}_{a;b}\, l^b = \pi l_a + (\bar{\epsilon} - \epsilon)\bar{m}_a - \bar{\kappa}\, n_a , \qquad \ldots (A_{3.9})$$

$$\delta\bar{m}_a = \bar{m}_{a;b}\, m^b = \mu l_a + (\bar{\alpha} - \beta)\bar{m}_a - \bar{\rho}\, n_a , \qquad \ldots (A_{3.10})$$

$$\bar{\delta}\bar{m}_a = \bar{m}_{a;b}\bar{m}^b = \lambda\, l_a - (\alpha - \bar{\beta})\bar{m}_a - \bar{\sigma}\, n_a , \qquad \ldots (A_{3.11})$$

$$\Delta\, \bar{m}_a = \bar{m}_{a;b}\, n^b = \nu\, l_a + (\bar{\gamma} - \gamma)\bar{m}_a - \bar{\tau}\, n_a . \qquad \ldots (A_{3.12})$$

(iv) The intrinsic derivatives of the congruence n_a :

$$Dn_a = n_{a;b}\, l^b = \pi\, m_a + \bar{\pi}\, \bar{m}_a - (\epsilon + \bar{\epsilon})n_a , \qquad \ldots (A_{3.13})$$

$$\delta n_a = n_{a;b}\, m^b = \mu\, m_a + \bar{\lambda}\, \bar{m}_a - (\epsilon + \bar{\epsilon})n_a , \qquad \ldots (A_{3.14})$$

$$\bar{\delta} n_a = n_{a;b}\, \bar{m}^b = \lambda\, m_a + \bar{\mu}\, \bar{m}_a - (\alpha + \bar{\beta})n_a , \qquad \ldots (A_{3.15})$$

$$\Delta\, n_a = n_{a;b}\, n^b = \nu\, m_a + \bar{\nu}\, \bar{m}_a - (\gamma + \bar{\gamma})n_a \qquad \ldots (A_{3.16})$$

(A_4) Projections of the Covariant Derivatives of the Null Congruences :

(i) Projections of $l_{a;b}$:

$$l^a l_{a;b} = 0 , \qquad \ldots (A_{4.1})$$

$$m^a l_{a;b} = \tau\, l_b - \rho\, m_b - \sigma\, \bar{m}_b + k n_b , \qquad \ldots (A_{4.2})$$

$$\bar{m}^a\, l_{a;b} = \bar{\tau}\, l_b - \bar{\rho}\, m_b - \bar{\sigma}\, \bar{m}_b + k n_b , \qquad \ldots (A_{4.3})$$

$$n^a l_{a;b} = (\gamma + \bar{\gamma})l_b - (\alpha + \bar{\beta})m_b - (\bar{\alpha} + \beta)\, \bar{m}_b + (\epsilon + \bar{\epsilon})\, n_b . \qquad \ldots (A_{4.4})$$

(ii) <u>Projections of</u> $m_{a;b}$:

$$l^a m_{a;b} = -\tau\, l_b + \rho\, m_b + \sigma\, \bar{m}_b - k\, n_b, \qquad \ldots (A_{4.5})$$

$$m^a m_{a;b} = 0, \qquad \ldots (A_{4.6})$$

$$\bar{m}^a m_{a;b} = -(\gamma - \bar{\gamma})\, l_b - (\bar{\beta} - \alpha)\, m_b - (\bar{\alpha} - \beta)\, \bar{m}_b - (\epsilon - \bar{\epsilon})\, n_b, \qquad \ldots (A_{4.7})$$

$$n^a m_{a;b} = \nu\, l_b - \bar{\mu}\, m_b - \bar{\lambda}\, \bar{m}_b + \bar{\pi}\, n_b \qquad \ldots (A_{4.8})$$

(iii) <u>Projections of</u> $\bar{m}_{a;b}$

$$l^a \bar{m}_{a;b} = -\bar{\tau}\, l_b + \bar{\rho}\, \bar{m}_b + \quad + \bar{\sigma}\, m_b - \bar{k}\, n_b, \qquad \ldots (A_{4.9})$$

$$m^a \bar{m}_{a;b} = -(\bar{\gamma} - \gamma)\, l_b - (\beta - \bar{\alpha})\, \bar{m}_b - (\alpha - \bar{\beta})\, m_b - (\bar{\epsilon} - \epsilon)\, n_b, \qquad \ldots (A_{4.10})$$

$$\bar{m}^a \bar{m}_{a;b} = 0, \qquad \ldots (A_{4.11})$$

$$n^a \bar{m}_{a;b} = \nu\, l_b - \mu\, \bar{m}_b - \lambda\, m_b + \pi\, n_b. \qquad \ldots (A_{4.12})$$

(iv) <u>Projections of</u> $n_{a;b}$:

$$l^a n_{a;b} = -(\gamma + \bar{\gamma})\, l_b + (\alpha + \bar{\beta})\, m_b + (\bar{\alpha} + \beta)\, \bar{m}_b - (\epsilon + \bar{\epsilon})\, n_b, \qquad \ldots (A_{4.13})$$

$$m^a n_{a;b} = -\bar{\nu}\, l_b + \bar{\mu}\, m_b + \bar{\lambda}\, \bar{m}_b - \bar{\pi}\, n_b, \qquad \ldots (A_{4.14})$$

$$\bar{m}_a n_{a;b} = -\nu\, l_b + \lambda\, m_b + \mu\, \bar{m}_b - \pi\, n_b, \qquad \ldots (A_{4.15})$$

$$n^a n_{a;b} = 0. \qquad \ldots (A_{4.16})$$

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