CHAPTER-4

NONLINEAR PROPAGATION OF HE_{11} LASER MODE IN STEP-INDEX OPTICAL FIBERS *****

CONTENTS

4.1 INTRODUCTION :

The nonlinear processes[1] taking place in optical fibers play an important role in the problem of optical communications. The characteristics of such processes have been summarised in chapter 3. The intensity dependent R.I. profile leads to the optical Kerr effect which causes small phase shifts in the propagating optical signals. The study of nonlinear process in fibers commonly uses step index silica core fibers because of their advantages as pointed out earlier. In the present work we consider the effect of the intensity dependent R.I. of a monomode step index uniform core fiber on the propagation of HE_{11} laser mode.

4.2 NONLINEAR PROPAGATION OF HE_{11} MODE :

When an optical single pulse is launched into a fiber, its optical power is distributed almost over all the modes of a fiber.[2] In the case of a monomode fiber the power will be transferred to the HE_{11} or Lp_{01} mode alone. Once the power is launched its radial distribution in the fiber takes place acccording to the intensity profile of HE_{11} mode in the fiber core. We will consider the fact that the Gaussian intensity profile of the HE_{11} mode will induce a refractive index profile having the same shape. This induced R.I. profile will modify the inherent step index R.I. profile of the fiber. Thus the original step index profile will be modified to a graded index type as shown in Fig. 4.1. Through such a fiber core the propagation of HE_{11} mode will have to be considered as nonlinear one. The transmitted pulse will suffer a waveform distortion which will result into the broadening of the pulse waveform. As a consequence the transmission capacity of the optical fiber as a communication channel will be limited.

In order to work out the effect of intensity dependent refractive index quantitatively, first we shall calculate the radial intensity distribution of HE_{11} mode in a uniform core step index fiber under weakly guiding approximation.

4.2.1 RADIAL INTENSITY DISTRIBUTION OF HE_{11} MODE :

We consider the axial component of the time averaged Poynting vector for HE_{11} mode[3].

$$I(r) = \frac{1}{2}\left(E_r H_\theta^* - H_r E_\theta^*\right) \qquad \text{.....(4.1)}$$

For weakly-guiding optical fibers we have, the following field components[4] in the core,

$$E_r = iA \frac{akn_o}{u} \frac{J_{\nu-1}\left(\frac{ur}{a}\right)}{J_\nu(u)} \qquad \text{.....(4.2)}$$

$$E_\theta = -A \frac{akn_o}{u} \frac{\nu}{|\nu|} \frac{J_{\nu-1}\left(\frac{ur}{a}\right)}{J_\nu(u)} \qquad \text{......(4.3)}$$

$$H_r = A \frac{ak\,n_o^2}{u}\left(\frac{\varepsilon_o}{\mu_0}\right)^{1/2} \frac{\nu}{|\nu|} \frac{J_{\nu-1}\left(\frac{ur}{a}\right)}{J_\nu(u)} \qquad \text{.....(4.4)}$$

$$H_\theta = iA \frac{ak\, n_o^2}{u}\left(\frac{\varepsilon_o}{\mu_o}\right)^{1/2} \frac{J_{\nu-1}\left(\frac{ur}{a}\right)}{J_\nu(u)} \qquad \ldots\ldots(4.5)$$

where,

ν = azimuthal mode number.

ν = order of the Bessel function of 1^{st} kind.

a=Radius of the fiber core.

k=wave number.

n_o=Refractive index of the core.

u=Normalized mode parameter.

For HE_{11} mode,

$\nu = 1$ and $|\nu| = 1$.

Hence the above relations take the following forms.

$$E_r = iA \frac{akn_o}{u} \frac{J_o\left(\frac{ur}{a}\right)}{J_1(u)} \qquad \ldots\ldots(4.6)$$

$$E_\theta = -A \frac{akn_o}{u} \frac{J_o\left(\frac{ur}{a}\right)}{J_1(u)}$$

$$E_\theta^* = -A^* \frac{akn_o}{u} \frac{J_o\left(\frac{ur}{a}\right)}{J_1(u)} \qquad \ldots\ldots(4.7)$$

$$H_r = A \frac{akn_o^2}{u}\left(\frac{\varepsilon_o}{\mu_o}\right)^{1/2} \frac{J_o\left(\frac{ur}{a}\right)}{J_1(u)} \qquad \ldots\ldots(4.8)$$

$$H_\theta = iA \frac{akn_o^2}{u}\left(\frac{\varepsilon_o}{\mu_o}\right)^{1/2} \frac{J_o\left(\frac{ur}{a}\right)}{J_1(u)}$$

$$H_\theta^* = -iA^* \frac{qkn_o^2}{u}\left(\frac{\varepsilon_o}{\mu_o}\right)^{\frac{1}{2}} \frac{J_o\left(\frac{ur}{a}\right)}{J_1(u)} \qquad \ldots\ldots(4.9)$$

From (4.6) and (4.9) :

$$E_r H_\theta^* = AA^* \frac{a^2 k^2 n_o^3}{u^2} \left(\frac{\varepsilon_o}{\mu_o}\right)^{1/2} \left| \frac{J_o \frac{(ur)}{a}}{J_1 (u)} \right|^2 \quad ......(4.10)$$

From (4.7) and (4.8) :

$$H_r E_\theta^* = -AA^* \frac{a^2 k^2 n_o^3}{u^2} \left(\frac{\varepsilon_o}{\mu_o}\right)^{1/2} \left| \frac{J_o \frac{(ur)}{a}}{J_1 (u)} \right|^2 \quad ......(4.11)$$

Putting (4.10) and (4.11) in equation (4.1) :

$$I(r) = AA^* \frac{a^2 k^2 n_o^3}{u^2} \left(\frac{\varepsilon_o}{\mu_o}\right)^{1/2} \left| \frac{J_o \frac{(ur)}{a}}{J_1 (u)} \right|^2 \quad .....(4.12)$$

In this last equation the factor AA^* is proportional to the power contained in the mode which is launched into the optical fiber.

Now for small argument u the approximate values of the Bessel functions involved in the above equations are given by :

$$J_o\left(\frac{ur}{a}\right) \simeq 1 - 1/4 \left(\frac{ur}{a}\right)^2$$

and, $J_1(u) \simeq u/2$

Putting these values in eq.(4.12) and approximating the modulus squared term to second order we can obtain following expression.

$$I(r) \simeq AA^* \; 4a^2k^2n_o^3\left(\frac{\epsilon_o}{\mu_o}\right)\left[\frac{1}{u^4} - \frac{1}{2u^2}\left(\frac{r}{a}\right)^2\right] \quad \text{......(4.13)}$$

The value of the modal parameter u can be obtained by using the following approximate relations between u and the normalized frequency v :

$$u(v) \quad \frac{2.4142\ v}{(1 + (4 + v^4))^{1/4}} \quad \text{.....(4.14)}$$

where $v = k\ n_o\ a(\ 2\Delta\ )^{1/2}$ .....(4.15)

Eq.(4.14) is the approximate analytic solution of the characteristic equation for the HE_{11}(LP_{01}) mode.

We take the following typical numerical values for various constants involved in the multiplying factor of equation (4.13).

$a = 3.25 \times 10^{-6}\ \mu m$

$n_o = 1.473$

$\epsilon_o = 8.85 \times 10^{-12}$

$\mu_o = 4\pi \times 10^{-7}$

We choose a typical power of 800 mw, which is launched axially into the monomode step index fiber. Hence we take $AA^* \propto 0.8w$. Substituting these numerical values in equation (4.13) we obtain :

$$I(r) \simeq 35.91 \times 10^{-15} \times k^2\left[1/u^4 - 1/2u^2(r/a)^2\right] \quad \text{....(4.16)}$$

Hence the intensity dependent part of R.I. of the fiber core is given by,

$$\delta n = 1/2\ n_2'\ I(r) \qquad ....(4.17)$$

where, $n_2' = 0.9 \times 10^{-3} \left(\frac{m^2}{v^2}\right)$ .....(4.18)

Using (4.16) and (4.17) we obtain. :

$$\delta n = 16.16 \times 10^{-18} \times k^2 \left[1/u^4 - 1/2u^2\ (r/a)^2 \right] \qquad .....(4.19)$$

Considering the contribution of this intensity dependent part, the effective refractive index of the fiber core is written as,

$$\delta n^2(r) = (n_o + \delta n)^2$$

$$n^2(r) \approx \delta n_o^2 + 2n_o\ \delta n$$

Using (4.19) and the numerical value of n_o we obtain

$$n^2(r) \approx 2.16973 + 47.6074 \times 10^{-18} \times k^2 \left[1/u^4 - 1/2u^2 (r/a)^2 \right] .....(4.20)$$

The graphical representation of this effective refractive index profile for a typical k value of 7.39 X 10^6 m^{-1} is depicted in Fig. (4.2). The computations of the refractive index profile have been worked out/ using computer program given in Table 4.1. In order to bring out this nonlinearity effect clearly we have applied the relations (4.16) and (4.17) to a variety of nonlinear crystals.[4] The respective effective refractive index profiles are pictorially shown in Figs. (4.3) to (4.7). Each of the refractive index profiles represents an induced graded index in a step index optical medium.

4.2.2 CALCULATIONS OF b-v CURVES :

In order to study the nonlinear dispersion relation, the normalized frequency v is assumed to be intensity dependent and is defined as,

$$v_1 = ak \left[(n_o + \delta n)^2 - n^2 \right]^{1/2}$$

where, n = R.I. cladding

Under weakly guiding approximation it takes the form given below:

$$v_1 \simeq v \left[1 + \delta n / n_o \Delta \right]^{1/2} \qquad \text{.....(4.21)}$$

where, $v = ak\ (n_o^2 - n^2)^{1/2}$

$$\Delta = n_o - n/n_o$$

and δn is given by eq.(4.19)

We assume that the relation (4.16) holds good when $v = v_1$ and the corresponding u value is represented by u_1. Thus

$$u_1(v_1) = \frac{(1 + \sqrt{2})\ v_1}{\left[1 + (4 + v_1^4)^{1/4}\right]} \qquad \text{.....(4.22)}$$

In order to make the results independent of particular fiber confugurations usually the following ratio is calculated.

$$b(v) = 1 - \left(\frac{u^2}{v^2}\right)$$

In the present case this relation can be written as,

$$b_1(v_1) = 1 - \left(\frac{u_1^2}{v_1^2}\right) \qquad \ldots.(4.23)$$

The behaviour of this intensity dependent propagation parameter as a function of the intensity dependent normalized frequency v_1 for various (r/a) values has been presented in Figs.(4.8) to (4.11). These plots form the most general version of the numerical results of the optical fiber dispersion relation i.e. eigen value equation for a comparative study the b-v curves without considering the effect of field dependent dielectric constant are also given. All the figures show that for a given (r/a) value the nonlinear b-v curve is distinctly different from the linear b-v curve. Particularly for large v-values the diversification is more clear. Like the linear curve the nonlinear curve attains an assumptotic value as $v \rightarrow \infty$. All the nonlinear curves lie below the linear curve. These different locations clearly show the effect of field dependent dielectric constant in the propagation of HE_{11} mode In order to bring out the changes in b and v values on account of the said effect we have presented the numerical results in Tables (4.2)--(4.3) and (4.4). It is seen that on account of intensity dependent refractive index, the changes in b and v values occur in the third and fourth decimal places.

Further as (r/a) value increases both b and v values regularly decrease in the third and fourth decimal places.

4.2.3 FIELD DISTRIBUTIONS OF HE_{11} MODE :

The WKB approximation has proved to be an important method[7] in the analysis of multimode fibers in general. The method can provide reasonably accurate field distributions without the necessity for lengthy numerical calculations.

Neglecting the axial, azimuthal and the time dependances the scalar wave equation in cylindrical polar co-ordinates is written as :

$$\frac{d^2\psi}{dr^2} + \frac{1}{r}\frac{d\psi}{dr} + \left[k^2 n^2(r) - \beta^2 - \frac{\ell^2}{r^2} \right] = 0 \quad .....(4.24)$$

Usually a trial solution of the form,

$$\psi = \psi_o \exp\left[iKS(r) \right]$$

is assumed in which S is expanded in powers of 1/K :

$$S(r) = S_o(r) + 1/K\, S_1(r) + \cdots \quad .....(4.25)$$

Hence,

$$\psi = \psi_o \exp\left[iK\, S_o(r) + iS_1(r) + \ldots \right] \quad ....(4.26)$$

Putting (4.26) in (4.24) and equating terms of similar order in K, we can obtain the zero and the first order WKB approximation as given below :

$$S_0(r) = 1/k \int \left[k^2 n^2(r) - \beta^2 - \ell^2/r^2 \right]^{1/2} dr \quad .....(4.27)$$

$$S_1(r) = i/4 \ln \left[r^2 n^2(r) - k^2 r^2/k^2 - \ell^2/k^2 \right] \quad .....(4.28)$$

Depending upon the sign of the radical in the equation (4.27), we obtain oscillatory or evanescent field solution in the fiber core.

$$\psi(r) = \frac{\psi_o}{(rq)^{1/2}} \exp\left(\pm i \int q dr \right) \quad q^2 > 0 \quad ....(4.29)$$

$$\psi(r) = \frac{\psi_o}{(rp)^{1/2}} \exp\left[\pm \int p dr \right] \quad P^2 > 0 \quad ...(4.30)$$

where, $q^2 = -P^2 = k^2 n^2(r) - \beta^2 - \ell^2/r^2$ ....(4.31)

We have applied this formal theory of the WKB approximation to the intensity dependent refractive index profile of a step index uniform core fiber. This profile can be treated as graded index type as has been made clear in the beginning of this chapter. The field distributions have been evaluated without considering the cladding effect. In order to calculate q from eqn.(4.31) We have assumed the effective refractive index of the fiber core to be given by,

$$n^2(r) \simeq \delta n_o^2 + 2n_o \delta n \quad ....(4.32)$$

By definition,

$$u^2 = (k^2 n_1^2 - \beta^2)\, a^2$$

Hence, we get,

$$\beta^2 = k^2 n_1^2 - u^2/a^2 \qquad \ldots(4.33)$$

Using these in eqn. (4.31) we obtain ;

$$q^2 = 2k^2 \ n_o \, \delta n + u^2/a^2$$

Further using eqn. (4.19) in the above expression and simplifing we get,

$$q^2 \simeq \left[47.6074 \times 10^{-18} \ (k/u)^4 + u^2/a^2\right] - \left[\ 23.8037 \times 10^{-18} \ (k^2/u)^2 \ \right] (r/a)^2$$

Let $a_1 = \left[47.6074 \times 10^{-18} \ (k/u)^4 + (u^2/a^2)\right]$

and,

$$a_2 = \left[23.8037 \times 10^{-18} (k^2/u)^2 \right]$$

Hence $q^2 \simeq a_1 - a_2 (r/a)^2$

$$q = \sqrt{a_2} \left[c_1^2 - (r/a)^2 \right]^{1/2}, \text{ where } c_1^2 = a_1/a_2$$

$$q dr = \sqrt{a_2} \int \left[c_1^2 - (r/a)^2\right]^{1/2} dr$$

$$= \sqrt{a_2} \left[\sqrt{1/2} \ (r/a) \sqrt{c_1^2 - (r/a)^2} + c_1^2 \text{ arc tan } (r/ac_1) / \sqrt{1 - (r/ac_1)^2} \right] \qquad \ldots(4.34)$$

(From 4.29) we have,

$$|\psi| = \frac{\psi_o}{(r.q)^{1/2}} \cos \int (q.dr), \quad q^2 > 0 \qquad \ldots(4.35)$$

Using (4.34) the value of the integral can be evaluated for a given k value and hence field distribution can be calculated as a function of r by assuming an arbitrary value $\psi_0 = 100$. For rapid computations of the field distributions a computer program has been written as given in Table (4.5). By considering various wave number values the field distributions of HE_{11} mode have been calculated in Figs. (4..12) — (4.16). For a given k-value the field distribution is seen to exhibit oscillatory behaviour in agreement with the reported results in the literature.[8] In order to have an idea of numerical values of the oscillatory fields, we have presented our results in tabular forms for typical k-values (Tables 4.6 - 4.8).

4.2.4 COMPUTATIONS OF DELAY TIME :

In optical fiber communication systems, the pulse-modulation schemes such as PCM (Pulse Code Modulation) is commonly used in modulating the carrier light. However, in such a scheme wave form distortion occurs during the travel of the pulse through the transmition medium viz. optical fiber. This distortion usually leads to the broadening of the pulse waveform at the recieving end. This limits the transmission capacity of optical fiber.

In general the pulse waveform distortion arises from the difference in the delay time of various components contained in the pulse. The 'component' means portions of light energy transmitted by various modes and various frequency components. The 'delay time' is defined as the time required to propagate 1 km length of an optical fiber. For uniform core fibers the delay time can be expressed as,[9]

$$t = \frac{N_1}{c} \frac{\left[1 - (1 + y/4)\, Qx\right]}{\left[1 - 2x\,\Delta\right]^{1/2}} \qquad \ldots\ldots(4.36)$$

Where the parameters, N_1 y and Q are defined as below :

N_1 represents group index and is defined by Gloge[5] as :

$$N_1 = n_o + k\,\frac{dn_o}{dk}$$

$$N_1 \approx n_o$$

The parameter y represents the difference between the material dispersions in core and cladding. However, in the present case we don't consider material dispersion so that y = o. The parameter Q is given by the relation

$$Q = (2\,v/u)\ du/dv$$

or $\ldots.(4.37)$

$$Q = \frac{2(1 - G_m)}{x(-G_m/H_m)}$$

where $G_m(w) = K_m^2(W)/k_{m-1}(W)\ K_{m+1}(W)$

and, $\ldots.(4.38)$

$$H_m(u) = J_m^2(u)/J_{m-1}(u)\ J_{m+1}(u)$$

For HE_{11} mode, m = 0

$$G_o = \frac{K_o^2(W)}{K\text{-}1(w)\ K_1(W)}$$

$$H_o = \frac{J_o^2(u)}{J_{-1}(u)\ J_1(u)}$$

For small arguments u and w, we can write

$$K_o(w) \simeq \ln\left[\frac{2}{\gamma W}\right], \qquad \gamma = 1.781$$

$$K_1(w) \simeq \frac{1}{W}$$

$$J_1(u) \simeq \frac{u}{2} \quad \text{and} \quad J_o(u) \simeq 1 - \frac{u^2}{4}$$

Using these relations alongwith the appropriate recurrence relations we can obtain,

$$G_o = \left[W \ln\left(\frac{1.123}{W}\right)\right]^2 \qquad \text{.....(4.39)}$$

and,

$$H_o = -\frac{4\ (1 - u^2/4)^2}{u^2}$$

With these values Eqn. (4.37) becomes,

$$Q = \frac{2\left[1 - G_o\right]}{x\left[1 - (G_o/H_o)\right]} \qquad \text{.....(4.40)}$$

Using (4.40) alongwith $N_1 \simeq n_o$, y = 0 and $x = v^2/u^2$ in Eq.(4.36) we can calculate the delay-time for HE_{11} mode.

We have calculated the delay-times for various (r/a) values by employing the intensity dependent normalized frequency v_1 given by Eq.(4.21). For rapid calculations a suitable computer program in BASIC language has been given in Table (4.9). We have illustrated the time-delay calculations graphically in Fig. (4.17)-(4.19). In order to understand clearly the effect of intensity dependent normalized frequency on the delay-time we have presented our results for some (r/a) values in Tables (4.11)-(4.13). For a comparative study the "linear" time delay characteristics are also given in Table (4.10)

4.3 SUMMARY :

In this chapter we have studied the effect of intensity dependent refractive index of a monomode step index uniform core fiber on the various characteristics of the propagation of HE_{11} Laser mode. We have assumed that the Gaussian intensity profile of the HE_{11} mode will induce a similar shaped refractive index profile in the fiber core. This induced profile will superpose on the original step index profile. As a result the fiber can be regarded as the one having a graded index type profile. Effectively speaking the propagation of HE_{11} mode takes place in a graded index fiber. Hence the relevant theory can be applied to the present case.

First we have calculated the radial intensity distribution of HE_{11} mode by considering the radial and azimuthal field components for a weakly guiding optical fiber having a step index type profile. Next we have defined a normalized frequency v_1 which depends on the intensity of the incoming mode in addition to it's wavelength and the fiber confugurations such as core, diameter, axial refractive index and the refractive index of the cladding. Using this new normalized frequency v_1 we have calculated corresponding parameter u_1 with the help of an approximate relation given by Gloge[5] under weakly guiding approximation. This has enabled us to plot nonlinear b_1-v_1 curves similar to the usual b-v curves which represent the dispersion relation for the propagating mode. The features of b_1-v_1 curves have been briefly discuss.

We have also calculated the field distributions of HE_{11} mode under WKB approximation by considering the effect of intensity dependent R.I. profile. Neglecting the cladding effect we have estimated oscillatory fields in the fiber core as functions of the radial distance in the fiber cross-section. These nonlinear field distributions are found to exhibit similar behaviour as reported in the literature[4] for a linear propagation of the concerned mode.

Finally, in view of the importance of the delay-time concept in optical fiber communication systems, we have evaluated the delay-time for HE_{11} mode by emplcying the above defined intensity dependent normalized frequency v_1. The delay-time characteristics as functions of v_1 have been illustrated both graphically and in tabular forms.

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TABLE 4.1

INTENSITY DEPENDENT R.I.PROFILE FOR HE_{11} MODE THROUGH STEP-INDEX FIBER

```
10Ø REM File Jath
20Ø REM Programming by Shri. D.S.Upare
30Ø REM Intensity Dependent R.I.Profile for HE11 Mode Through Step Index Fiber
40Ø REM R1=r/a,a=A=core radius,k=wave number,no=R.I. of core,n=R.I.of cladding
50Ø NO=1.473
60Ø  N=1.45
70Ø A=3.25E-Ø6
80Ø LPRINT "r1","n^2(r)"
90Ø INPUT K
100Ø LPRINT "k=",K
110Ø FOR R1=.1 TO .6 STEP .Ø5
120Ø V=A*K*(NO^2-N^2)^.5
130Ø U=2.4142*V/(1+(4+V^4)^.25)
140Ø DN=1.616E-17*K^2*((1/U^4)-(1/(2*U)^2)*(R1)^2)
150Ø NR=NO^2+2*NO*DN
160Ø LPRINT R1,NR
170Ø NEXT R1
180Ø END
```

TABLE 4.2

LINEAR b-v CURVE FOR HE_{11} MODE

b	v
-1.701412×10^{38}	0
0.0641377	1
0.3993379	2
0.6423217	3
0.7683458	4
0.8385577	5
0.8812306	6
0.9090132	7
0.928088	8
0.941742	9
0.9518484	10
0.959537	11
0.9655214	12
0.9702703	13
0.9741017	14
0.9772376	15
0.9798366	16
0.9820146	17
0.9838579	18
0.9854317	19
0.9867861	20

TABLE 4.3

NONLINEAR b_1-v_1 CURVE FOR r/a = 0.2

k (m^{-1})	b_1	v_1
593346.6	4.582048×10^{-3}	0.5010182
1780039	0.2219837	1.503054
2966731	0.5417698	2.505089
4153424	0.7159894	3.507126
5340117	0.8087226	4.509162
7120155	0.8816231	6.012215
9493540	0.9283354	8.016286
1.305362×10^7	0.9596809	11.0224
1.602045×10^7	0.9723855	13.52757
2.017377×10^7	0.9820806	17.03461
2.373385×10^7	0.986335	20.04071

TABLE 4.4

NONLINEAR b_1-v_1 CURVE FOR r/a= 0.4

$k(m^{-1})$	b_1	v_1
593346.6	4.561434×10^{-3}	0.5004367
1780039	0.2213414	1.50131
2966731	0.5410744	2.502182
4153424	0.7154883	3.503055
5340117	0.808362	4.503928
7120155	0.8813879	6.005237
9493540	0.9281875	8.006983
1.305362×10^{7}	0.9595949	11.0096
1.602045×10^{7}	0.9723258	13.51187
2.017377×10^{7}	0.9820412	17.01484
2.373385×10^{7}	0.9868059	20.01746

TABLE 4.5

INTENSITY DEPENDENT R.I PROFILE AND FIELD DISTRIBUTION FOR HE_{11} MODE

THROUGH STEP-INDEX FIBER

```
10 REM File:Corfield
20 REM Programming by Shri D.S.Upare
30 REM Intensity Dependent R.I.Profile & Field Distribution
40 REM HE11 Through Step Index Fiber
50 REM D=Relative Index Difference,NC=R.I.of Core,N=R.I.of Cladding
60 REM A=Radius of Core,K1=wave Number,R1=r/a,a=A
70 NS=EFFECTIVE R.I.OF CORE
80 SO=100
90 D=.0156143
100 NC=1.473
110 N=1.45
120 A=3.25E-06
130 INPUT K1
140 B1=.0012932
150 V=A*K1*(NC^2-N^2)^.5
160 U=2.4145*V/(1+(4+V^4)^.25)
170 B=1-(U/V)^2
180 A1=2.1701573#-(NC)^2*((N/NC)+B*D)^2
190 C1=(A1/B1)^.5
200 B2=K1*NC*((N/NC)+B*D)^2
210 LPRINT "k1=";K1
220 LPRINT "r1","ns","S"
230 FOR R1=0 TO 1 STEP .05
240 NS=2.1701573#-B1*R1^2
250 K2=K1/.0719221
260 Q=(K1^2*NS-B2^2)^.5
270 R2=R1/C1
280 I=K2*(R1*C1*(1-R2^2)^.5)+K2*C1^2*ATN(R2/(1-R2^2)^.5)
290 S=(SO/(R1*A*Q)^.5)*COS(I)
300 LPRINT R1,NS,S
310 NEXT R1
320 END
```

TABLE (4.6),(4.7)and (4.8)
OSCILLATARY FIELD VALUES

K (r/a)	8900194 ψ	1.008689×10^7 ψ	1.127358×10^7 ψ	1.364696×10^7 ψ	1.483366×10^7 ψ	1.839373×10^7 ψ	1.958043×10^7 ψ
0.1	186.4647	171.4877	17.51021	133.1071	77.74395	172.6551	186.9458
0.2	-104.584	-31.34492	65.70061	126.079	60.22324	51.73427	134.1155
0.3	123.3548	-120.6839	89.36411	-118.3738	37.11591	3.795903	99.88567
0.4	85.57338	38.77282	99.53489	94.67869	50.60907	80.37214	51.47722
0.5	-89.95309	91.51198	-11.73276	-45.30641	-83.40187	72.76541	42.3208
0.6	54.26689	-50.09722	-79.18315	16.47813	-71.12846	72.28339	48.91592
0.7000001	-60.40659	23.34961	62.54496	39.59536	-80.4057	-23.64824	29.21871
0.8000001	-76.69396	-55.67228	38.65309	-36.58925	-53.03916	-58.81668	-18.31962
0.9000001	-48.73851	-0.1552791	-13.03359	20.30781	-13.58961	35.84998	-11.7109

TABLE 4.9

TIME DEALY CALCULATIONS FOR HE_{11} MODE (LINEAR AND NONLINEAR CASE)

```
10 REM Time-Delay Calculations- HE11 Mode(Linear & Nonlinear Case)
20 REM Programming By Shri D.S.Upare
30 REM No=R.I.of Core,N=R.I. of Cladding, A= Radius of Core,K=Wave Number
40 REM r1 means r/a,a=A
50 NO=1.473
60 N=1.45
70 D=(NO-N)/NO
80 A=3.25E-06
90 INPUT K
100 V=8.46E-07*K
110 U=(2.42428*V)/(1+(4+V^4)^.25)
120 PRINT "k=";K
130 PRINT "r1","v1","ct"
140 FOR R1=0 TO 1 STEP .05
150 DN=4.76074E-17*K^2*((1/U^4)-.5*(R1/U)^2)
160 V1=K*A*(NO^2+DN-N^2)^.5
170 U1=(2.4142*V1)/(1+(4+V1^4)^.25)
180 X=(U1/V1)^2
190 W=(V1^2-U1^2)^.5
200 Z=W*LOG(1.123/W)
210 Q=2*(1-Z^2)/(X*(1+(U1^2*Z^2)/(4*(1-(U1^2/4))^2)))
220 Y=0
230 CT=NO*(1-D*(1+(Y/4))*Q*X)/(1-2*X*D)^.5
240 PRINT R1,V1,CT
250 NEXT R1
260 END
```

TABLE 4.10

TIME DELAY CALCULATIONS HE_{11} (LINEAR CASE)

$K \times 10^7$	V	ct
5.0	42.3	1.478379
4.0	33.84	1.47806
3.0	25.38	1.477578
2.0	16.92	1.476792
1.0	8.46	1.475704
0.9	7.614	1.475691
0.8	6.768	1.475781
0.7	5.922	1.476057
0.6	5.076	1.476689
0.5	4.23	1.47803
0.4	3.384	1.480825
0.3	2.538	1.483219
0.2	1.692	1.448346
0.15	1.269	1.459326
0.10	0.8459	1.454017
0.07	0.5922	1.45

TABLE(4.11 & 4.12)

TIME DELAY CALCULATIONS HE_{11}(NONLINEAR)

r/a $k \times 10^7 m^{-1}$	0.1 v_1	ct	0.2 v_1	ct
2.5	21.229	1.477016	21.211	1.477015
2.0	16.940	1.476585	16.931	1.476584
1.5	12.68	1.476045	12.675	1.476044
1.0	8.440	1.47557	8.439	1.47557
0.9	7.594	1.475575	7.593	1.475575
0.8	6.749	1.475686	6.748	1.475686
0.7	5.904	1.475992	5.904	1.475992
0.6	5.060	1.476663	5.059	1.476663
0.5	4.216	1.478057	4.216	1.478057
0.4	3.372	1.48092	3.372	1.478092
0.3	2.529	1.483101	2.529	1.483101
0.2	1.686	1.448048	1.686	1.448049
0.15	1.265	1.459023	1.264	1.459023
0.10	0.843	1.454441	0.843	1.454441
0.07	0.591	1.451039	0.591	1.4551039
0.05	0.422	1.450091	0.422	1.450091

TABLE 4.13

TIME DELAY CALCULATIONS HE_{11}(NONLINEAR)

r/a	0.4		0.5	
$k \times 10^7\ m^{-1}$	v_1	ct	v_1	ct
2.5	21.159	1.47701	21.120	1.477007
2.0	16.904	1.476581	16.883	1.476578
1.5	12.664	1.476042	12.655	1.476041
1.0	8.435	1.47557	8.432	1.47557
0.9	7.590	1.475575	7.588	1.475575
0.8	6.746	1.475687	6.745	1.475687
0.7	5.902	1.475993	5.901	1.475993
0.6	5.058	1.476664	5.058	1.476665
0.5	4.215	1.478058	4.215	1.478059
0.4	3.372	1.480922	3.372	1.480923
0.3	2.529	1.483099	2.529	1.483097
0.2	1.686	1.448052	1.686	1.448054
0.15	1.264	1.459023	1.264	1.459023
0.10	0.843	1.454441	0.843	1.454441
0.07	0.590	1.451039	0.590	1.451039
0.05	0.422	1.450091	0.422	1.450091

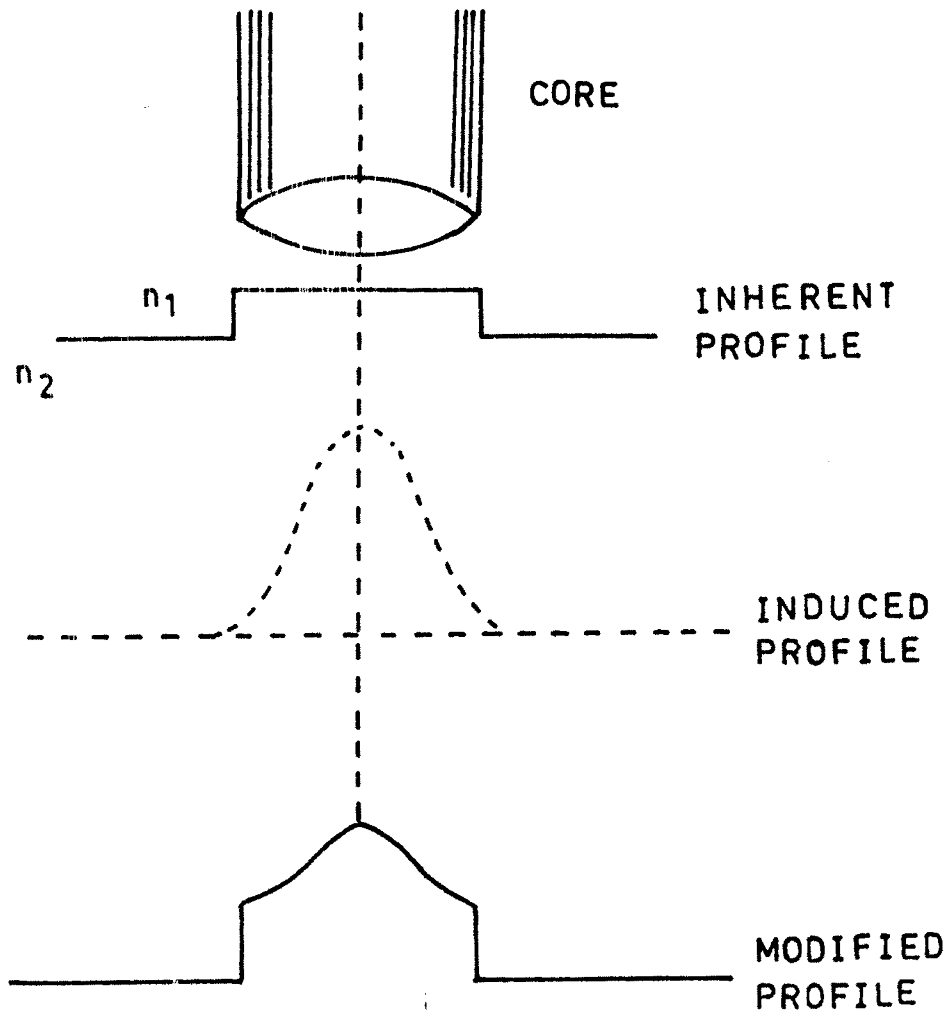


FIG. 4-1 — STEP INDEX PROFILE TO GRADED INDEX TYPE.

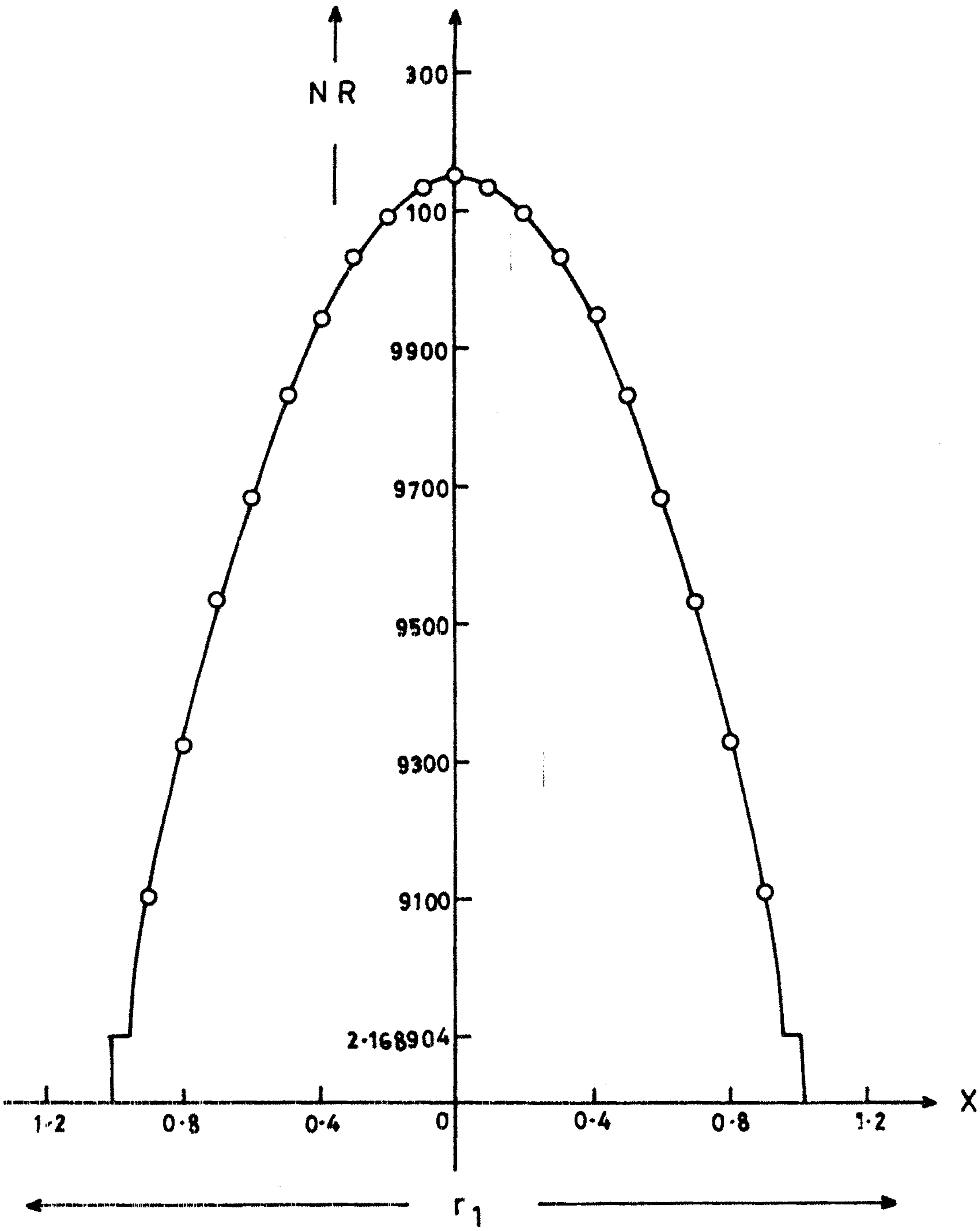
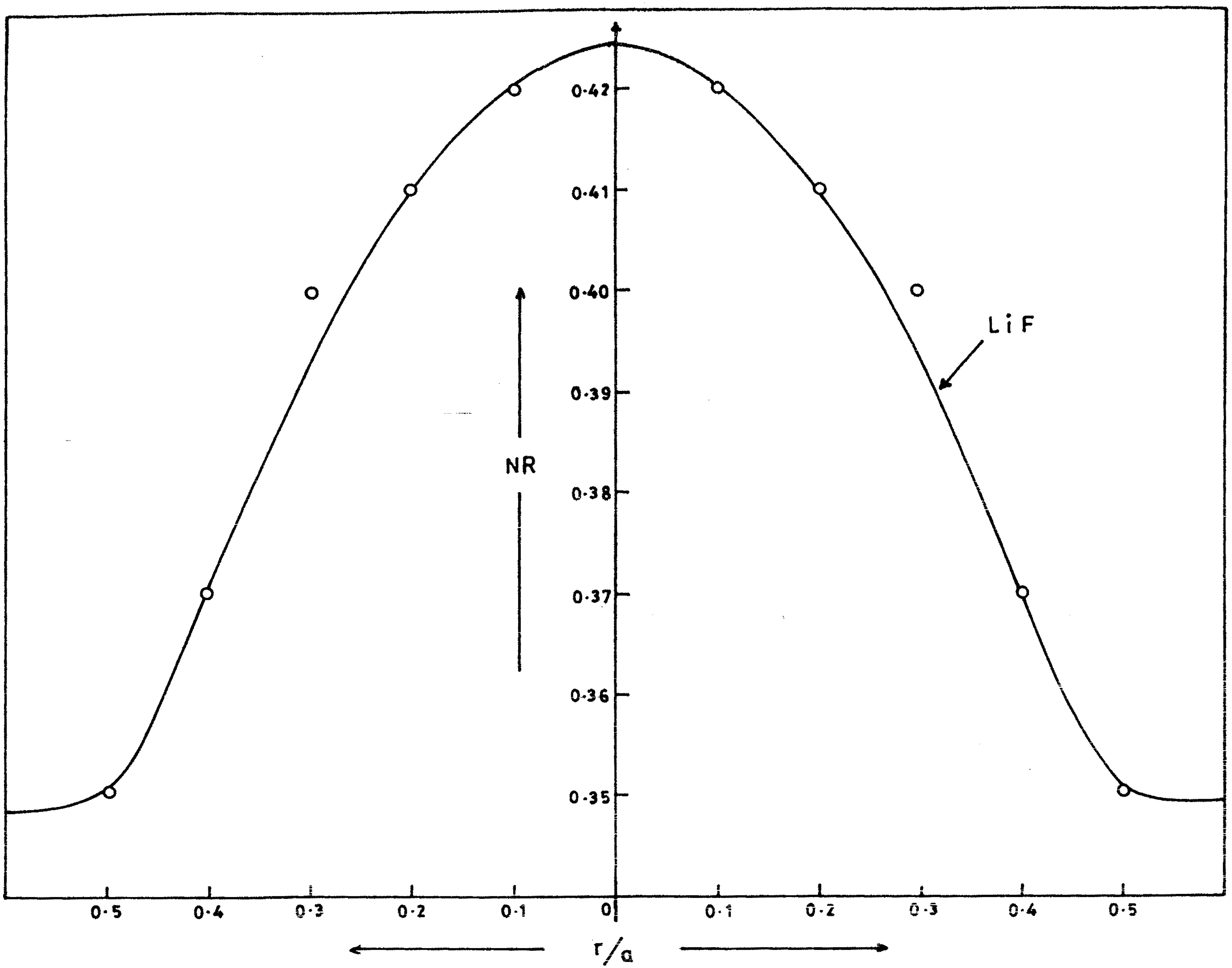


FIG. 4·2–TYPICAL INTENSITY DEPENDENT R.I. PROFILE FOR HE_{11} MODE PROPAGATION.


0·42
0·41
0·40
0·39
0·38
0·37
0·36
0·35
NR
LiF
0·5
0·4
0·3
0·2
0·1
0
0·1
0·2
0·3
0·4
0·5
r/a

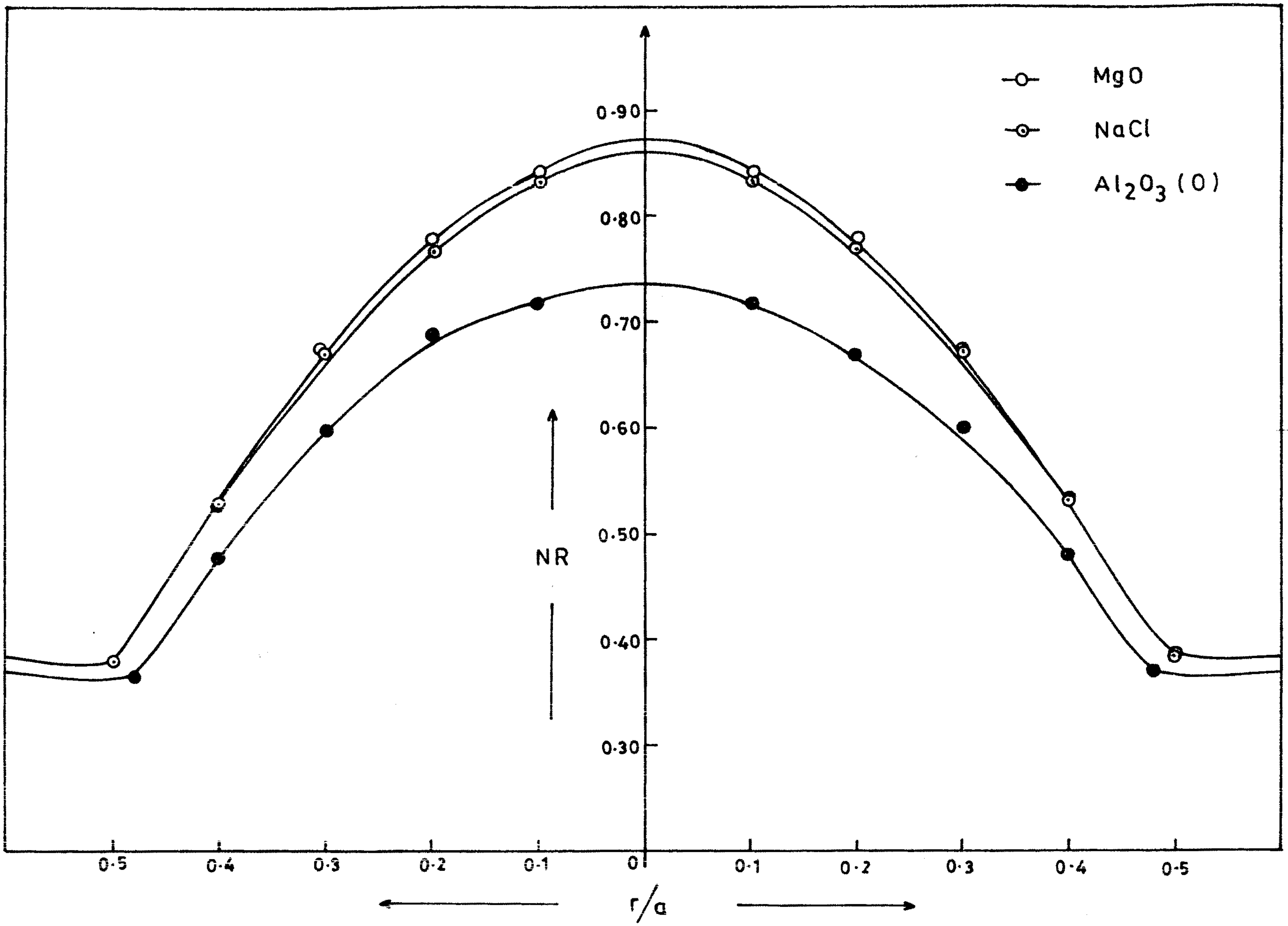


FIG. 4·4 — REFRACTIVE INDEX PROFILE FOR NaCl, MgO AND Al_2O_3 (O).

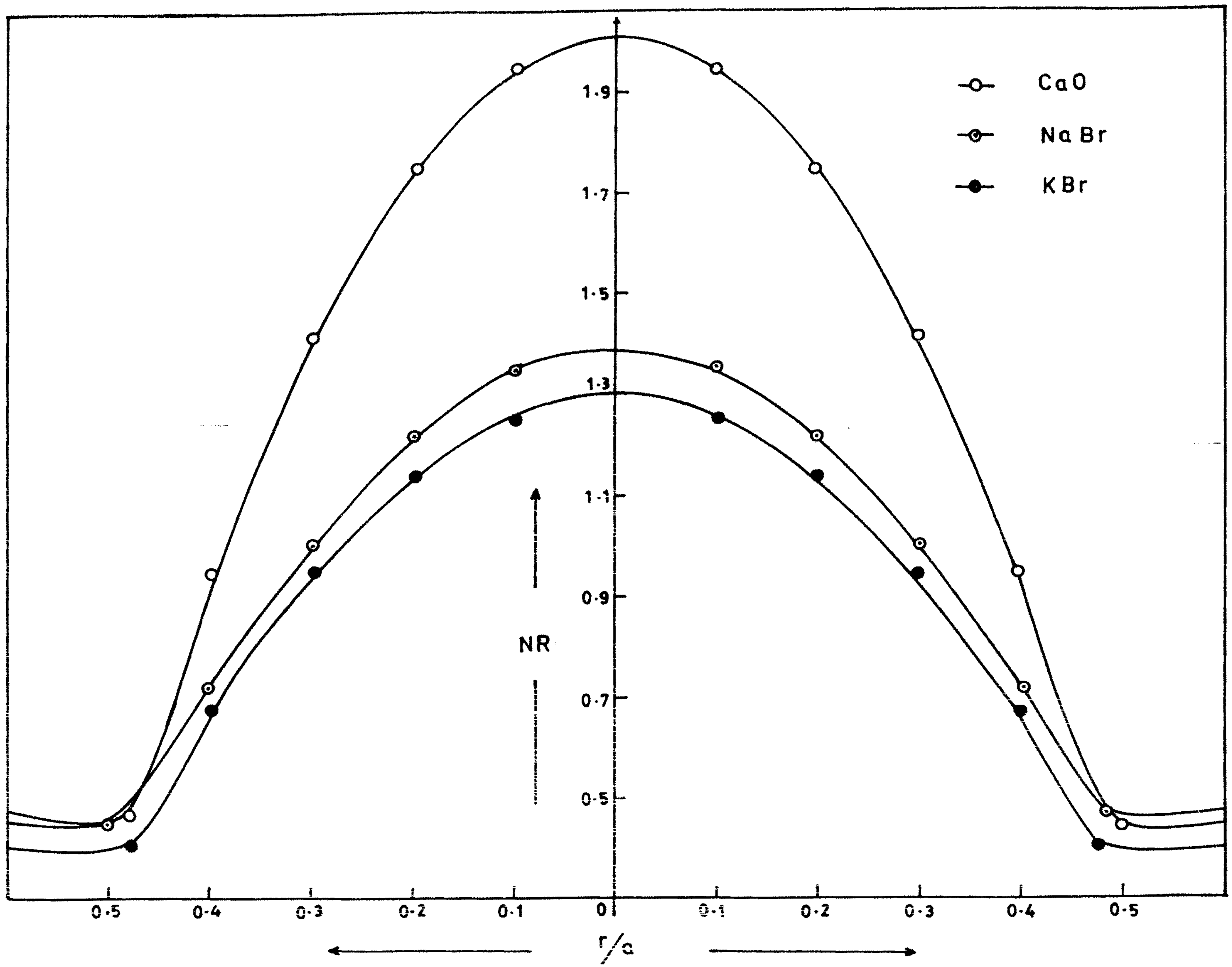


FIG. 4·5 — INTENSITY DEPENDENT R.I. PROFILE FOR NaBr, KBr AND CaO.

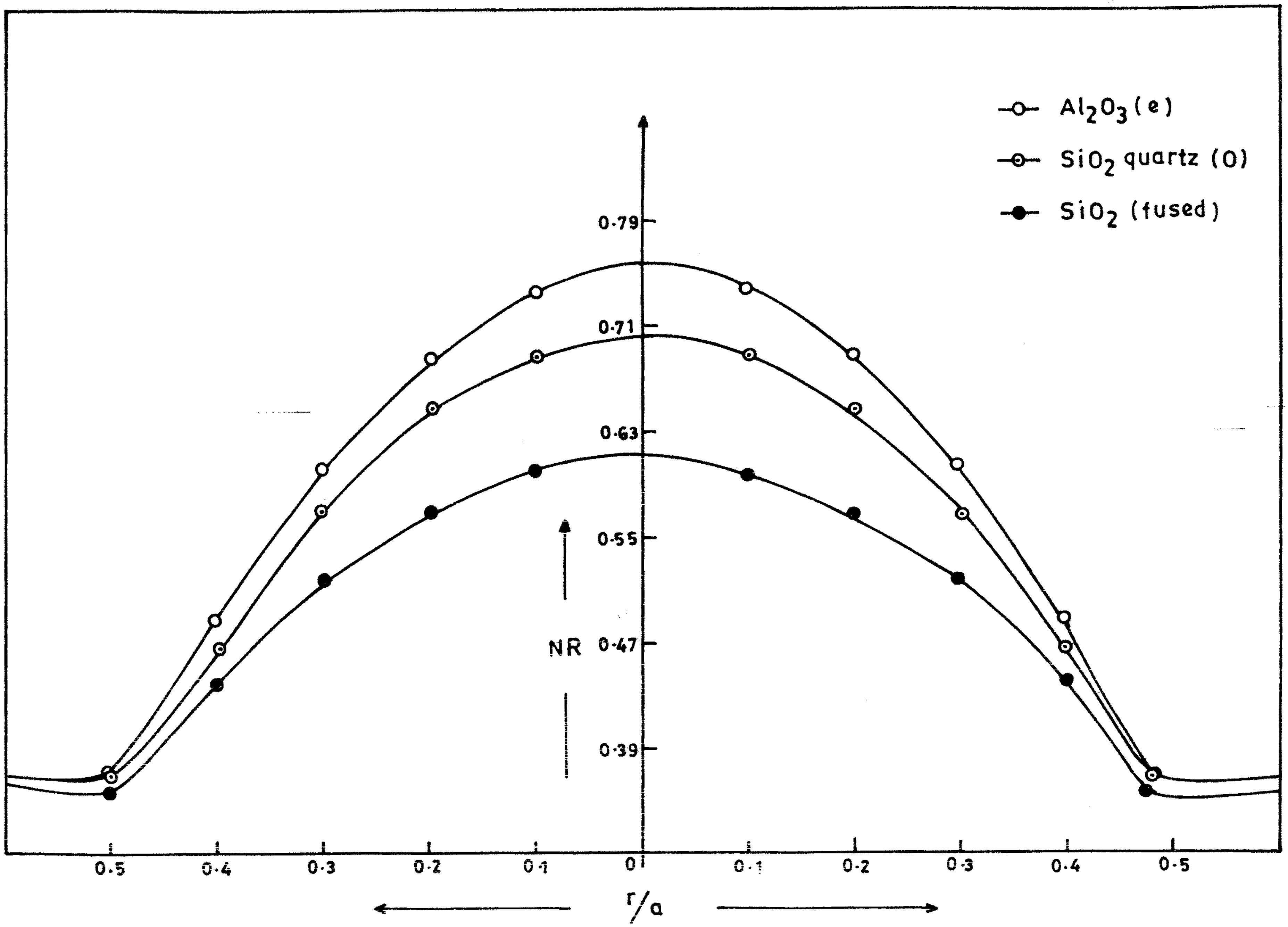


FIG. 4·6 — REFRACTIVE INDEX PROFILE FOR Al_2O_3(e), SiO_2(fused) AND SiO_2 quartz (O).

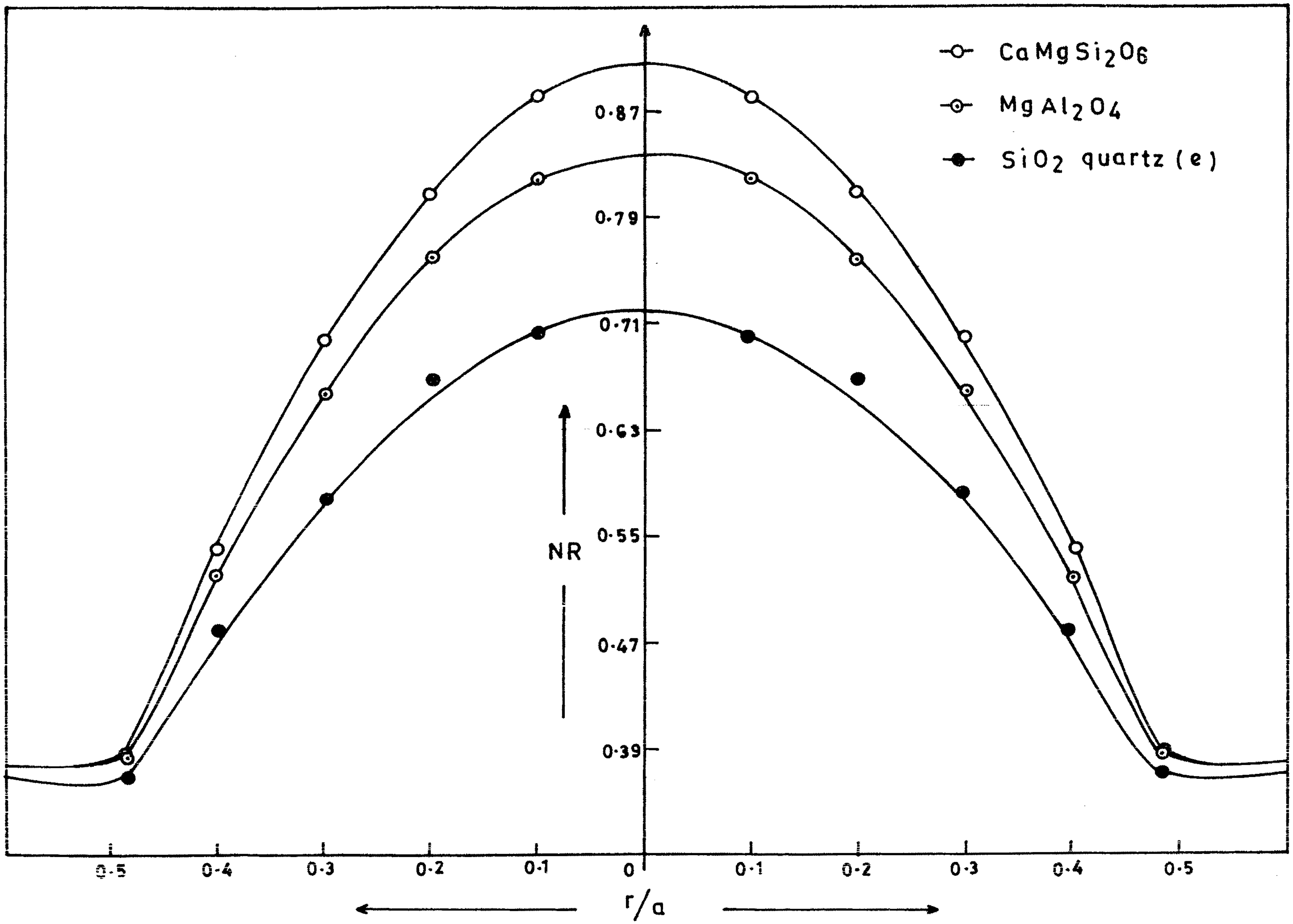


FIG. 4·7 – REFRACTIVE INDEX PROFILE FOR SiO_2 quartz (e), $MgAl_2O_4$ AND $CaMgSi_2O_6$.

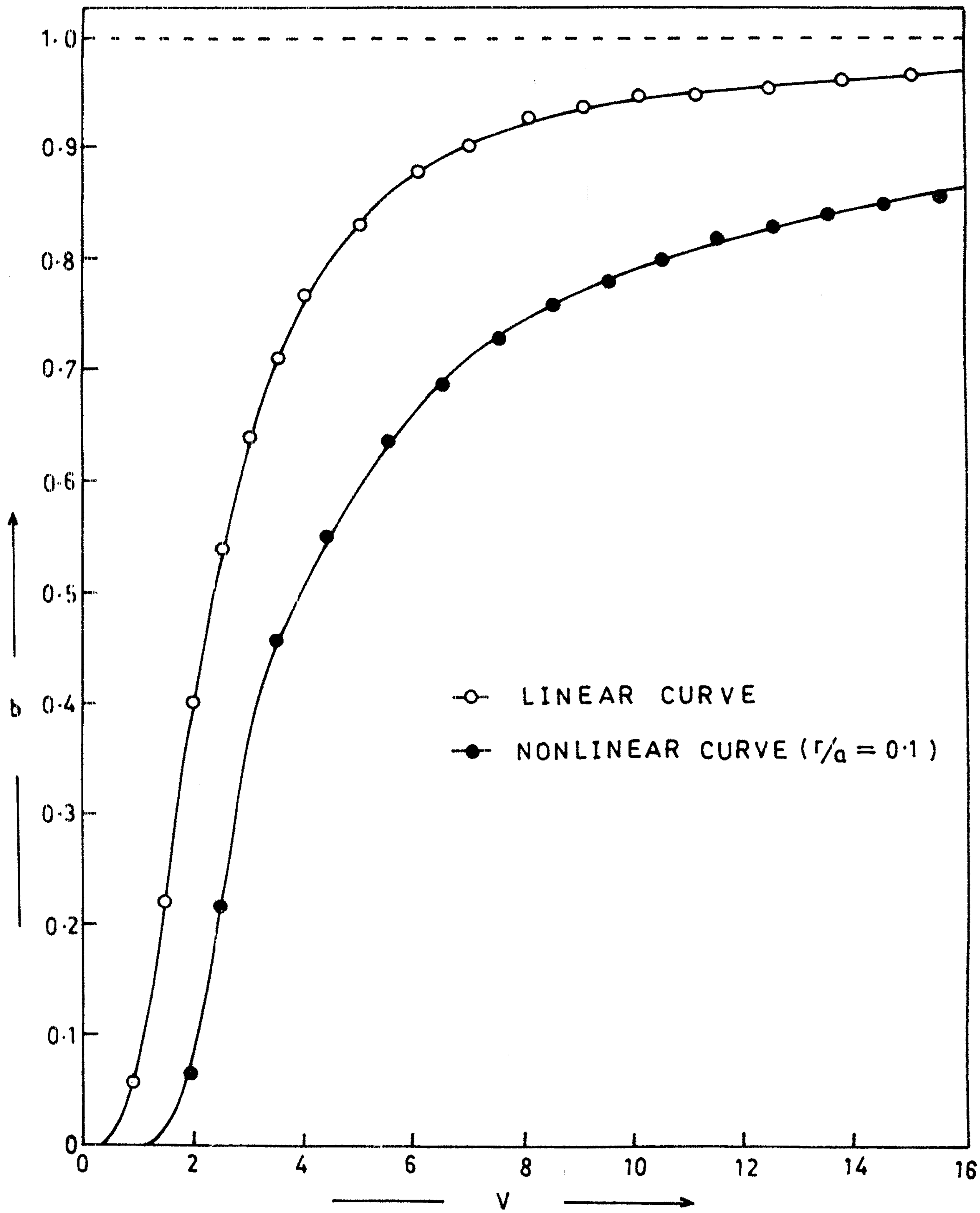


FIG. 4·8 — b – V CURVES FOR HE_{11} MODE .

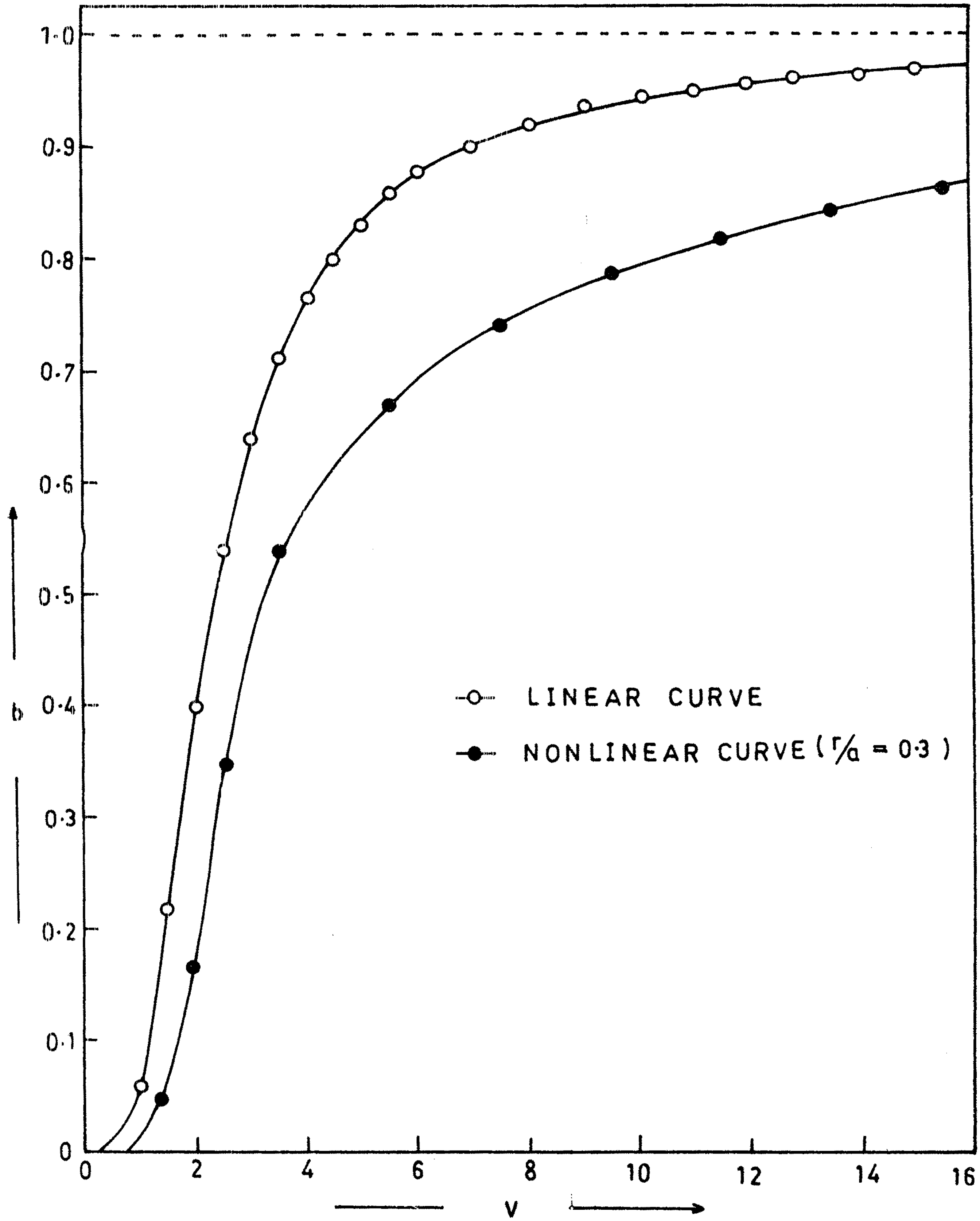


FIG. 4·9 — b–V CURVES FOR HE_{11} MODE .

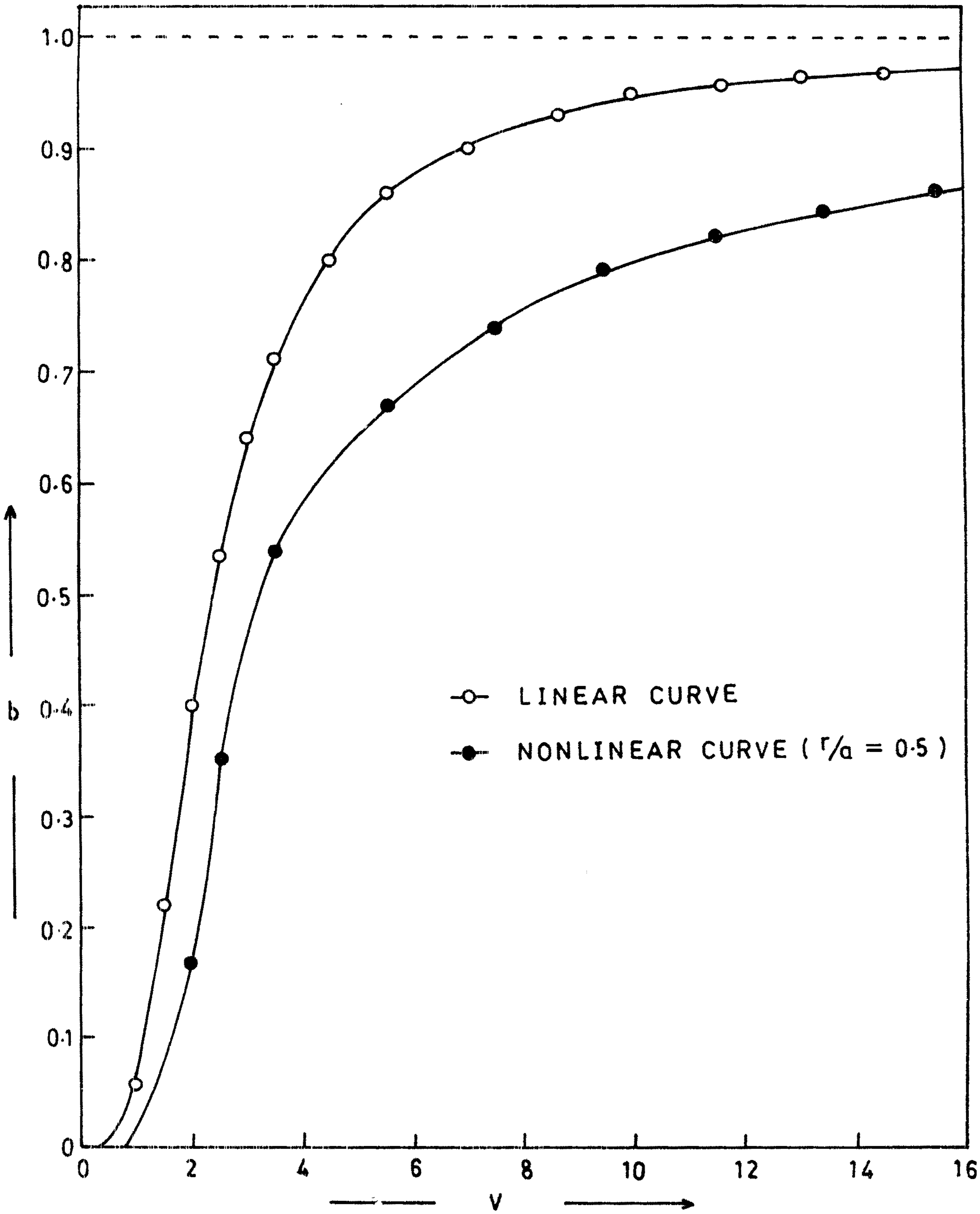


FIG. 4.10 — b – V CURVES FOR HE_{11} MODE .

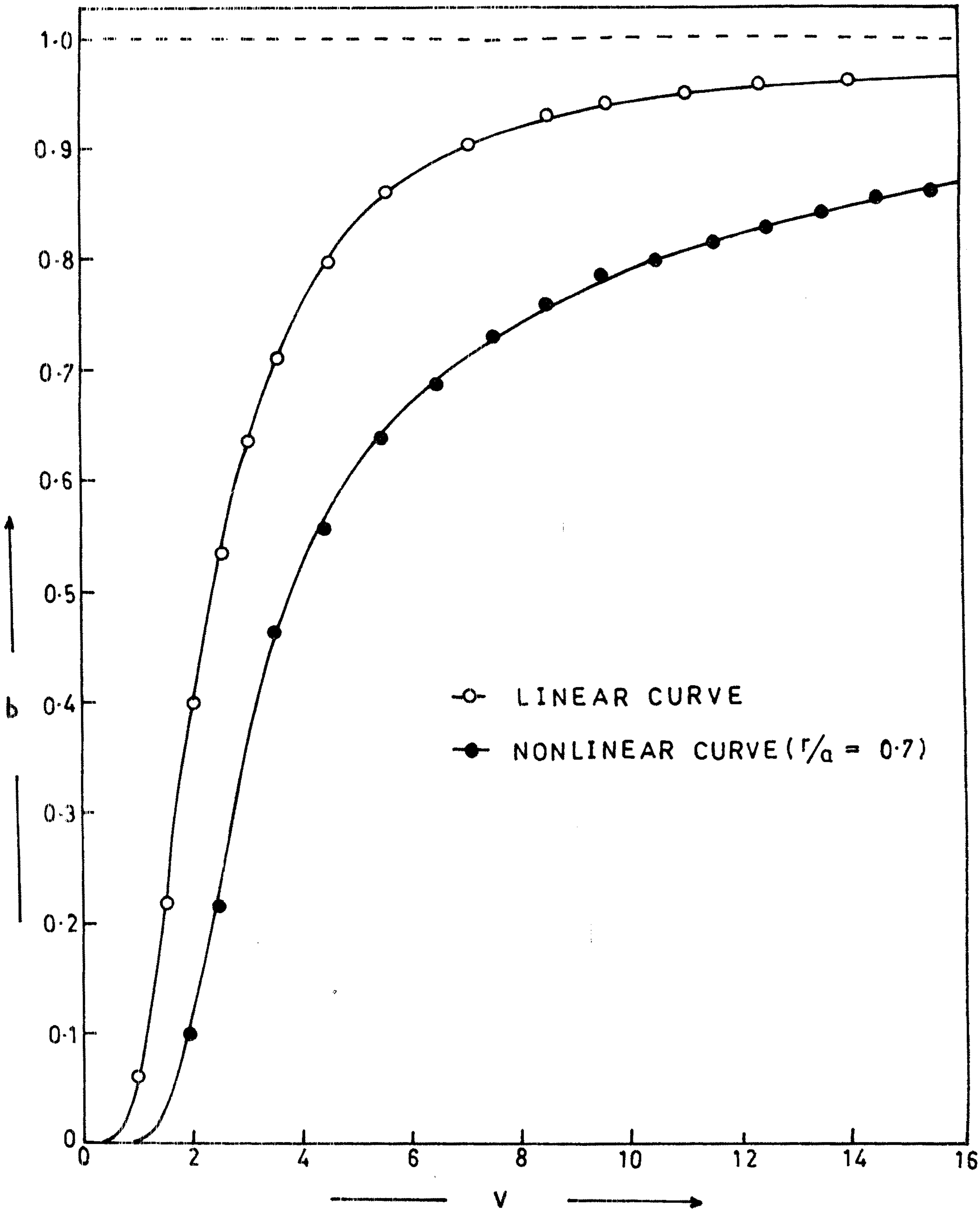


FIG. 4·11 — b – V CURVES FOR HE_{11} MODE.

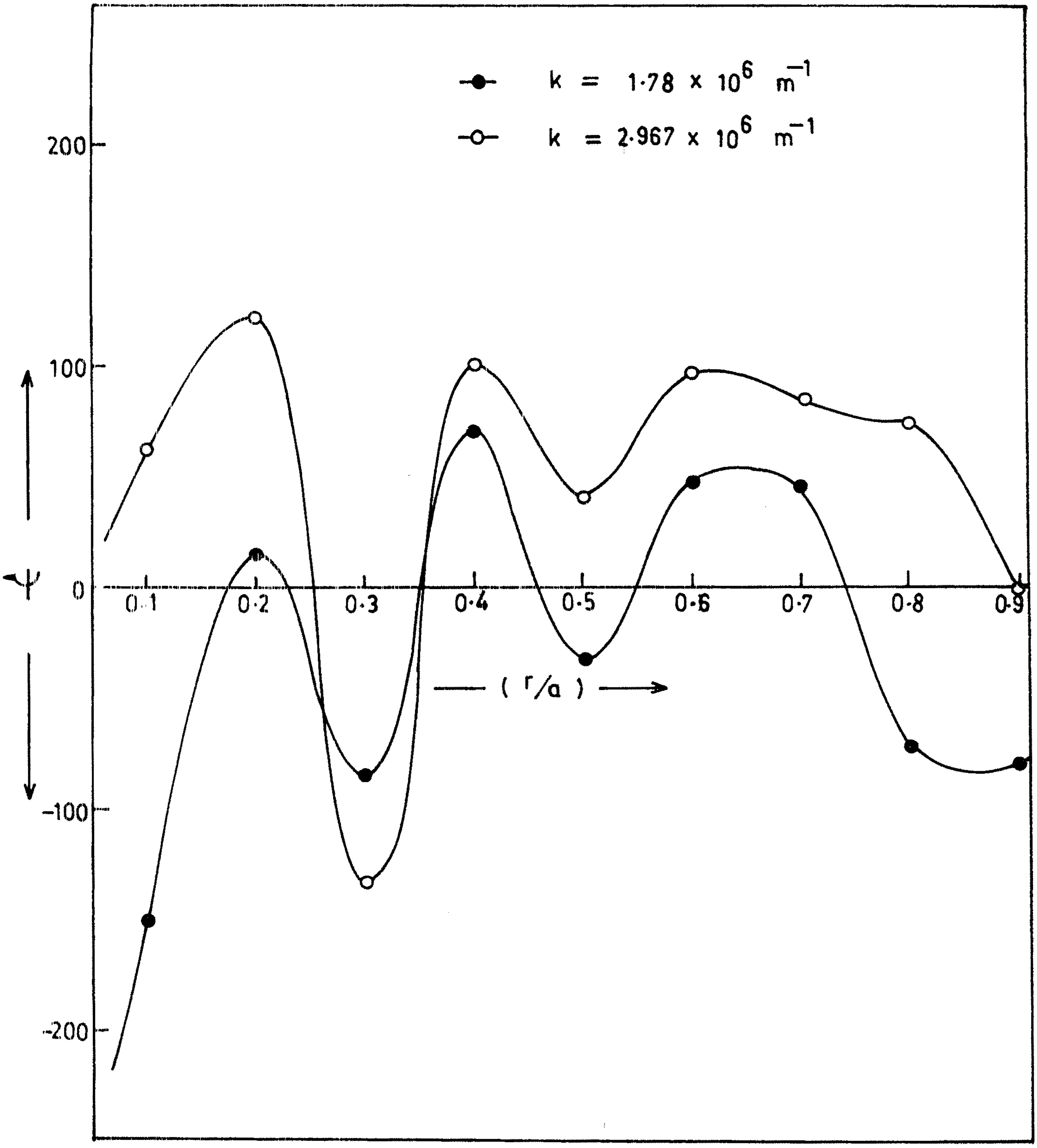


FIG. 4·12 — NONLINEAR FIELD DISTRIBUTIONSOF HE_{11} MODE FOR $k = 1{\cdot}78 \times 10^{6}\ m^{-1}$ AND $2{\cdot}967 \times 10^{6}\ m^{-1}$.

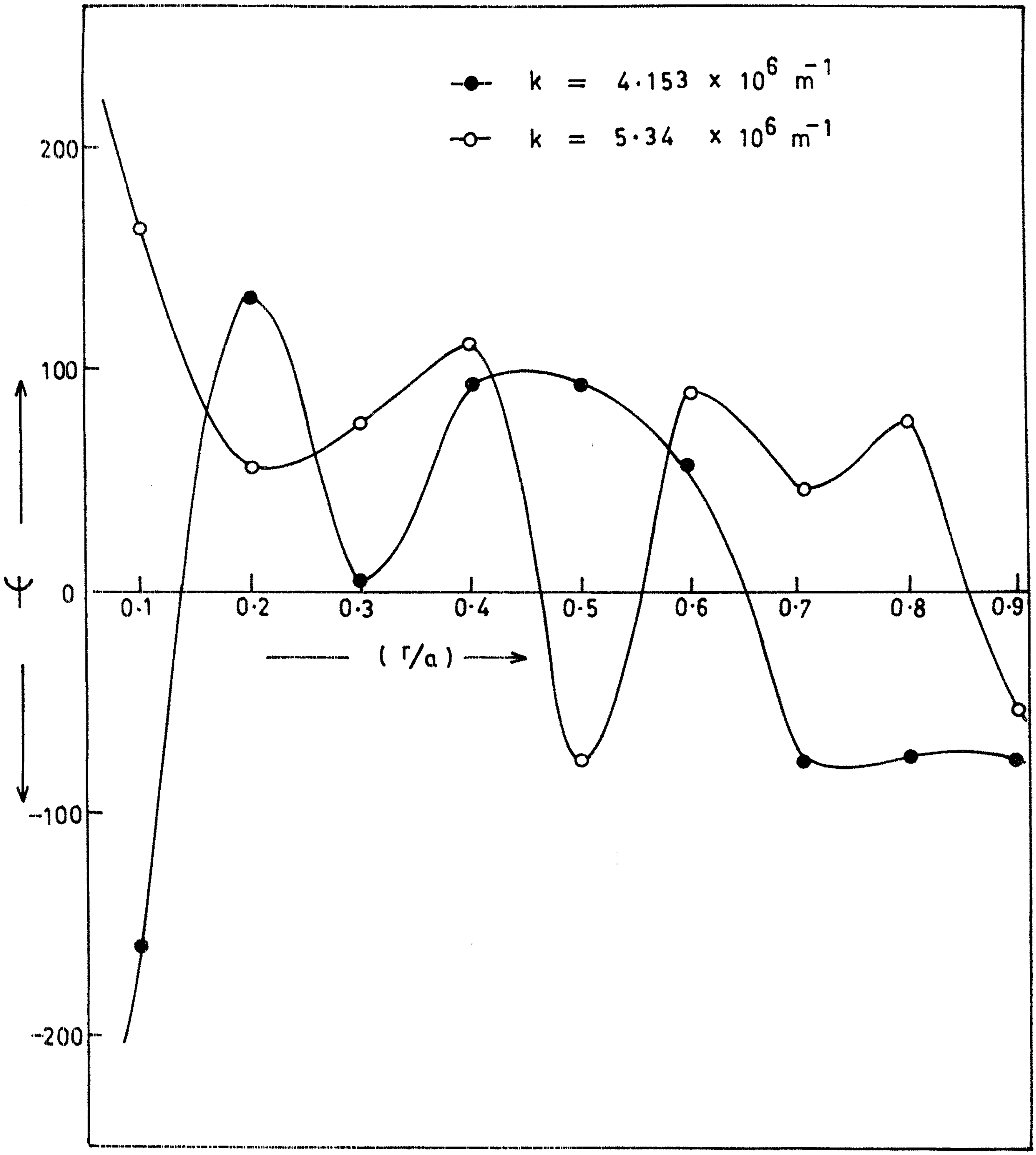


FIG. 4·13 – NONLINEAR FIELD DISTRIBUTIONS OF HE_{11} MODE FOR $k = 4{\cdot}153 \times 10^{6}\ m^{-1}$ AND $5{\cdot}34 \times 10^{6}\ m^{-1}$.

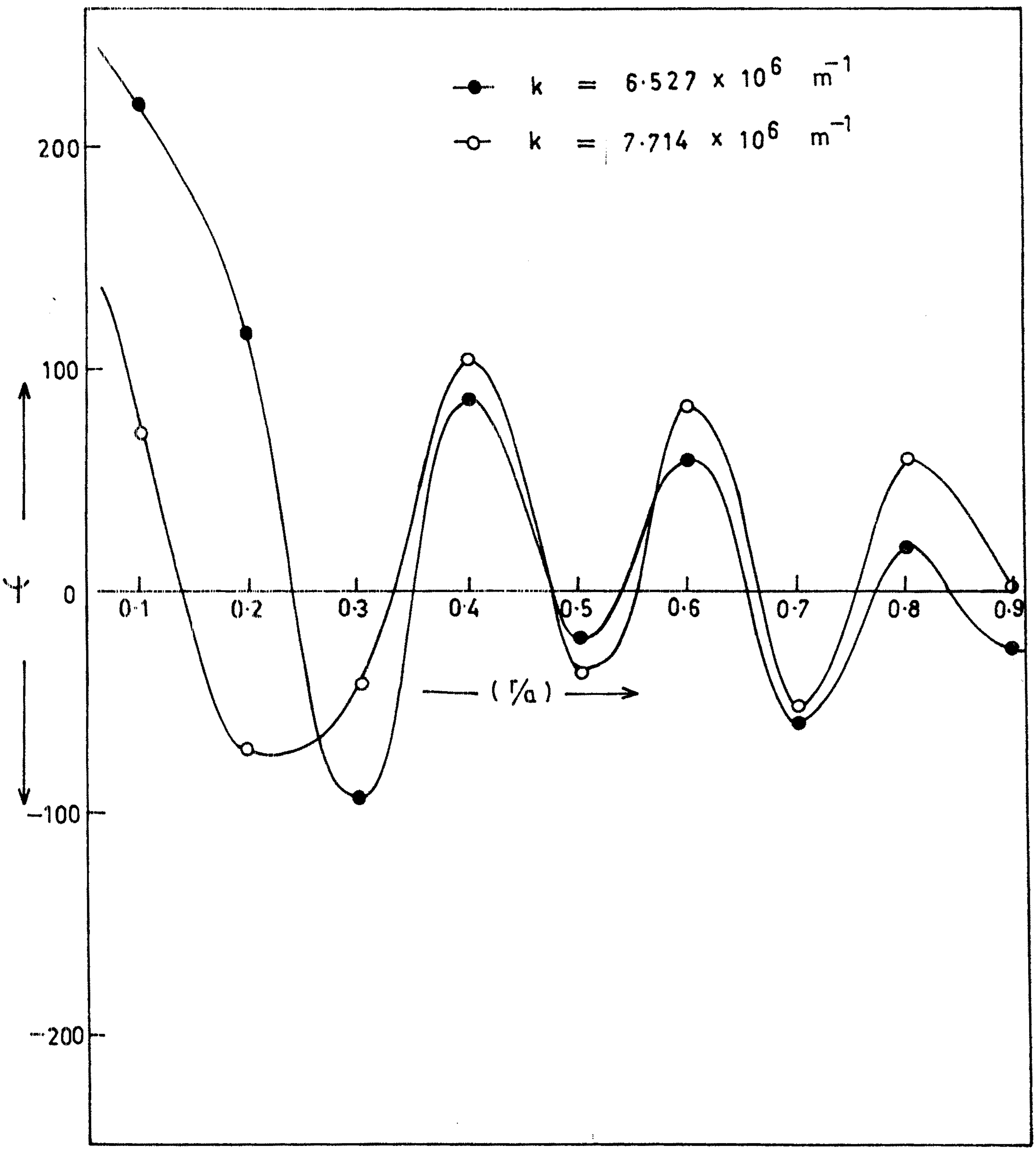


FIG. 4·14 – NONLINEAR FIELD DISTRIBUTIONS OF HE_{11} MODE FOR $k = 6{\cdot}527 \times 10^6\ m^{-1}$ AND $7{\cdot}714 \times 10^6\ m^{-1}$.

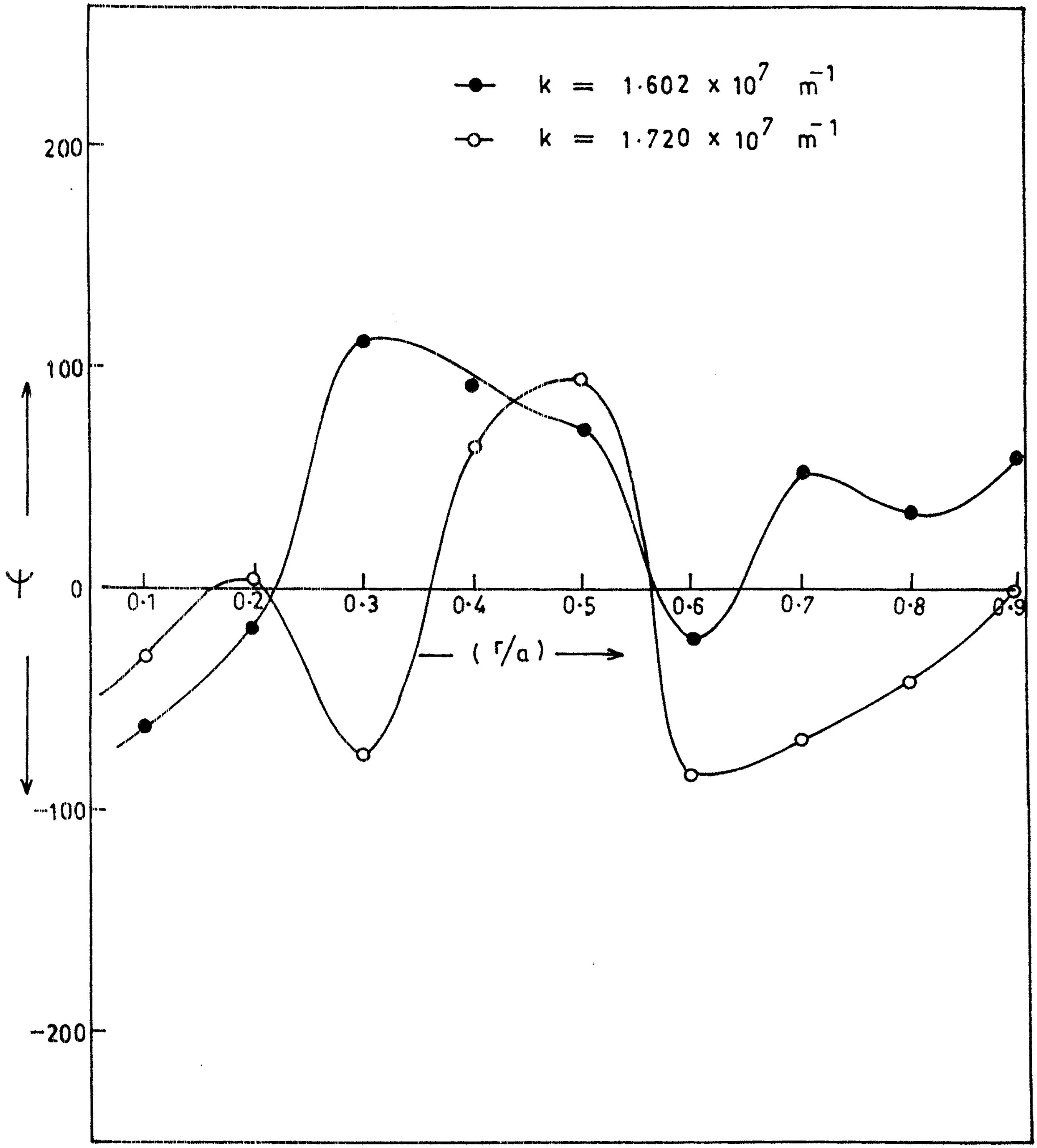


FIG. 4·15 — NONLINEAR FIELD DISTRIBUTIONS OF HE_{11} MODE FOR $k = 1{\cdot}602 \times 10^7 \; m^{-1}$ AND $1{\cdot}720 \times 10^7 \; m^{-1}$.

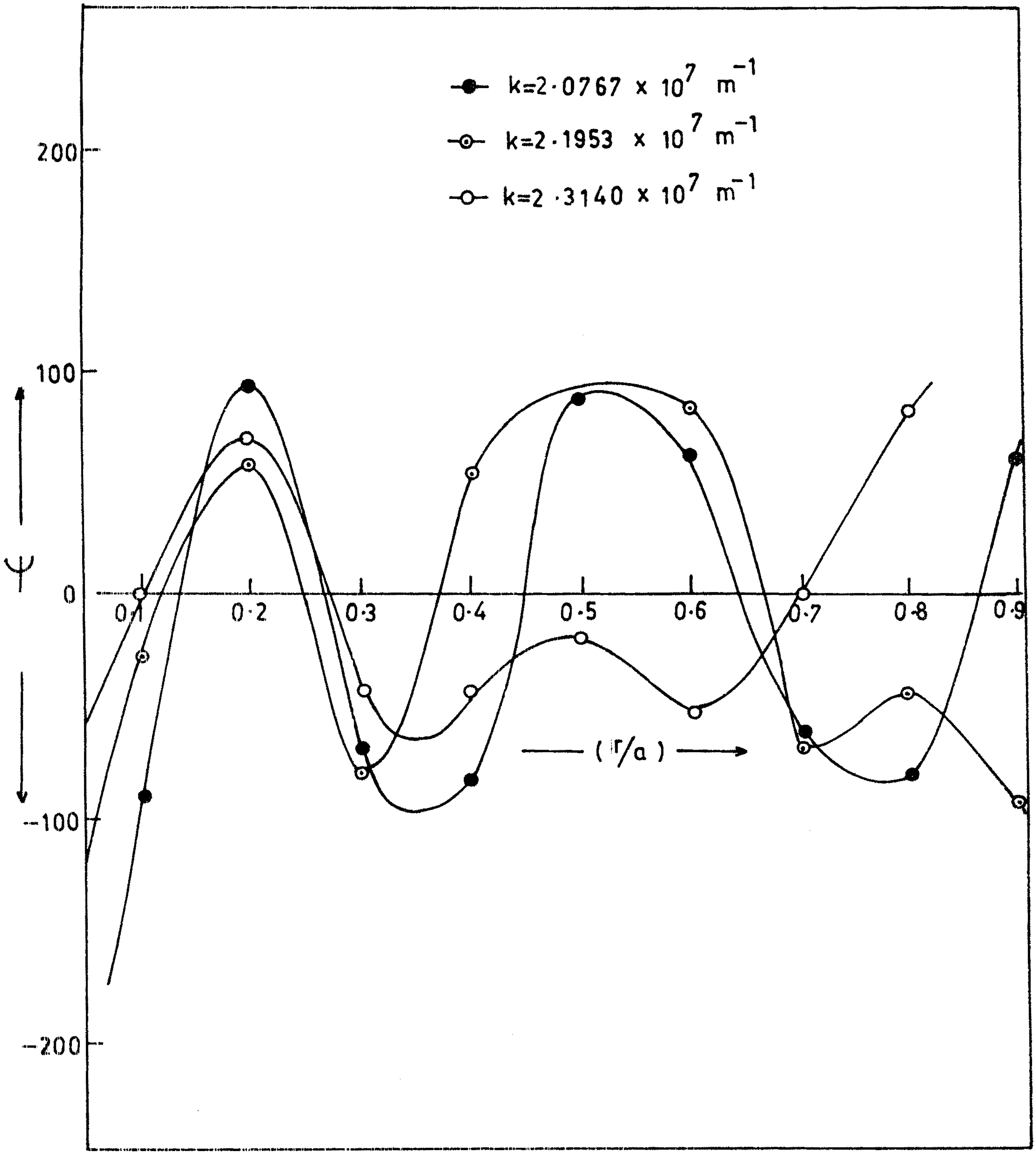


FIG. 4·16 — NONLINEAR FIELD DISTRIBUTIONS OF HE_{11} MODE FOR $k = 2{\cdot}0767 \times 10^7 m^{-1}$, $2{\cdot}1953 \times 10^7$ AND $2{\cdot}3140 \times 10^7 m^{-1}$.

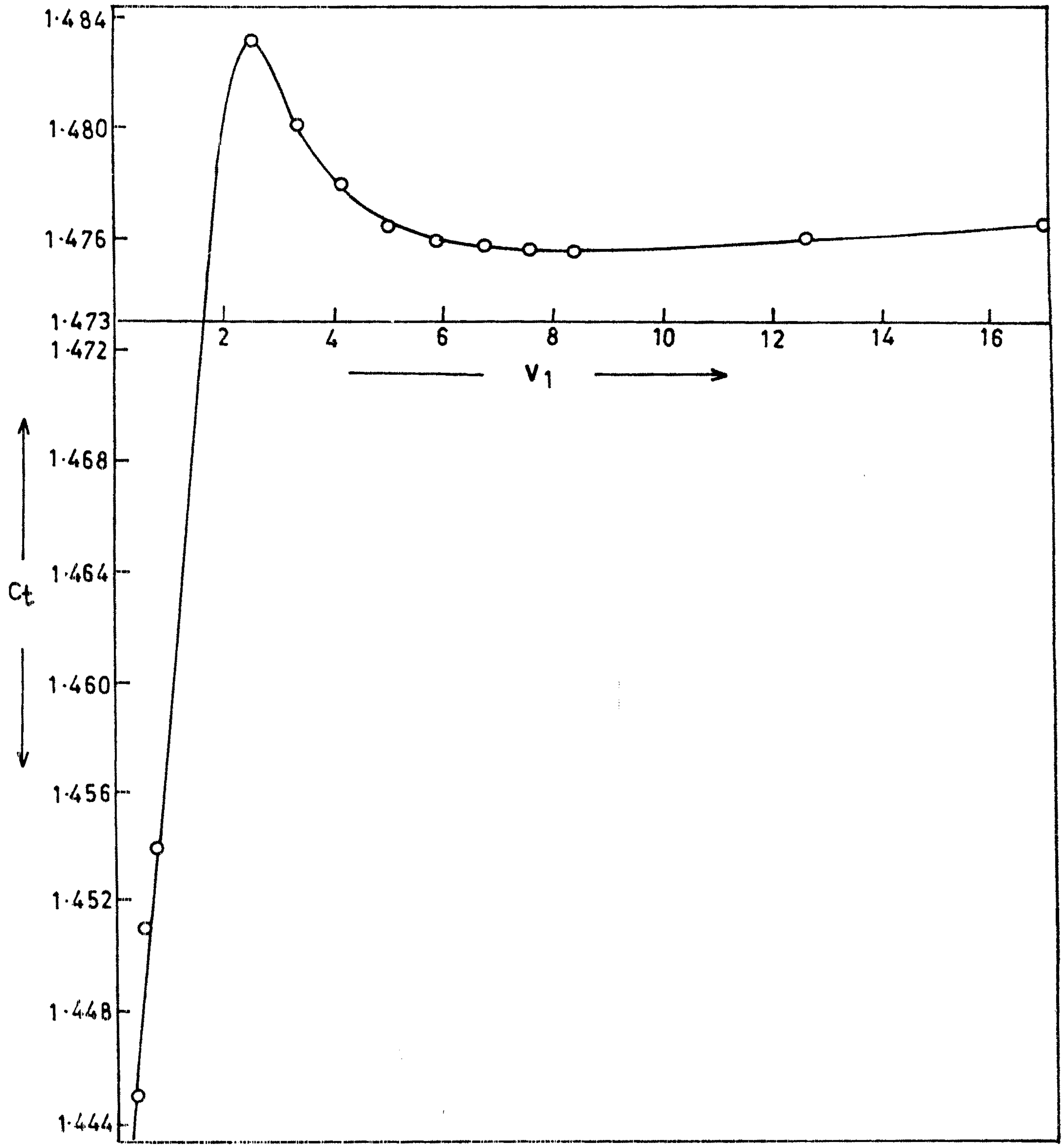


FIG. 4·17 — NORMALIZED DELAY TIME FOR HE_{11} MODE WITH $r/a = 0{\cdot}3$.

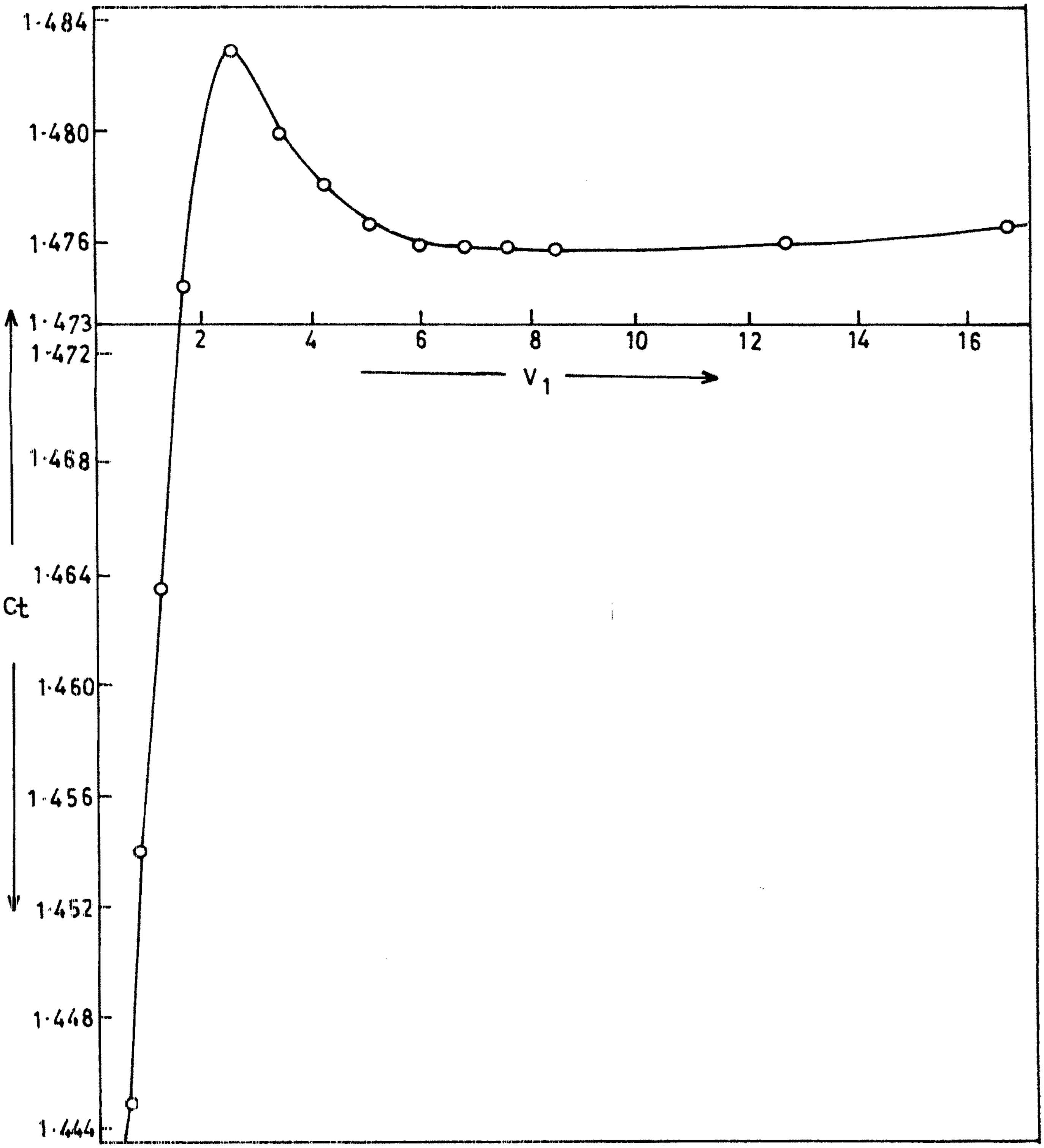


FIG. 4·18 — NORMALIZED DELAY TIME FOR HE_{11} MODE WITH $r/a = 0{\cdot}6$.

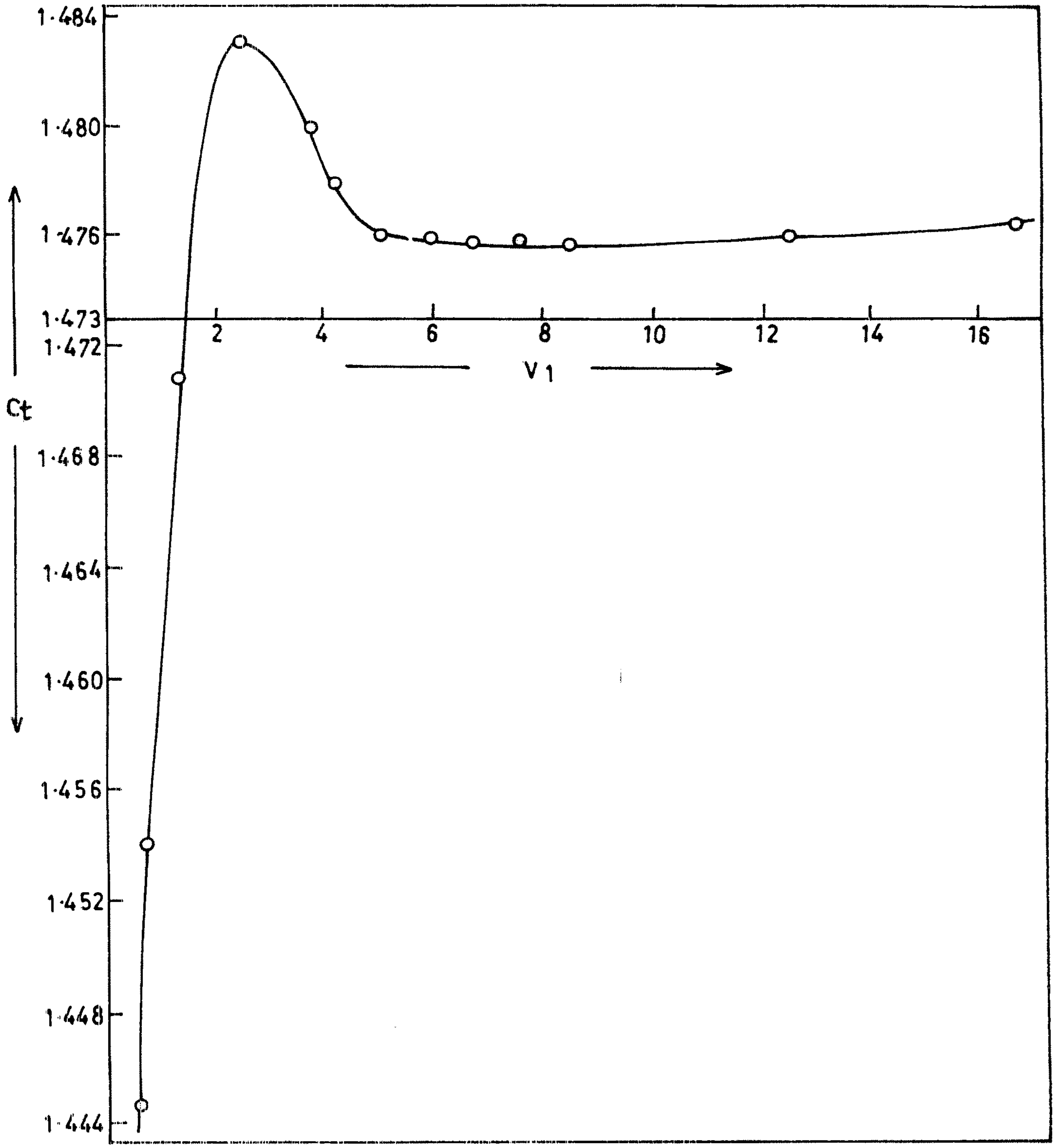


FIG. 4·19 — NORMALIZED DELAY TIME FOR HE_{11} MODE WITH $r/a = 0 \cdot 9$.