CHAPTER-.2

THEORIES OF OPTICAL FIBERS

CONTENTS

The analysis of wave propagation in optical fibers is carried out normally by the standard approach of wave theory.[1] The ray theory is also useful for this purpose though it is less general. The latter theory is advantageous in making the physical process of the picture more clear and comprehensible. At the outset we shall briefly summarize the salient features of the ray theory. Next we will consider the wave theory or mode theory in somewhat more detail.

2.1 RAY THEORY[1] :

This theory can be applied to uniform core multimode fibers and relatively thin graded-core fibers. However, the theory cannot be applied to single-mode or monomode fibers. The progapation of light is considered in terms of total internal reflection. The structure of optical fiber allows the core refractive index (n_0) be more than that of the clandding (n) and so the condition for total internal reflection is easily satisfied. Fig. 2.1 illustrates how light transmits through an uniform core fiber by a series of total internal reflections taking place at the core-cladding boundary.

In order that the total internal reflections should occur the rays are required to be entered the optical fiber with sufficiently shallow grazing angle (less than critical angle θ_c) called the acceptance angle θ_a(Fig. 2.2). The rays entering at this angle are called the meriodinal rays. Other rays making greater angles we known as skew rays which are not totally internally reflected. The condition for the total internal reflection of meriodinal rays is expressed as

$$\text{Sin } \theta_o < n_o \sqrt{2\Delta}$$

where θ_o = angle of incidence

$$\Delta \simeq \frac{n_o - n}{n_o} = \text{relative}$$

refractive index difference

Another useful quantity in the ray theory is the numerical aperture (NA) defined as

$$NA = (n_o^2 - n^2)^{1/2}$$

or $\quad NA = \text{Sin}(Q_a)$

or $\quad NA = n_o \; (2\,\Delta)^{1/2}$

NA measures the light collecting ability of a fiber. It is independent of the dimensions of the fiber.

The Skew rays follow a helical path through the fiber (Fig. 2.3). The acceptance angle for a Skew ray is given by,

$$\text{Sin } \theta_{as} = \frac{n_o}{n_{air} \text{Cos}\gamma} \left(1 - \frac{n^2}{n_o^2} \right)^2$$

where γ = angle between the core radius and the ray normal to the core axis.

The acceptance condition for the Skew rays is therefore written as,

$$n_{air} \sin \theta_{as} \cos \gamma = NA$$

Depending upon the value of $\cos \gamma$ the Skew rays are accepted at larger axial angles in a given fiber, than the meridional rays.

In non-uniform core fibers both meridional and Skew rays may exist, but the Skew rays follow more or less deformed paths.

BASIC EQUATIONS :

In cylindrical co-ordinates the ray equations determining the ray path in an inhomogeneous medium are written as,

$$\frac{d}{ds}\left(n\frac{dr}{ds}\right) - nr\left(\frac{d\theta}{ds}\right)^2 = \frac{dn}{dr} \qquad .....(2.1)$$

$$n\left(\frac{dr}{ds}\right)\left(\frac{d\theta}{ds}\right) + \frac{d}{ds}\left(nr\frac{d\theta}{ds}\right) = 0 \qquad ....(2.2)$$

$$\frac{d}{ds}\left(n\frac{dz}{ds}\right) = 0 \qquad .....(2.3)$$

In writting these component equations, an axially symmetric and uniform refractive index distribution has been assumed. Combining these equations appropriately and integrating under certain initial conditions one can obtain.

$$Z = \int N_i \{ [\frac{n(r)}{n_i}]^2 + [1 - (\frac{r_o}{r})^2] (X_i M_i - Y_i L_i) - N_i^2 \}^{-1/2} \, dr \qquad \ldots.(2.4)$$

where N_i = direction cosine of incident ray

n_i = refractive index at the incident point

L_i =, M_i = directions in a plane $\perp$ to Z axis

X_i =, Y_i Co-ordinates of the fiber end where the light ray is incident

Eq.(2.4) useful to compute the path of any ray provided we know the refractive index distribution n(r) and the launch conditions X_i, Y_i, L_i and M_i

e.g. with $n^2(r) = n_o^2 [1 - (2r)^2]$ we can obtain the following ray path equation for meriodinal rays

$$r = C \sin (\frac{\alpha n_o Z}{n_i N_i} + \psi) \qquad \ldots..(2.5)$$

where ψ = constant

$$C = \alpha^{-1} [1 - N_i^2 (1 - \alpha^2 N_i^2)]^{1/2}$$

Eq.(2.5) represents the path of an undulating ray having a period length of

$$\Lambda = (\frac{2\pi N_i}{\alpha}) [1 - (\alpha r_o)^2]^{1/2}$$

$$\simeq \frac{2\pi}{\alpha} \qquad \ldots..(2.6)$$

It is found that the period length Λ is a constant for a refractive index profile close to $n^2(r)$ given above. As a result the average axial velocity of the ray becomes independent of the launch conditions.

2.2 ELECTROMAGNETIC MODE THEORY[1,2] :

A mode is defined as an electromagnetic wave which propagates along a waveguide with a well defined phase velocity, group velocity cross-sectional intensity distribution and polarization. Modes characterize the waveguide structure in terms of its e.m. resonances. Each mode can be described as a superposition of uniform plane waves propagating at a fixed angle with the guide axis .

WAVE EQUATIONS :

For axially symmetric waves propagating through optical fibers, the wave equations are written in terms of cylindrical co-ordinates.

$$\frac{\partial^2 E_z}{\partial r^2} + 1/r \frac{\partial E_z}{\partial r} + 1/r^2 \frac{\partial^2 E_z}{\partial \theta^2} + \beta_t^2 \; E_z = 0 \quad .....(2.7)$$

and,

$$\frac{\partial^2 H_z}{\partial r^2} + 1/r \frac{\partial H_z}{\partial r} + 1/r^2 \frac{\partial^2 H_z}{\partial \theta^2} + \beta_t^2 \; H_z = 0 \quad ....(2.8)$$

By separating the variables with E_z(or H_z) = $R_z(r)$ Θ_zO one can obtain the general solutions of the two equations as.

$$\Theta_z(\theta) = \begin{matrix} \cos(n\theta + \phi) \\ \sin(n\theta + \phi) \end{matrix} \quad .....(2.9)$$

$$R_z(r) = \left.\begin{matrix} A\, J_n(\beta_t r) + A'\, N_n(\beta_t r), & \beta_t > 0 \\ C\, K_n(1|\beta_t| r) + C'\, A_n(|\beta_t| r), & \beta_t < 0 \end{matrix}\right| \quad ....(2.10)$$

J_n and N_n = n^{th} order Bessel functions of first kind
K_n and I_n = n^{th} order Bessel functions of 2^{nd} kind.

The functions J_n are oscillatory with their amplitude gradually decreasing. These are suitable to represent the core-fields. On the other hand the exponentially decaying K_n functions can well represent the fields in the cladding. The other functions are insignificant in the theory of uniform-core fiber.

The phase and group velocities of a plane monocromatic light wave are defined as,

$V_P = W/\beta$ and $V_g = \delta W/\delta\beta$

If the propagation takes place in an infinite medium of refractive index n_o, then

$\beta = n_o W/C$

So that, $V_P = C/n_o$ and $V_g = C/N_1$

Where $N_1 = (n_o - \lambda dn_o/d\lambda)$ = group index of the guide.

HYBRID MODES OF OPTICAL FIBER[3] :

The optical fibers usually satisfy the condition $\Delta << 1$ for weakly guiding approximation. Under this condition the fibers have additional modes called the 'Hybrid modes' which are denoted by $HE_{\ell m}$ and $EH_{\ell m}$.

These modes arise depending upon whether the components of H or E makes the larger contribution to the transverse field. Since Δ is very small. Usually the HE-EH mode pairs occur. These pairs have almost identical propogation constants. Such modes are said to be degenerate. All the modes in optical fibers are commonly designated by the symbol $LP_{\ell m}$. The subscripts ℓ and m are related to the electric field intensity profile for a particular LP mode. LP stands for linearly polarised waves. The intensity distributions of a few lower order LP modes are shown in Fig. (2.4).

MODAL PARAMETERS :

In the mode theory the following three modal parameters are very important in discussing the propagation of particular mode.

$$u^2 = (K^2 n_o^2 - \beta^2) a^2$$

$$W^2 = (\beta^2 - K^2 n^2) a^2$$

and,

$$V^2 = K^2 a^2 (n_o^2 - n^2) \qquad .....(2.11)$$

Here, β is called the longitudinal propagation constant while V is known as the normalized frequency. In terms of these parameters the propagation of the given mode is discussed with the help of the following relation.

$$b = 1 = u^2/v^2$$

where, b = normalized propagation constant.

The propagation constants are determined with the help of the following condition at the core cladding bouncary. Under weakly guiding approximation such a boundary condition for hybrid modes is expressed in a simpler from given below;

$$\frac{J'_n(u)}{uJ_n(u)} + \frac{K'_n(w)}{Wk_n(W)} = \pm n\ (1/u^2 + 1/w^2) \quad .....(2.12)$$

This equation gives two sets of solutions for the positive and negative signs. When the sign is positive we obtain.

$$- J_1(u)/uJ_o(u) = K_1(W)/WK_o(W)$$

This applies to the EH mode. On the other hand for negative sign; we get,

$$J_{n-1}(u)/\ uJ_n(u) = K_{n-1}(W)/WK_n(W) \quad .....(2.13)$$

This is applicable to the HE mode.

CUTOFF FREQUENCIES :

As pointed out above, Bessel function Kn decays exponencially. Therefore, when $Kn < \beta$ is satisfied the electromagnetic field in the cladding decays exponentialy. This means the electromagnetic energy can be assumed to be confined in the core. However, for $\beta = Kn$ it is found that the field in the cladding does not decay i.e. the energy can not be confined in core. The condition $\beta = Kn$ is called the cutoff condition and the corresponding light frequency is known as the cutoff frequency. In optical fibers energy transmission is possible at frequencies well above the cutoff frequency. At such frequencies the transmission modes are known as 'propagation modes."

For $\beta < Kn$, we obtain spatially oscillatory solutions in the cladding region. This leads to drastic radiation losses, so that the wave can not propagate axially. Such a state of the wave is known as the "radiation mode". However, there is a possiblity of wave propagating over certain distance into the cladding. In spite of the fact that, $\beta < Kn$, such a state corresponds to the "leaky modes."

At cutoff frequencies W = 0, so that, u = v. The normalized cutoff frequencies for the $LP_{m\ell}$ modes are defined as,

$$V_c = J_{(m-\ell)\ell}$$

Where, the radial mode number ℓ represents the number of radial variations of the field in the core region, while the mode number m denotes the number of variations of E_x or E_y field component observed in one rotation.

2.3 ANALYSIS OF STEP INDEX FIBERS ON MODE THEORY[3,4]:

The single mode step index fiber is advantageous over the multimode step index fiber due to its low inftermodal dispersion and the subsequent increase in the maximum bandwidth. In multimode step index fiber, considerable dispersion occurs due to various group velocities of the propagating modes. Due to the oscillatory Bessel functions $J_\nu(x)$ involved in the modal equation, there will be m roots of the equation for a given ν value. The roots are designated by $\beta_{\nu m}$ and the corresponding modes are either $TE_{\nu m}$, $TM_{\nu m}$ $EH_{\nu m}$ or $HE_{\nu m}$. For the dielectric fiber waveguide all modes are hybrid modes except those for which $\nu = 0$. The number of modes which can exist in the optical fiber waveguide is given by the expression,

$$M = \frac{2\pi^2 a^2}{2}\left(n_o^2 - n^2\right) V^2/2 \qquad \text{.....(2.14)}$$

It can also be represented in terms of a normalized propagation constant defined by,

$$b = \frac{a^2 W^2}{V^2} = \frac{(\beta/K)^2 - n^2}{n_o^2 - n^2}$$

The graphical representation of this constant as a function of normalized frequency V is illustrated in Fig.2.5 for a first few low-order modes.

Various modes in the optical fiber waveguide obey different cut-off conditions in terms of the Bessel functions J_ν (ua) e.g.. All the EH $_{\nu m}$ and HE $_{\nu m}$ modes with $\nu \geqslant 2$ are cut-off when,

$$\left(\frac{n_o^2}{n^2} + 1 \right) J_{\nu-1}(ua) = \left(\frac{ua}{\nu-1}\right) J_\nu(ua) \qquad .....(2.15)$$

For $V \geqslant 2.405$, the lowest order Bessel function J_0 is zero so that all modes (except HE_{11}) are cut-off. However, the HE_{11} mode has no cutoff. It ceases to exist only when the core diameter becomes zero.

The e.m. energy of a guided mode is carried partially in the core and partially in the cladding. As the cut-off is approached the field penetrates further into the cladding region and a greater percentage of energy travels in the cladding. According to Gloge,[4] the relative powers in the core and cladding of a step index fiber for a particular mode ν are given by,

$$\frac{P_{core}}{P} = \left(1 - \frac{u^2}{V^2} \right) \left[1 - \frac{J_\nu^2(ua)}{J_{\nu+1}(ua)\, J_{\nu-1}(ua)} \right. \qquad .....(2.16)$$

and,

$$\frac{P_{clad}}{P} = 1 - \frac{P_{core}}{P} \quad .....(2.17)$$

Were P = total power in the mode ν. The fractional power flow in the cladding of a step index fiber as a function of normalized frequency V is depicted in Fig. 2.6.

2.4 ANALYSIS OF GRADED-INDEX FIBERS ON MODE THEORY[3]:

In the graded-index fiber design, the core refractive index (n_o) decreases continuously with the radial distance (r) from the center of the fiber, but is generally constant in the cladding. The index variation is represented differently in the core and cladding regions as given by the following expressions,

$$n(r) = n_o \, [1 - 2\,\Delta(r/a)^{\alpha}]^{1/2}, \text{ core } (r < a) \quad .....(2.18)$$

$$= n_o \, (1-2\,\Delta)^{1/2} \approx n, \text{ cladding } (r \geq a) \quad .....(2.19)$$

The dimensionless parameter α defines the shape of the index profiles as illustrated in Fig. 2.7. These are called α - profiles. For $\alpha = 2$, e.g., we have the commonly used parabolic profile.

The numerical aperture (NA) in graded index fibers is a function of position across the core end face as against the constant NA for the step index fibers.

$$NA(r) = [\, n^2(r) - n^2]^{1/2} \simeq NA(0)\, \sqrt{1-(r/a)^{\alpha}},$$

$$r \leq a \qquad \text{.....(2.20)}$$

$$= 0, \; r > a \qquad \text{.....(2.21)}$$

Where NA(0) = axial NA $\simeq n_0 \sqrt{2\Delta}$

Thus NA(r) decreases from NA(0) value to zero as we move from fiber axis to the core-cladding boundary.

The analysis of graded-index fibers by mode theory is widely carried out by the WKB method. In this method, the solution of the modal equation is given an asymptotic representation with the help of a parameter which varies slowly over the desired range of the modal equation e.g. the parameter chosen for the present case is the refractive index n(r) which varies only slightly over distances of the order of an optical wavelength.

Expanding the parameter S(r) in the form of a series

$$S(r) = S_0 + 1/K\, S_1 + \; ....$$

and putting into the wave equation we obtain, after integration,

$$KS_0 = \int_{r_1}^{r_2} [k^2 n^2(r) - \beta^2 - \ell^2/r^2]^{1/2} \, dr \qquad \text{....(2.22)}$$

where $\ell = \nu - 1$

If the radical in the integrand is greater than zero, so is real so that a given mode is bound in the fiber core. For a given mode ν_1, there are two values r_1 and r_2 for which the radical is zero Guided modes exist for r values lying between r_1 and r_2. For other r values, so becomes imaginary so that we have decaying fields. We shall again discuss WKB method in chapter 4.

The number of bound modes in a graded index fiber is given by,

$$m(\beta) = a^2k^2n_o^2 \,\Delta\alpha/\alpha + 2 \left[\frac{kn_o^2 - \beta^2}{2\Delta k^2 n_o^2} \right]^{2+\alpha/\alpha} \quad \ldots.(2.23)$$

All bound modes in a fiber must have $\beta \geqslant kn$ which leads to the maximum number of bound modes to be equal to ,

$$M = \alpha/\alpha + 2\, a^2k^2n_o^2\Delta \quad \ldots.(2.24)$$

2.5 SUMMARY :

In this chapter first we have studied in brief the distinguishing features of the ray theory for uniform core fibers. This theory though approximate gives a clear physical understanding of the propagation of light in optical fibers in terms of meridional and Skew rays. However, the analysis of the phenomena is most commonly worked out on the basis of e.m. wave theory or mode theory. We have also given a recipe of this theory elucidating the important concepts involved in it. The application of this theory to analyse the light propagation in step index fibers forms the next topic of discussion in this chapter. A summary of the essential features of the analysis have been briefly outlined. Next the analysis of the wave propagation in the Graded-Index fibers is presented. These two analysis have been covered in this chapter, because the present work is concerned with the propagation of HE_{11} and HE_{21} laser modes through the step-Index Fibers behaving like the Graded-Index Fibers on account of the intensity dependent R.I. profiles.

R E F E R E N C E S

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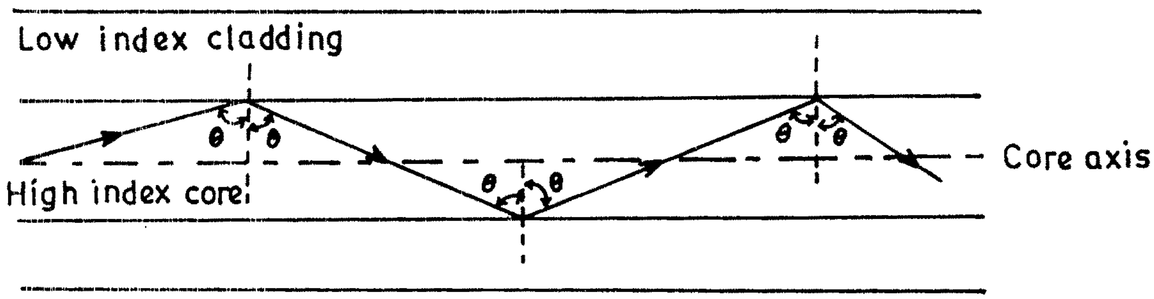

FIG. 2.1 — THE TRANSMISSION OF A LIGHT RAY IN A PERFECT OPTICAL FIBER.

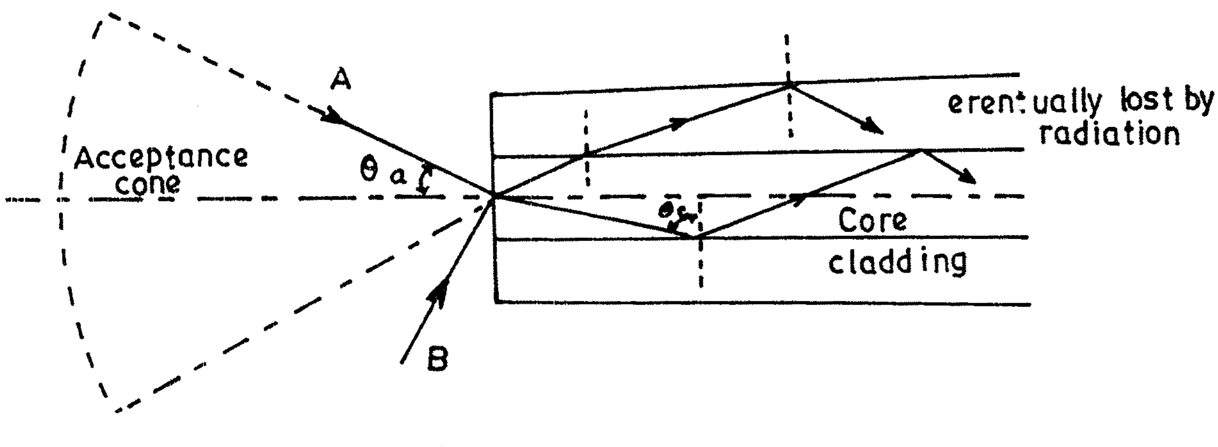

FIG. 2.2 — THE ACCEPTANCE ANGLE θ_a WHEN LAUNCHING LIGHT INTO AN OPTICAL FIBER.

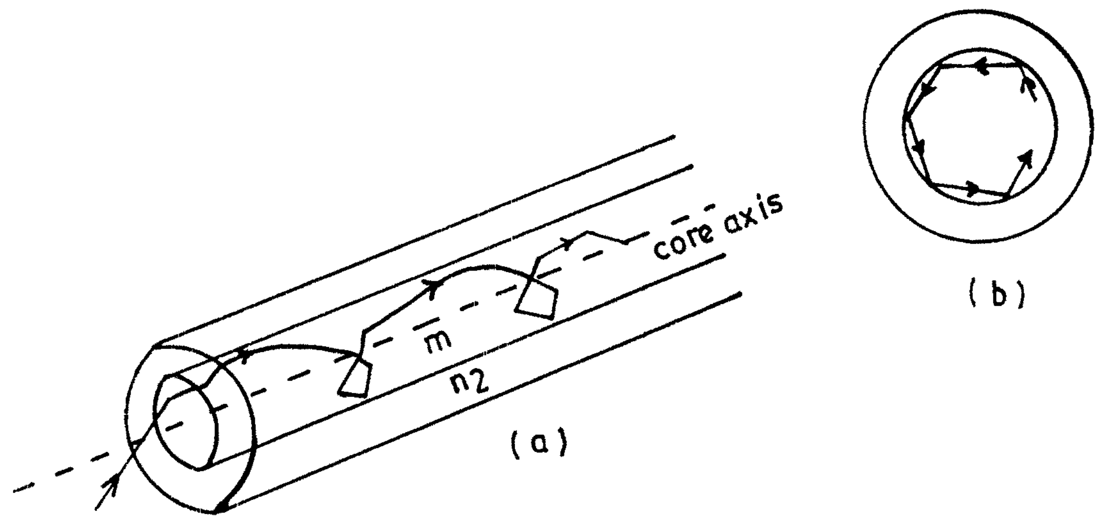

FIG. 2·3 — THE HELICAL PATH TAKEN BY A SKEW RAY IN AN OPTICAL FIBER.

Lp-mode designations	Traditional designations	
LP_{01}	HE_{11}	$n = 1$, $m = 0$, $l = 1$
LP_{11}	TE_{01}	$n = 0$, $m = 1$, $l = 1$
	TM_{01}	$n = 0$, $m = 1$, $l = 1$
	HE_{21}	$n = 2$, $m = 1$, $l = 1$
LP_{21}	EH_{11}	$n = 1$, $m = 2$, $l = 1$
	HE_{31}	$n = 3$, $m = 2$, $l = 1$

Intensity distribution of Ex

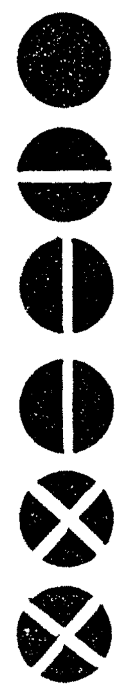

FIG. 2·4 — THE ELECTRIC FIELD CONFIGURATION FOR THE THREE LOWEST Lp MODES.

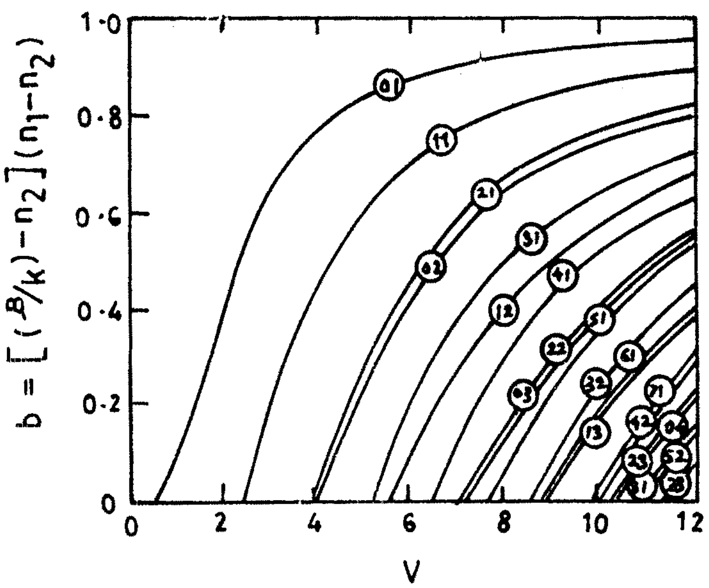

FIG. 2·5 — THE NORMALIZED PROPAGATION PARAMETER b AS A FUNCTION OF THE V NUMBER. WHEN $\nu \neq 1$. THE CURVE NUMBERS νm GIVE THE HE_{2m} , TE_{0m} & TM_{0m} MODES.

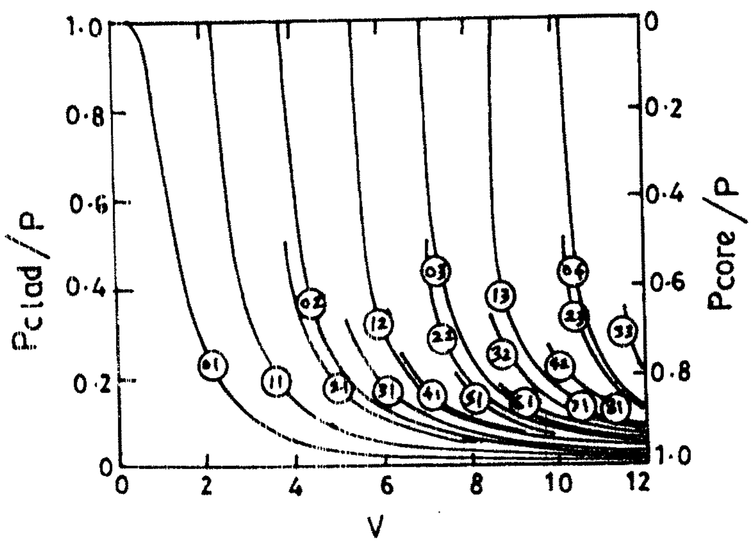

FIG. 2·6 — FRACTIONAL POWER FLOW IN THE CLADDING OF A STEP-INDEX OPTICAL FIBER AS A FUNCTION OF V. WHEN $\nu \neq 1$, THE CURVE NUMBERS νm DESIGNATE THE $HE_{\nu+1\,m}$ AND $EH_{\nu-1\,m}$ MODES. FOR $\nu=1$, THE CURVE NUMBERS νm GIVE THE HE_{2m} TE_{0m} & TM_{0m} MODES.

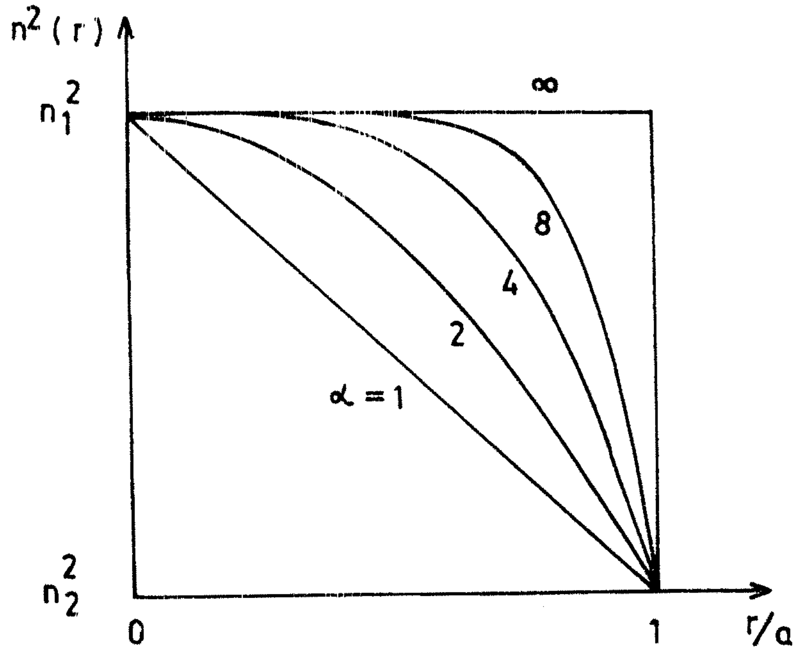

FIG. 2·7 — REFRACTIVE INDEX DISTRIBUTIONS FOR THE SO-CALLED 'ALPHA-PROFILES'.