CHAPTER - II

Charged Perfect Fluid in Null Electromagnetic Field

1. Introduction :

The properties of relativistic null fluid are investigated by Vaidya (1951), while the universes filled with relativistic dust distribution are studied by Ozsvath (1966). The work of Bonnor and Vaidya (1970) is mainly concerned only with relativistic charged distribution with a null current. The spherical symmetry is used in the work of Vaidya (1951) whereas, the homogeneous space times are utilised in the work of Ozsvath. The study of stationary null electromagnetic field coexisting with dust distribution viewed with the angle of pure radiation fields with propogation vectors as killing vectors is due to Banarji (1970). A study of orthonormal congruences in charged dust is carried out by Gumaste in 1984. This study has become feasible due to a tetrad whose members have immediate physical significance. This work also explores the Fermi Walker transport of the stress tensors of certain known fluid distributions.

The main aim of this Chapter is to project some characteristic features of the space time filled with charged perfect fluid coexisting with null electromagnetic field. The well known Newman Penrose formalism emboiding a complex null tetrad is used in the text of this Chapter.

The Section 2 deals with the form of the stress energy tensor of charged perfect fluid in null electromagnetic field under the specific choice of time like vector expressible in terms of real null tetrad vectors. The comprehensive decomposition of this stress energy tensor is also given in this section. The Ricci tensor and its N.P. concomitants together with Ricci scalars are presented in Section 3. The Maxwell's equations and the relevant consequences are discussed in Section 4. In Section 5 we give Ricci identities and N.P. equations relevant to the charged perfect fluid in null electromagnetic field. The next Section 6 contains Bianchi identities and associated consequences by keeping in view the space time of charged perfect fluid in null electromagnetic field. The energy balance equations and equations of continuity are derived in the last Section 7.

2. The form of stress energy tensor :

The special choice for the flow vector :

The time like flow vector u^a which is unitary in character is expressible through tetrad vectors as their linear combination. We mainly use the two expressions given by

$$\text{(A)} \quad u^a = 2^{-1/2} (l^a + n^a) \qquad \dots (2.1)$$

$$\text{(B)} \quad u^a = 2^{-1/2} (m^a + \bar{m}^a) \qquad \dots (2.2)$$

Note that the simultaneous use of these two expressions for the time like vector u^a is ambiguous. We mainly use the type (A) in all further derivations.

Stress Energy Tensor for Null Electromagnetic Field :

By the choice of expression (2.1) for the flow vector the expression (4.22) of Chapter I for the stress energy tensor for null electromagnetic field is given by,

$$T_{ab}\,(em) = \frac{1}{2}\,|\phi|^2\, l_a l_b\,. \qquad \ldots (2.3)$$

The stress energy tensor for perfect fluid :

The well known form of the stress energy tensor for relativistic perfect fluid is

$$T_{ab}\,(p.f\,) = (\,\rho^* + p)\, u_a u_b - p g_{ab} \qquad \ldots (2.4)$$

where p is the isotropic pressure,

ρ^* is the proper energy density.

This form for the expression (2.1) becomes

$$T_{ab} = \frac{1}{2}\,(\rho^* + p)\, l_a l_b + (\,\rho^* + p)\, n_a n_b +$$

$$+ 2(\,\rho^* - p)\, l_{(a} n_{b)} + 4p\, m_{(a}\bar{m}_{b)} \qquad \ldots (2.5)$$

Now by coupling together the expressions (2.3) and (2.5) for the stress energy tensor for null electromagnetic field and relativistic perfect fluid we write the final form of the stress energy tensor of charged perfect fluid

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in null electromagnetic field given by

$$T_{ab} = \frac{1}{2} \; A \, l_a l_b + B \, n_a n_b + C \, l_{(a} n_{b)} + 4P \, m_{(a} \bar{m}_{b)} \qquad \ldots .(2.6)$$

$$\text{where } A = (\tfrac{1}{4} |\phi|^2 + \rho^* + p), \qquad \ldots (2.7)$$

$$B = (\rho^* + p), \qquad \ldots (2.8)$$

$$C = 2 \; (\rho^* - p). \qquad \ldots (2.9)$$

This form (2.6) is used throughout the future calculations.

<u>The decomposition of stress energy tensor</u> :

Following Grot and Eringen (1966) we have the decomposition of the stress energy tensor in the form

$$T_{ab} = \bar{T} \, u_a u_b + T'_a \, u_b + T''_b \, u_a + \frac{1}{T_{ab}} \, , \qquad \ldots (2.10)$$

where

$$\bar{T} = T_{ab} \, u^a u^b \, , \qquad \ldots (2.11)$$

$$T'_a = T_{\frac{1}{a} c} \, u^c \, , \qquad \ldots (2.12)$$

$$T''_b = T_{c \frac{1}{b}} \, u^c \, , \qquad \ldots (2.13)$$

$$\frac{1}{T_{ab}} = T_{\frac{1}{a} \frac{1}{b}} \, . \qquad \ldots (2.14)$$

Note that,

$$T_{\frac{1}{c} c} = h^b_a \; T_{bc}$$

$$= (\delta^b_a - u^b u_a) T_{bc} \qquad \ldots (2.15)$$

The physical meanings of these components are as follows :

(i) Kinetic Energy Momentum tensor :

The tensor $\bar{T} u_a u_b$ appearing in the above expression is the kinetic energy momentum. $\bar{T}$ is then mean energy density.

Note that $\bar{T} \neq T = T^a_a$.

(ii) Heat flow vector :

The 4-vector T^1_a is the heat flow vector.

(iii) Non-mechanical momentum :

The 4-vector T''_a is called the non-mechanical momentum.

(iv) Relativistic Stress :

The 4-tensor T^{ab} is the relativistic stress tensor or pressure tensor. For the stress energy tensor T_{ab}, the heat flow vector and non-mechanical momentum are identical as it is symmetric.

The values of these components pertaining to charged perfect fluid in null electromagnetic field characterized by the stress energy tensor (2.6) are as follows

(i) Mean energy density :

$$\bar{T} = \frac{1}{4} |\phi|^2 + \rho^* \qquad \ldots (2.16)$$

(ii) Heat flow vector :

$$T'_a = -\frac{1}{4}|\phi|^2 \hat{s}_a \qquad \ldots (2.17)$$

(iii) Non-mechanical momentum :

$$T''_a = -\frac{1}{4}|\phi|^2 \hat{s}_a \qquad \ldots (2.18)$$

(iv) Pressure tensor :

$$T_{ab} = (\frac{1}{4}|\phi|^2 + p) \hat{s}_a \hat{s}_b + 2P\, m_{(a}\bar{m}_{b)} \qquad \ldots (2.19)$$

Therefore (2.10) can be written as

$$T_{ab} = (\rho^* + \frac{1}{4}|\phi|^2) u_a u_b - \frac{1}{2}|\phi|^2 \hat{s}_{(a}u_{b)} +$$

$$+ (\frac{1}{4}|\phi|^2 + p) \hat{s}_a \hat{s}_b + 2p\, m_{(a}\bar{m}_{b)} \qquad \ldots (2.20)$$

where

$$\hat{s}_a = \frac{\bar{s}}{|\bar{s}|} = 2^{-\frac{1}{2}} (n^a - l^a) \qquad \ldots (2.21)$$

3. Ricci tensor expression in N.P. version :

To write the value of Ricci tensor for charged perfect fluid in null electromagnetic field we use the expression (2.6) of the stress energy tensor in the Einstein's field equations given in conventions, so that we have the final expression for the Ricci tensor

$$R_{ab} = -2^{-1}\Big[(\,|\phi|^2 + \rho^* + p)\, l_a l_b + (\rho^* + p) n_a n_b + $$

$$+ 4p\, l_{(a} n_{b)} + 2(\rho^* - p)\, m_{(a}\bar{m}_{b)}\Big] \qquad \ldots (3.1)$$

From this expression we can find the value of the Ricci scaler,

$$R = (\rho^* - 3p). \qquad \ldots (3.2)$$

We utilize the Ricci tensor expression (3.1) and the Ricci scaler expression (3.2) to derive ten independent Ricci scalers as follows :

$$\phi_{00} = \frac{1}{4}(\rho^* + p), \qquad \ldots (3.3)$$

$$\phi_{01} = 0\,, \qquad \ldots (3.4)$$

$$\phi_{10} = 0\,, \qquad \ldots (3.5)$$

$$\phi_{02} = 0\,, \qquad \ldots (3.6)$$

$$\phi_{20} = 0, \qquad \ldots (3.7)$$

$$\phi_{11} = \frac{1}{8}(\rho^* + p), \qquad \ldots (3.8)$$

$$\phi_{12} = 0 \qquad \ldots (3.9)$$

$$\phi_{21} = 0 \qquad \ldots (3.10)$$

$$\phi_{22} = \frac{1}{4}(|\phi|^2 + \rho^* + p) \qquad \ldots (3.11)$$

$$\Lambda = \frac{1}{24}(\rho^* - 3p) \qquad \ldots (3.12)$$

4. Electromagnetic Field Equations :

Null electromagnetic field :

$$F_{ab} F^{ab} = F_{ab} F^{*ab} = 0 \qquad \ldots (4.1)$$

where * denotes the dual of the tensor. Under these conditions the equation (4.20) of Chapter I giving the value of electromagnetic field tensor in terms of Maxwell's scalers produces

$$\phi_0 = \phi_1 = 0, \qquad \phi_2 \neq 0 \qquad \ldots (4.2)$$

Hence the field tensor F_{ab} has the form

$$F_{ab} = \phi_2 \; l_{[a}m_{b]} + \bar{\phi}_2 \; l_{[a}\bar{m}_{b]} \qquad \ldots (4.3)$$

i.e. $$F_{ab} = \phi \; l_{[a}m_{b]} + \text{C.C.} \qquad \ldots (4.4)$$

where $\phi_2 = \phi$

Then by making use of Maxwell's equations

$$F^{ab}{}_{;b} = J^a$$

Provides the value of the electric current J in the form

$$J^a = I_2 \, l^a + I_0 n^a - \bar{I}_1 m^a - I_1 \bar{m}^a \, . \qquad \ldots (4.5)$$

Here the scalers I_0, I_1, I_2 are source scalers out of which I_0, I_2 are real and I_1, $\bar{I}_1$ are complex conjugates of each other. It follows from (4.5)

$$I_0 = J^a l_a \, , \qquad \ldots (4.6)$$

$$I_2 = J^a n_a \,, \qquad \ldots (4.7)$$

$$I_1 = J^a m_a \,, \qquad \ldots (4.8)$$

and

$$J^a J_a = 2(\, I_0 I_2 \; - I_1 \bar{I}_1). \qquad \ldots (4.9)$$

Thus when the electromagnetic field is null with the principle null direction l^a the N.P. version of Maxwell's equations is given by

$$D\phi - (\,\rho - 2\,\epsilon\,)\phi = \frac{1}{2}\,\bar{I}_1 \,, \qquad \ldots (4.10)$$

$$\delta\,\phi - (\tau - 2\beta\,)\,\phi = \frac{1}{2}\,I_2 \,, \qquad \ldots (4.11)$$

$$K\phi \;=\; \frac{1}{2}\,I_0 \,, \qquad \ldots (4.12)$$

$$\sigma\,\phi = \frac{1}{2}\,I_1 \,, \qquad \ldots (4.13)$$

and
$$\Delta\, I_0 + DI_2 - \delta\bar{I}_1 - \bar{\delta}I_1 = (\gamma + \bar{\gamma} - \mu - \bar{\mu}\,)\,I_0 + (\bar{\beta} - \alpha - \bar{\tau}\,)I_1 + (\beta-\alpha-\tau)\bar{I}_1 + (\,\rho + \bar{\rho}\,)I_2 \ldots . (4.14)$$

The electric current is always conservative therefore we write

$$J^a{}_{;a} = 0. \qquad \ldots (4.15)$$

This with equation (4.5) yields

$$I_{2;a}\, l^a + I_2 l^a{}_{;a} + I_{0;a}\, n^a + I_0 n^a{}_{;a} - \bar{I}_{1;a} m^a - \bar{I}_1 m^a{}_{;a} + \quad + I_{1;a}\, \bar{m}^a + I_1\, \bar{m}^a{}_{;a} = 0,$$

i.e. $(DI_2 + \Delta I_0 - \delta\bar{I}_1 + \bar{\delta}I_1) + (\epsilon + \bar{\epsilon} - \rho - \bar{\rho})\, I_2 +$

$+ I_0\, (\mu + \mu - \gamma - \bar{\gamma}) - \bar{I}_1\, (\bar{\pi} - \bar{\alpha} + \beta - \tau) +$

$+ I_1\, (\pi - \alpha + \bar{\beta} - \tau) = 0.$ ... (4.16)

Lorentz fource :

The general relativistic Lorentz fource K_b has the expression (Bressan 1978)

$$K_b = F_{ab}\, J^a \, . \qquad \text{... (4.17)}$$

If we use equation (4.5) in this, then the value of the Lorentz fource is given by

$$K_b = \frac{1}{2}\, \phi\, I_0 m_b + \frac{1}{2}\, \bar{\phi}\, I_0\, \bar{m}_b - \frac{1}{2}\, \bar{\phi}\, \bar{I}_1 l_b - \frac{1}{2}\, \phi I_1 l_b . \qquad \text{...(4.18)}$$

THEOREM (1)

The Lorentz fource generated by charged perfect fluid in null electromagnetic field vanishes if and only if $I_0 = 0$ and $\phi\, I_1 = \bar{\phi}\, \bar{I}_1$.

PROOF : Necessary Part : Let the Lorentz fource produced by the charged perfect fluid in null electromagnetic field be zero. This with (4.18) after contraction with $\bar{m}^b$ gives

$$\phi\, I_0 = 0,$$

i.e. $I_0 = 0, \quad \phi \neq 0$ ... (4.19)

Also by transvecting equation (4.18) with n^b we get

$$\phi \, I_1 = \bar{\phi} \; \bar{I}_1. \qquad \ldots (4.20)$$

Hence equations (4.19) and (4.20) give the necessary conditions.

<u>Sufficient part</u> :

Let $I_1 = 0$
and $\phi \, I_1 = \bar{\phi} \; \bar{I}_1$

These when substituted in the expression of Lorentz fource (4.18) we get

$$K_b = 0.$$

Thus the proof is complete.

<u>Freedom condition and Maxwell's equations</u> :

We start with a reasonable assumption that the null congruence l^a is geodesic. This implies that

$$l^a{}_{;b} \, l^b = 0. \qquad \ldots (4.21)$$

This in N.P. version becomes

$$(\epsilon + \bar{\epsilon}) \, l_a - \bar{K} \, m_a - K \, \bar{m}_a = 0. \qquad \ldots (4.22)$$

It follows from (4.22) that the necessary and sufficient conditions for the null congruence l^a to be geodesic are

$$\epsilon \; + \; \bar{\epsilon} = 0, \; K = 0. \qquad \ldots (4.23)$$

Hence for this geodesic null congruence the Maxwell's equation (4.12) gives

$$I_0 = 0.$$

This value when substituted in (4.9) we get

$$J^a J_a = -2\,|I_1|^2 . \qquad \ldots (4.24)$$

The non-zero source term is called a null source according as J^a is null. Hence for null source we have the condition

$$I_1 = 0, \quad I_2 \neq 0. \qquad \ldots (4.25)$$

The 4-current vector for such a null source is then can be written as

$$J^a = I_2\, l^a. \qquad \ldots (4.26)$$

Further the Maxwell equation (4.13) under the condition (4.25) provides the condition

$$\sigma \phi = 0$$

i.e. $\sigma = 0 \qquad \ldots (4.27)$

as $\phi \neq 0$

Thus the Maxwell's equations for the null electromagnetic field with null source are reduced to

$$k = \sigma = 0, \qquad \ldots (4.28)$$

$$D\phi - (\rho - 2\epsilon)\,\phi = 0, \qquad \ldots (4.29)$$

$$\delta\phi - (\tau - 2\beta)\,\phi = \frac{1}{2} I_2 . \qquad \ldots (4.30)$$

Note : (1) The value of Lorentz fource produced by charged perfect fluid in null electromagnetic field under source term as null becomes

$$K_b = 0 \qquad \ldots (4.31)$$

Hence the Lorentz fource vanishes.

Note :(2) The expression for the conservation of current described by (4.16) following the condition that, the source term is null produces the result

$$DI_2 + I_2 (- \rho - \bar{\rho}) = 0 \qquad \ldots (4.32)$$

5. N.P. Concomitants of Bianchi and Ricci Identities :

(A) Bianchi Identities :

The Bianchi identities for the charged perfect fluid in the null electromagnetic field using (3.3) to (3.12) are given as

$$(B_1) \quad \delta \psi_0 - D \psi_1 - \frac{1}{4} \delta(\rho^* + P) = (4\alpha-\pi) \psi_0 - 2(2 \rho + \tau) \psi_1 + 3K \psi_2 + \frac{1}{4} (\rho^* + P) (\bar{\pi} - 2\bar{\alpha} - 2\beta - K),$$

$$(B_2) \quad \Delta \psi_0 - \delta \psi_1 = (4\gamma - \mu) \psi_0 - 2(2 \tau + \beta) \psi_1 + 2 \sigma \psi_2 - \frac{1}{4} (\rho^* + P) (\bar{\lambda} - \sigma),$$

$$(B_3) \quad 3(\bar{\delta} \psi_1 - D \psi_2) + \frac{1}{4} D(\rho^* + P) - \frac{1}{4} \Delta (\rho^*+P) =$$

$$= 3\lambda\Psi_0 - 9\rho\Psi_2 + 6(\alpha - \pi)\Psi_1 + 6\kappa\Psi_3 +$$

$$+ \frac{1}{4}(\rho^* + P)(\bar{\mu} - 2\mu - 2\gamma - 2\bar{\gamma} + 2\bar{\rho} - \rho),$$

$$(B_4)\qquad 3(\Delta\Psi_1 - \delta\Psi_2) - \frac{1}{4}\delta(\rho^* + p) = 3\nu\Psi_0 + 6(\gamma-\mu)\Psi_1 -$$

$$- 9\tau\Psi_2 + 6\sigma\Psi_3 + \frac{1}{4}(\rho^* + P)(\tau + 2\bar{\pi} - \bar{\gamma} - 2\kappa) -$$

$$- \frac{1}{2}\kappa|\phi|^2 ,$$

$$(B_5)\qquad 3(\bar{\delta}\Psi_2 - D\Psi_3) + \frac{1}{4}\bar{\delta}(\rho^* + P) = 6\lambda\Psi_1 - 9\pi\Psi_2 +$$

$$+ 6(\epsilon - \rho)\Psi_3 + 3\kappa\Psi_4 + \frac{1}{4}(\rho^* + P)(\pi + 2\bar{\tau} - 2\nu - \bar{\kappa}) -$$

$$- \frac{1}{4}\bar{\kappa}|\phi|^2 ,$$

$$(B_6)\qquad 3(\Delta\Psi_2 - \delta\Psi_3) + \frac{1}{4}D(\rho^* + P + |\phi|^2) - \frac{1}{4}\Delta(\rho^* + P) =$$

$$= 6\nu\Psi_1 - 9\mu\Psi_2 + 6(\beta - \tau)\Psi_3 + 3\sigma\Psi_4 +$$

$$+ \frac{1}{4}(\rho^* + P)(2\mu - \mu + \bar{\rho} - 2\rho - 2\epsilon - 2\bar{\epsilon}) +$$

$$+ \frac{1}{4}|\phi|^2(\bar{\rho} - 2\rho - 2\epsilon - 2\bar{\epsilon}),$$

$$(B_7)\qquad \bar{\delta}\Psi_3 - D\Psi_4 = 3\lambda\Psi_2 - 2(\alpha + 2\pi)\Psi_3 + (4\epsilon - \rho)\Psi_4 +$$

$$+ \frac{1}{4}(\rho^* + P)(\lambda - \bar{\sigma}) - \frac{1}{4}\bar{\sigma}|\phi|^2 ,$$

$$(B_8)\qquad \Delta\Psi_3 - \delta\Psi_4 + \frac{1}{4}\bar{\delta}(|\phi|^2 + \rho^* + P) = 3\nu\Psi_2 -$$

$$- 2(\gamma + 2\mu)\Psi_3 + (4\beta - \tau)\Psi_4 +$$

$$+ \frac{1}{4} (\rho^* + P) (\bar{\tau} - 2\bar{\beta} - 2\alpha - \nu) +$$

$$+ \frac{1}{4} |\phi|^2 (\bar{\tau} - 2\bar{\beta} - 2\alpha),$$

(B_9) $$D(\rho^* - p) + \frac{1}{4} \Delta(\rho^* + P) = (\rho^* + P)(2\gamma + 2\gamma - \mu - \bar{\mu} + $$

$$+ \rho - \bar{\rho}),$$

(B_{10}) $$\delta (-2P) = (\rho^* + P) (\nu + \pi - \tau - K) - K |\phi|^2$$

(B_{11}) $$D(|\phi|^2 + \rho^* + P) + \Delta(\rho^* - P) = |\phi|^2(\rho + \bar{\rho} - 2\epsilon - 2\bar{\epsilon}) +$$

$$+ (\rho^* + p) (\rho + \bar{\rho} - \mu - \bar{\mu} - 2\epsilon - 2\bar{\epsilon}) .$$

<u>The Ricci identities and the 18 N.P. Equations</u> :

The Ricci identity

$$z^i{}_{a;bc} - z^i{}_{a;cb} = z^i{}_d R_{abc}{}^d ,$$

in terms of rotation coefficients becomes

$$R_{hijk} = \gamma_{hij;k} - \gamma_{hik;j} - \gamma_{hmj} \gamma_i{}^m{}_k + \gamma_{hmk} W_i{}^m{}_j +$$

$$+ \gamma_{hi}{}^m (\gamma_{mjk} - \gamma_{mkj}). \qquad \ldots (5.1)$$

We cite below a set of eighteen equations (known as N.P. equations) using equations (3.3) to (3.12)

(NP_1) : $$D \sigma - \bar{\delta} k = (\delta^2 + \sigma \bar{\sigma}) +(\epsilon + \bar{\epsilon}) \sigma - \bar{K} - K(3\alpha+\bar{\beta}-\pi) +$$

$$+ \frac{1}{4} (\sigma^* + P),$$

(NP_2) : $D\sigma - \delta k = (\rho + \bar{\rho})\sigma + (3\epsilon - \bar{\epsilon})\sigma -$

$- (\tau - \bar{\pi} + \bar{\alpha} + 3\beta)\kappa + \Psi_0,$

(NP_3) : $D\tau - \Delta\kappa = (\tau + \bar{\pi})\rho + (\bar{\tau} + \pi)\sigma + (\epsilon - \bar{\epsilon}) -$

$- \kappa\ (3\gamma + \bar{\gamma}) + \Psi_1,$

(NP_4) : $D\alpha - \bar{\delta}\epsilon = (\rho + \bar{\epsilon} - 2\epsilon)\alpha + \beta\bar{\sigma} - \bar{\beta}\epsilon - \kappa\lambda -$

$- \bar{\kappa}\gamma + (\epsilon + \rho)\pi,$

(NP_5) : $D\beta - \delta\epsilon = (\alpha + \pi)\sigma + (\bar{\rho} - \bar{\epsilon})\beta - (\mu + \gamma)\kappa -$

$- (\bar{\alpha} - \bar{\pi}) + \Psi_1,$

(NP_6) : $D\gamma - D\epsilon = (\tau + \bar{\pi})\alpha + (\bar{\tau} + \pi)\beta - (\epsilon + \bar{\epsilon})\gamma -$

$- (\gamma + \bar{\gamma})\epsilon + \tau\pi - \nu\kappa + \Psi_2 -$

$- \frac{1}{24}(\rho^* - p) + \frac{1}{8}(\rho^* + p),$

(NP_7) : $D\lambda - \bar{\delta}\pi = (\rho\lambda + \bar{\rho}\mu) + \pi^2 + (\alpha - \beta) - \nu k -$

$- (3\epsilon - \bar{\epsilon}),$

(NP_8) : $D\mu - \delta\pi = (\bar{\rho}\mu + \rho\lambda) + \pi\bar{\pi} - (\epsilon + \bar{\epsilon})\mu -$

$- \pi(\bar{\alpha} - \beta) - \nu k + \Psi_2 + \frac{1}{12}(\rho^* - 3p),$

$$(NP_9) : D\nu - \Delta\pi = (\pi + \bar{\tau})\mu + (\bar{\pi} + \tau)\lambda + (\gamma - \gamma)\pi -$$

$$- (3\epsilon + \bar{\epsilon})\nu + \Psi_3 .$$

$$(NP_{10}) : \Delta\lambda - \bar{\delta}\nu = -(\mu + \bar{\mu})\lambda - (3\gamma - \bar{\gamma})\lambda +$$

$$+ (3\alpha + \bar{\beta} + \pi - \tau)\nu - \Psi_4 .$$

$$(NP_{11}) : \delta\rho - \bar{\delta}\sigma = \rho(\bar{\alpha} + \beta) - \sigma(3\alpha - \bar{\beta}) + (\rho - \bar{\rho}) +$$

$$+ (\mu - \bar{\mu})k - \Psi_1 .$$

$$(NP_{12}) : \delta\alpha - \bar{\delta}\beta = (\mu\rho - \lambda\sigma) + \alpha\bar{\alpha} + \beta\bar{\beta} - 2\alpha\beta + \gamma(\rho - \bar{\rho}) +$$

$$+ ((\mu - \bar{\mu}) - \Psi_2 + \frac{1}{24}(\rho^* - 3p) +$$

$$+ \frac{1}{8}(\rho^* + p),$$

$$(NP_{13}) : \delta\lambda - \bar{\delta}\mu = (\rho - \bar{\rho})\nu + (\mu - \bar{\mu})\pi + \mu(\bar{\alpha} + \beta) +$$

$$+ \lambda(\bar{\alpha} - 3\beta) - \Psi_3 ,$$

$$(NP_{14}) : \delta\nu - \Delta\mu = (\mu^2 + \lambda\bar{\lambda}) + (\gamma + \bar{\gamma})\mu - \nu\pi +$$

$$+ (\tau - 3\beta - \bar{\alpha})) + \frac{1}{4}(\phi^2 + \rho^* + p),$$

$$(NP_{15}) : \delta\gamma - \Delta\beta = (\tau - \bar{\alpha} - \beta)\gamma + \mu\tau - \sigma\nu - \epsilon\nu -$$

$$- \beta(\gamma - \bar{\gamma} - \mu) + \alpha\bar{\lambda} ,$$

$$(NP_{16}) : \delta\tau - \Delta\sigma = (\mu\sigma + \bar{\lambda}\rho) + (\tau + \beta - \bar{\alpha}) -$$

$$- (3\gamma - \bar{\gamma})\sigma - K\nu ,$$

$$(NP_{17}) : \Delta\rho - \bar{\delta}\tau = -(\rho\bar{\mu} + \sigma\lambda) + (\bar{\beta} - \alpha - \bar{\tau})\tau + $$
$$+ (\gamma + \bar{\gamma})\rho + \nu\kappa - \psi_2 - \frac{1}{12}(\rho^* - 3p).$$

$$(NP_{18}) : \Delta\alpha - \bar{\delta}\gamma = (\rho + \epsilon)\nu - (\tau + \beta)\lambda + (\bar{\gamma} - \bar{\mu})\alpha +$$
$$+ (\bar{\beta} - \bar{\tau})\gamma - \psi_3.$$

Energy Balance Equation :

The identities

$$T^{ab}{}_{;b} = 0 \qquad \ldots (5.2)$$

for the charged perfect fluid yields the local 'energy balance' equations in the form

$$\Big[D(|\phi|^2 + \rho^* + p) + \Delta(\rho^* - p) + (\rho^* + p)(2\epsilon + 2\bar{\epsilon} - \rho -$$
$$- \bar{\rho} + \mu + \mu) + |\phi|^2 (2\epsilon + 2\epsilon - \rho - \bar{\rho})\Big] l^a +$$
$$+ \Big[D(\rho^* - p) + \Delta(\rho^* + p) + (\rho^* + p)(\mu + \bar{\mu} - 2\gamma - 2\bar{\gamma}) +$$
$$+ (\rho^* - p)(-\rho - \bar{\rho}) - 2p(\rho - \bar{\rho})\Big] n^a +$$
$$+ \Big[-\bar{\kappa}(\phi^2 + \rho^* + p) \pm (\rho^* - p)(\pi - \bar{\tau}) + (\rho^* + p)\nu +$$
$$+ 2\bar{\delta}p + 2p(\pi - \bar{\tau})\Big] m^a + \text{c.c.} = 0. \qquad \ldots (5.3)$$

The equation (5.3) gives the following relations :

$$\Big[D\,|\phi|^2 + \rho^* + p) + \Delta(\rho^* - p) + (\rho^* + p)(2\epsilon + 2\bar{\epsilon} - \rho - \bar{\rho} + \mu + \bar{\mu}) + |\phi|^2 (2\epsilon + 2\bar{\epsilon} - \rho - \bar{\rho})\Big] = 0. \quad \ldots (5.4)$$

$$D(\rho^* - p) + \Delta(\rho^* + p) + (\rho^* + p)(\mu + \bar{\mu} - 2\gamma - 2\bar{\gamma}) + (\rho^* - p)(-\rho - \bar{\rho}) - 2p(\rho - \bar{\rho}) = 0, \quad \ldots (5.5)$$

$$-\kappa(|\phi|^2 + \rho^* + p) + (\rho^* - p)(\pi - \bar{\tau}) + (\rho^* + p)\,\nu + 2\bar{\delta}\,p + 2p(\pi - \bar{\tau}) = 0. \quad \ldots (5.6)$$

<u>Equation of continuity</u> :

The equation of continuity

$$T^{ab}{}_{;b}\,u_a = 0. \quad .. (5.7)$$

in the N.P. version is given by

$$(D + \Delta)\rho^* + \rho^*(\mu + \bar{\mu} + \epsilon + \bar{\epsilon} - \rho - \bar{\rho} - \gamma - \bar{\gamma}) + p(\epsilon + \bar{\epsilon} + \mu + \bar{\mu} - \gamma - \bar{\gamma} - \rho + \bar{\rho}) + \frac{1}{2} D\,|\phi|^2 = 0. \quad \ldots (5.8)$$

<u>Consequences of freedom conditions</u> :

We recall from Section 4 the conditions for geodesic null coggruence as follows :

$$\epsilon + \bar{\epsilon} = \kappa = \sigma = 0$$
$$D\phi - (\rho - 2\epsilon)\phi = 0$$
$$\Delta\phi - (\tau - 2\beta)\phi = \frac{1}{2} I_2 \quad \ldots (5.9)$$

If we employ these conditions in the equations (5.4) to (5.8) emerging from energy balance equation and continuity equation we get

$$D\,(|\phi|^2 + \rho^* + p) + \Delta\,(\rho^* - p) + (\rho^* + p)(\mu + \bar\mu - \rho - \bar\rho) + + |\phi|^2\,(-\rho - \bar\rho) = 0, \qquad \ldots (5.10)$$

$$D\,(\rho^* - p) + \Delta\,(\rho^* + p) + (\rho^* + p)(\mu + \bar\mu - 2\gamma - 2\bar\gamma) + + (\rho^* - p)(-\rho - \bar\rho) - 2p\,(\rho - \bar\rho) = 0, \qquad \ldots (5.11)$$

$$(\rho^* - p)(\pi - \bar\tau) + (\rho^* + p)\,\nu + 2\bar\delta\,p + 2p(\pi - \bar\tau) = 0, \qquad \ldots (5.12)$$

$$(D + \Delta)\,\rho^* + \rho^*\,(\mu + \bar\mu - \rho - \bar\rho - \gamma - \bar\gamma) + p\,(\mu + \bar\mu - \gamma - \bar\gamma - - \rho + \bar\rho) + \frac{1}{2} D\,|\phi|^2 = 0. \qquad \ldots (5.13)$$

6. Special types of flows :

(a) Geodesic Flow : We recall the conditions in N.P. version from Chapter I necessary for geodesic flow.

$$G_1 = \epsilon + \bar\epsilon + \gamma + \bar\gamma = 0, \qquad \ldots (6.1)$$

$$G_4 - G_3 = \pi - \bar\kappa - \bar\tau + \nu = 0. \qquad \ldots (6.2)$$

The equation of continuity for this type of flow gives

$$(D + \Delta)\,\rho^* + \rho^*\,(\mu + \bar\mu - \rho - \bar\rho - 2\gamma - 2\bar\gamma) + P(\mu + \bar\mu - \rho - \bar\rho - 2\gamma - 2\gamma) = 0 \qquad \ldots (6.3)$$

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(b) <u>Killing Flow</u> : We write the necessary conditions for this flow by Chapter I

$$\epsilon + \bar{\epsilon} = \gamma + \bar{\gamma} = 0,$$

$$\bar{\tau} + \nu + \alpha + \bar{\beta} = 0,$$

$$\alpha + \bar{\beta} + \pi - \bar{k} = 0,$$

$$\bar{\rho} - \mu + \rho - \bar{\mu} = 0,$$

$$\bar{\sigma} - \lambda = 0. \qquad \ldots (6.4)$$

The equation of continuity for this type of flow then becomes

$$(D + \Delta)\,\rho^* + 2\,\bar{\rho}\,p + \frac{1}{2}\,D\,|\phi|^2 = 0$$

$$\text{i.e., } (D + \Delta)\,\rho^* = -2\,\rho^*\,p - \frac{1}{2}\,D\,|\phi|^2\,. \qquad \ldots (6.5)$$

This shows that ρ^* is invariant provided the variation in $|\phi|^2$ is proportional to isotropic pressure p of the fluid. Also the equation (5.4) produces

$$D\,(\,|\phi|^2 + \rho^* + p) + \Delta\,(\,\rho^* - p) = 0. \qquad \ldots (6.6)$$

Hence we conclude from this that, under the regularity condition $\rho^* = P$ the quantity $(2\,\rho^* + |\phi|^2\,)$ is conservative.

(c) <u>Rigid flow</u> : The N.P. version of the necessary conditions for this flow are

$$\epsilon + \bar{\epsilon} - \gamma - \bar{\gamma} = 0,$$

$$\rho + \bar{\rho} - \mu - \bar{\mu} = 0,$$

$$\bar{\tau} + \nu + \pi - \bar{k} + 2(\,\bar{\alpha} + \beta) = 0,$$

$$\bar{\sigma} - \lambda = 0. \qquad \ldots (6.7)$$

We note that, the continuity equation for the charged fluid in null electromagnetic field has the same form under killing flow and Rigid flow of the fluid.

7. Jacobi Deviation Equation :

The expressions relative velocity and relative acceleration of the separation vector ^{a} of two infinitesimally neighbouring curves as measured in the three space orthogonal to time-like congruence u^{a} are derived by Hawking and Ellis (1973) in the form

$$\perp \frac{D}{\partial s} (\perp Y^{a}) = u^{a}{}_{;b} \perp Y^{b} . \qquad \ldots (7.1)$$

$$h^{a}{}_{b} \frac{D}{\partial s} (h^{b}_{c} \frac{D}{\partial s} \perp Y^{c}) = - R^{a}{}_{bcd} \perp Y^{c} u^{b} u^{d} +$$

$$+ h^{a}{}_{b} u^{b}{}_{;c} \perp Y^{c} + \dot{u}^{a} \dot{u}_{c} \perp Y^{c} . \qquad \ldots (7.2)$$

respectively.

Here $\perp$ is projection operator which projects the quantity in three space with the help of three space operator $h^{a}{}_{b}$. For the choice of time-like vector $u^{a} = 2^{-\frac{1}{2}} (l^{a} + n^{a})$ we derive the equations (7.1) and (7.2) in N.P. version. Hence the equation of relative velocity (7.1) becomes

$$\perp \frac{D}{\partial s} \perp Y^{q} = 2^{-1/2} \Big[(\gamma + \bar{\gamma})(l^{a} l_{b} - n^{a} l_{b}) +$$

$$+ (\epsilon + \bar{\epsilon}) (l^{a} n_{b} + n^{a} n_{b}) + \Big\{ - (\alpha + \bar{\beta}) l^{a} m_{b} +$$

$$+ (\nu - \bar{\tau})\, m^a l_b + (\bar{\sigma} - \lambda)\, m^a m_b + (\bar{\rho} - \mu)\, m^a \bar{m}_b +$$

$$+ (\pi - k) m^a n_b + (\alpha + \bar{\beta}) n^a m_b \Big\} + \text{c.c.} \Big] \qquad \ldots (7.3)$$

The geodesic deviation equation for charged perfect fluid in null electromagnetic field can be written by using N.P. formalism in the following way

$$h^a_b \frac{\mathcal{D}}{\partial s} \left(h^b_c \frac{D}{\partial s} \perp Y^c \right) = -\frac{1}{2} \Big[\Psi_1 \Big\{ (l_c - n_c)\, \bar{m}^a +$$

$$+ (l^a - n^a)\, m_c \Big\} + \text{c.c.} + \Psi_2 \Big\{ \bar{m}^a m_c - (l^a + n^a)(l_c + n_c) +$$

$$+ m^a \bar{m}_c \Big\} + \text{c.c.} - \Psi_3 \Big\{ (l_c n_c) m^a + (l^a n^a)\, m_c \Big\} - \text{c.c.} \Big] \perp Y^c -$$

$$- \frac{1}{24} \Big[(-2\rho^* + 18p)\, l^a l_c + (-2\rho^* + 18p) n^a n_c +$$

$$+ (-6\rho^* + 30p)\, l^a n_c + (-6\rho^* + 30p)\, n^a l_c + 2(4\rho^* +$$

$$+ 3|\phi|^2 + 12p)\, m^{(a} m_{c)} \Big] + \Big[-\Big\{ \delta^a_b - 2^{-1} (l^a + n^a)$$

$$(l_b + n_b) \Big\} \Big\{ 2^{-1} (\gamma + \bar{\gamma} + \epsilon + \bar{\epsilon})\, (l^b - n^b) +$$

$$+ (\nu + \pi - k - \bar{\tau}) m^b + \text{c.c.} \Big\} \Big] \perp Y^c +$$

$$+ \frac{1}{4} \Big[\Big\{ (\gamma + \bar{\gamma} + \epsilon + \bar{\epsilon})\, (l^a - n^a) + (\nu + \pi - \bar{k} - \tau)\, m^a +$$

$$+ \text{c.c.} \Big\} \Big\{ (\gamma + \bar{\gamma} + \epsilon + \bar{\epsilon})\, (l^a - n^a) +$$

$$+ (\nu + \pi - k - \bar{\tau})\, m^c \Big\} + \text{c.c.} \Big] . \qquad \ldots (7.4)$$

<u>Remarks</u> : (1). For the relativistic charged dust the flow becomes geodesic and then the equations (7.3) and (7.4) are

just as the equations derived by Gumaste (1984) for charged dust distribution.

(2) From equation (7.4) one can derive the necessary and sufficient conditions (kinematical and dynamitical) for relative acceleration to vanish.

8. WEYL TENSOR FIELD :

We have introduced the concept of Weyl conformal curvature tensor C_{abcd} and the associated scalars. So we write Riemannian curvature tensor in terms of Weyl tensor, Ricci tensor and Ricci scalars through the relation

$$R_{abcd} = C_{abcd} + \frac{1}{2}\left(g_{ac}R_{bd} - g_{ad}R_{bc} - g_{bc}R_{ad} + g_{bd}R_{ac}\right) + \frac{1}{6}\left(g_{ad}\,g_{bc} - g_{ac}g_{bd}\right)R. \qquad \ldots (8.1)$$

The first term on R.H.S. describes the free gravitational field and the remaining terms exhibit the gravitational field due to matter. The trace free weyl tensor characterising free gravitational field plays an important role in the description, algebraic classification of matter fields Carmeli (1982).

By making use of Weyl tensor some new concepts as electric type component and magnetic type component are introduced by E.N.Glass (1975). According to him we have the defining expression for the electric part and magnetic part of Weyl tensor are respectively given by

$$E_{ac} = C_{abcd}\, u^b u^d , \qquad \dots (8.2)$$

$$B_{ac} = C^{*}_{abcd}\, u^b u^d . \qquad \dots (8.3)$$

i.e. in N.P. formalism

$$\begin{aligned} E_{ac} = & - (\psi_2 + \bar{\psi}_2) \left[V_a V_c - m_{(a}\bar{m}_{c)} \right] + \\ & + (\bar{\psi}_1 - \psi_3) \left[l_{(a} m_{c)} - m_{(a} n_{c)} \right] + C.C. - \\ & - \frac{1}{2} (\bar{\psi}_0 + \psi_4) m_a m_c - C.C. \end{aligned} \qquad \dots (8.4)$$

$$\begin{aligned} B_{ac} = H_{ac} = & - i (\psi_2 - \bar{\psi}_2) \left[V_a V_c - m_{(a}\bar{m}_{c)} \right] - \\ & - i (\bar{\psi}_1 + \psi_3) \left[l_{(a} m_{c)} - m_{(a} n_{c)} \right] - C.C. + \\ & + \frac{1}{2} i (\bar{\psi}_0 - \psi_4) m_a m_c + C.C. \end{aligned} \qquad \dots (8.5)$$

where * denotes dual tensor field. It follows from these expressions that both the defined tensors are symmetric, trace free and orthogonal to u^a.

The Weyl tensor also can be expressed in terms of electric type and magnetic type components by utilizing the Bianchi identities as

$$\begin{aligned} C_{abcd} = & (g_{abef}\, g_{cdij} - \eta_{abef}\, \eta_{cdij})\, u^e u^i E^{fj} - \\ & - (g_{abef}\, \eta_{cdij} + \eta_{abef}\, g_{cdij})\, u^e u^i B^{fj} . \end{aligned} \qquad \dots (8.10)$$

<u>Note</u> : All the above expressions can be rewritten in the

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N.P. formalism by replacing the value of time-like vector u in terms of real tetrad vectors.

9. <u>Gravitational Tidal force</u> :

Non-uniformaties in gravitational fields are called tidal forces, these tidal forces prevent the construction of global inertial frames. It follows from geodesic, deviation equation that the tidal forces of a gravitational field (which cause trajectories of neighbouring particles to diverge) can be represented by curveture of the space time in which particle follow geodesic Schutz (1985).

The gravitational tidal force has the mathematical description as $C_{abcd}\, u^b u^d$. Hence the gravitational tidal force is described by electric type component as defined in earlier Section 8.

Claim : If the gravitational tidal force is divergence free then

(i) $(\Psi_2 + \bar{\Psi}_2) = 0$, or

(ii) $(\rho + \bar{\rho}) + 2(\epsilon + \bar{\epsilon}) = (\mu + \bar{\mu}) + 2(\gamma + \bar{\gamma})$.

<u>Proof</u> : We have the divergence relation (Glass 1974).

$$\nabla^c E_{c\alpha} = R_{d[\alpha;b]}\, u^d u^b + \frac{1}{12}\, \gamma^b_\alpha\, R_{,b} - \theta^b E_{\alpha b} + 3 w^b B_{\alpha b} - u_\alpha\, \sigma^{cd} E_{cd} .$$

This when contracted with u^a we get

$$\nabla^c E_{c\alpha} = u_\alpha \, \sigma^{cd} E_{cd} \qquad \ldots (9.1)$$

This is N.P. formalism becomes

$$\nabla^c E_{c\alpha} = \frac{1}{2\sqrt{2}} \left(\Psi_2 + \bar{\Psi}_2 \right) \Big[(\rho + \bar{\rho}) - (\mu + \bar{\mu}) + + (\epsilon + \bar{\epsilon}) - 2 (\gamma + \bar{\gamma}) \Big]$$

Hence when gravitational tidal force is divergence free then

$$\nabla^c E_{c\alpha} = 0$$

which interns implies from above equation that

$$(\Psi_2 + \bar{\Psi}_2) \Big[(\rho + \bar{\rho}) - (\mu + \bar{\mu}) + 2(\epsilon + \bar{\epsilon}) - 2(\gamma + \bar{\gamma}) \Big] = 0$$

i.e. $(\Psi_2 + \bar{\Psi}_2 = 0)$ or

$$(\rho + \bar{\rho}) + 2 (\epsilon + \bar{\epsilon}) = (\mu + \bar{\mu}) + 2(\gamma + \bar{\gamma}).$$

Hence the required result.

APPENDIX

(A_1). **Einstein Scalars** : We have the value of Einstein tensor, $G_{ab} = R_{ab} - \frac{1}{2} Rg_{ab}$. So we calculate the independent scalars for G_{ab} as

(a) $G_{00} = - \frac{1}{2} G_{ab}\, l^a l^b$

$= \frac{1}{4} (\rho^* + p)$,

(b) $G_{01} = - \frac{1}{2} G_{ab}\, l^a m^b$

$= 0$,

(c) $G_{02} = - \frac{1}{2} G_{ab}\, m^a m^b$

$= 0$,

(d) $G_{10} = \frac{1}{2} G_{ab}\, l^a \bar{m}^b$

$= 0$,

(e) $G_{11} = - \frac{1}{4} G_{ab}\, (l^a n^b + m^a \bar{m}^b)$

$= \frac{1}{8} (\rho^* + p)$,

(f) $G_{12} = - \frac{1}{2} (G_{ab}\, n^a m^b$

$= 0$,

(g) $G_{20} = - \frac{1}{2} G_{ab}\, \bar{m}^a \bar{m}^b$

$= 0$,

(h) $G_{21} = -\frac{1}{2} G_{ab} n^a \bar{m}^b$

$= 0,$

(i) $G_{22} = -\frac{1}{2} G_{ab} n^a n^b$

$= \frac{1}{4} (|\phi|^2 + \rho^* + p).$ ... $(A_{1.1})$

We observe that, Einstein scalars are same as Ricci scalars.

(A_2) (a) Stress energy tensor of perfect fluid in null electromagnetif field for the choice (B)

$u^a = 2^{-1/2} (m^a + \bar{m}^a)$ ... $(A_{2.1})$

for above choice

$T_{ab\,(em)} = \frac{1}{2} |\phi|^2 \; l_a l_b ,$ ... $(A_{2.2})$

$T_{ab\,(p.f)} = \frac{1}{2} (\rho^* + P) \left[m_a \bar{m}_b + \bar{m}_a \bar{m}_b \right] -$

$- 2p \; l_{(a} n_{b)} - (\rho^* + p) \; m_{(a} \bar{m}_{b)},$ ... $(A_{2.3})$

Therefore stress energy tensor of perfect fluid in null electromagnetic field for above choice of u^a is given by

$T_{ab} = 2^{-1} \left[|\phi|^2 \; l_a l_b + (\rho^* + p)(m_a m_b + \bar{m}_a \bar{m}_b) - \right.$

$\left. - 4p \; l_{(a} n_{b)} - 2 (\rho^* - p) \; m_{(a} \bar{m}_{b)} \right]$ ... $(A_{2.4})$

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(A_2) (b) Ricci tensor expressing in N.P. version for the choice of $u^a = 2^{-1/2}\ (m^a + \bar{m}^a)$;

The expression of Ricci tensor for charged perfect fluid in null electromagnetic field for above choice of u^a using Einstein's field equations is given by

$$R_{ab} = -2^{-1}\Big[\ |\phi|^2\, l_a l_b + (\rho^* + p)\ (m_a m_b + \bar{m}_a \bar{m}_b) - 2(\rho^* - p)\ l_{(a}n_{b)} - 2\ (\rho^* - p)\ m_{(a}\bar{m}_{b)}\Big]. \qquad \ldots (A_{2.5})$$

(A_3) Ricci scalar for the choice $u^a = 2^{-1/2}\ (m^a + \bar{m}^a)$

From T_{ab} given by ($A_{2.4}$)

$$T^a_a = T = R = (\rho^* - 3p) \qquad \ldots (A_{3.1})$$

We use the Ricci expression given in(A_2) and R to calculate ten independent Ricci scalars as follows,

$$\phi_{00} = \phi_{01} = \phi_{10} = \phi_{12} = \phi_{21} = 0,$$

$$\phi_{02} = (1/4)(\rho^* + p),$$

$$\phi_{11} = (-1/8)(\rho^* + p),$$

$$\phi_{20} = (1/4)\ (\rho^* + p),$$

$$\phi_{22} = 1/4\ |\phi|^2$$

$$= 1/24\ (\rho^* - 3p). \qquad \ldots (A_{3.2})$$