CHAPTER - 4

A STUDY OF NONLINEAR SURFACE WAVES AND GUIDED WAVES IN PLANAR OPTICAL WAVEGUIDE

4.1 Introduction :

In the recent past considerable interest has been developed in the optics of layered media with nonlinear dielectric properties. Many interface phenomena[1] are found to be associated with reflection and refraction of a strong plane wave from the surface of a nonlinear medium having intensity-dependent refractive index. In the presence of negative nonlinearity, it has been shown that longitudinally inhomogeneous travelling waves (LITW) can be excited such that the intensity and angle of propagation would vary perpendicular to the interface. Similarly nonlinear surface waves (NSW) can be excited[2] at the interface if the nonlinear medium has a lower refractive index with positive optical Kerr coefficient. Kaplan[1] has obtained conditions for the excitation of all the possible types of waves in a medium having negative nonlinearity. The above mentioned two types of waves give rise to a number of nonlinear interface phenomena like bistable reflectivity and hysteresis, nonlinear self-deflection of refracted rays and self-induced transparancy of nonlinear interface.

The salient feature[3] of the nonlinear surface wave is that its dispersion relation contains the square of the electric field as a parameter apart from the frequency and the wave vector. Further, such waves can be directly excited by bounded light beam which is incident on the interface. Tomlinson[2] has obtained an exact solution of Maxwell's equations which describe the propagation of s-polarised (i.e. TE) nonlinear surface waves. He has shown that an interface between two dielectric media (with the lower refractive index material having a positive nonlinearity) can support a two-dimensional optical surface wave which propagates along the interface with a constant shape and intensity.

Apart from these studies the propagation phenomena of nonlinear film-guided waves (NGW) have been investigated by considering multilayer dielectric structures[4-8]. Dispersion relations and power flow expressions for such modes have been obtained. Holland's theoretical investigations[8] have shown that in a symmetric structure there exist nonlinear guided waves with symmetric field profiles. In addition there exist asymmetric guided waves which are closely related to the surface waves supported at an interface.

In view of their unique features and potential applications in all the integrated optical devices, the investigation of surface waves and nonlinear guided waves has attracted considerable interest. In the present work we have first revisited Tomlinson's analysis[2] with a view to get aquainted with the method of analysing the surface wave propagation along a nonlinear interface. In order to examine the intensity effect on the guided waves we have next investigated the nonlinear propagation of TE waves in both the high- and low-refractive index guiding layers.

4.2 Revisiting Tomlinson's Analysis[2] of Surface Wave at a Nonlinear Interface (Present Work) :

We consider an interface coinciding with x-y-plane and separating linear and nonlinear dielectric media which are on negative and positive z-sides respectively (Fig.4.1). The dielectric constant of the nonlinear medium is assumed to be

$$\epsilon(x,y,z) = \epsilon_o + \Delta\epsilon + \epsilon_2|E(x,y,z)|^2 \qquad \ldots\ldots (4.1)$$

where ϵ_o = dielectric constant of linear medium

$\Delta\epsilon$ = zero-field refractive index difference between the two media and

ϵ_2 = nonlinear coefficient.

We assume that the interface supports a two-dimensional optical surface wave which propagates along x-axis independent of y-co-ordinate. The existance of surface wave demands that $\Delta\epsilon < 0$ and $\epsilon_2 > 0$ i.e. the lower refractive index material (i.e. nonlinear medium) has a positive optical Kerr coefficient. The electric field of such a wave is supposed to be described by the scalar wave equation

$$\nabla^2 E + \epsilon \frac{\omega^2}{c^2} E = 0 \qquad \ldots\ldots (4.2)$$

We guess the following solutions of this equation for the two media under consideration.

$$E(x,y,z) = E_{1o}\exp(ik_x x)\exp(k_{1z}z) , \quad z < 0 \quad \ldots (4.3)$$

$$E(x,y,z) = E_{2o}\exp(ik_x x)\ \mathrm{sech}\,[k_{2z}(z-z_o)] , \quad z > 0 \quad \ldots (4.4)$$

The functional forms for the solutions have been chosen so as to consider the propagation of the surface wave along the interface with a constant shape and amplitude. With these expressions we solve the wave equation viz. Eq.(4.2) for the field parameters related to the beam shape and associated critical powers.

4.2.1 Field Parameters :

Linear Medium ($z < 0$) :

Differentiating Eq.(4.3) twice with respect to x and z separately we can obtain

$$\frac{\partial^2 E}{\partial x^2} = -k_x^2 E \quad , \quad \frac{\partial^2 E}{\partial z^2} = -k_{1z}^2 E$$

so that

$$\nabla^2 E = (k_{1z}^2 - k_x^2) E$$

Putting this in Eq.(4.2) we get the condition

$$(-k_x^2 + k_{1z}^2 + k_o^2) E = 0 \qquad \ldots\ldots (4.5)$$

where k_o is defined as

$$k_o = (\epsilon_o)^{1/2} \omega / c \qquad \ldots\ldots (4.5a)$$

This condition is satisfied only when

$$k_{1z}^2 = k_x^2 - k_o^2 \qquad \ldots\ldots (4.6)$$

Nonlinear Medium ($z > 0$) :

Differentiating Eq.(4.4) twice with respect to x and z we obtain the following expressions.

$$\frac{\partial^2 E}{\partial x^2} = -k_x^2 E$$

$$\frac{\partial E}{\partial z} = -E_{2o} \exp(ik_x x) k_{2z} \operatorname{sech}[..] \tanh[..]$$

$$\frac{\partial^2 E}{\partial z^2} = k_{2z}^2 \left[E_{2o} \exp(..) \operatorname{sech}[..] \right] \left\{ 1 - 2 \operatorname{sech}^2[..] \right\}$$

This is simplified further using Eq.(4.4)

$$\frac{\partial^2 E}{\partial z^2} = k_{2z}^2 E - \frac{2 k_{2z}^2 E}{E_o^2} |E|^2$$

Hence we obtain

$$\nabla^2 E = - k_x^2 E + k_{2z}^2 E - \frac{2 k_{2z}^2 E}{E_o^2} |E|^2 \qquad \ldots\ldots \quad (4.7a)$$

Now

$$\epsilon(x,y,z) \frac{\omega^2}{c^2} E =$$

$$= \left[\epsilon_o - \Delta\epsilon + \epsilon_2 |E|^2 \right] \frac{\omega^2}{c^2} E$$

$$= \epsilon_o \frac{\omega^2}{c^2} E - \left(\frac{\Delta\epsilon}{\epsilon_o}\right) \left(\epsilon_o \frac{\omega^2}{c^2}\right) E + \frac{\epsilon_2 |E|^2}{\epsilon_o} \left(\epsilon_o \frac{\omega^2}{c^2}\right) E$$

$$= k_o^2 E - \psi_c^2 k_o^2 E + \frac{\epsilon_2 k_o^2}{\epsilon_o} |E|^2 E \qquad \ldots (4.7b)$$

where we have used Eqs.(4.1) & (4.5a) with $\Delta\epsilon < 0$ and the definition of ψ_c as given below.

$$\psi_c^2 = \frac{|\Delta\epsilon|}{\epsilon_o} \qquad \ldots\ldots \quad (4.7c)$$

Physically $\sin^{-1}\psi_c$ represents the total internal reflection as measured from the interface in the limit of zero intensity.

Putting the three Eqs.(4.7) in Eq.(4.2) and simplifying we get

$$(- k_x^2 + k_{2z}^2 + k_o^2 - \psi_c^2 k_o^2) E + \left(- \frac{2 k_{2z}^2}{E_o^2} + \frac{\epsilon_2}{\epsilon_o} k_o^2 \right) |E|^2 E = 0$$

$$\ldots\ldots \quad (4.8)$$

This will hold good provided the following two conditions are satisfied.

$$k_{2z}^2 = k_x^2 - k_o^2 (1 - \psi_c^2) \qquad \ldots\ldots (4.9a)$$

$$k_{2z}^2 = \frac{\epsilon_2 E_{2o}^2}{2|\Delta\epsilon|} k_o^2 \psi_c^2 \qquad \ldots\ldots (4.9b)$$

The above assumed solutions (Eqs. 4.3 & 4.4) should satisfy the boundary conditions that E and $\partial E / \partial z$ must be continuous at the interface $z = 0$. For the continuity of E at $z = 0$ we get

$$E_{1o} = E_{2o} \operatorname{sech}(k_{2z} z_o) \qquad \ldots\ldots (4.10)$$

Differentiating Eqs.(4.3) and (4.4) with respect to z and equating the values at $z = 0$.

$$E_{1o} k_{1z} = E_{2o} k_{2z} \operatorname{sech}(-k_{2z} z_o) \tanh(-k_{2z} z_o)$$

where we have used the trigonometric relations

$$\operatorname{sech}(-\theta) = \operatorname{sech}(\theta) \quad \text{and} \quad \tanh(-\theta) = -\tanh(\theta)$$

Simplifying the above expression further we obtain

$$E_{1o} = E_{2o} \frac{k_{2z}}{k_{1z}} \frac{\sinh(k_{2z} z_o)}{\cosh^2(k_{2z} z_o)} \qquad \ldots\ldots (4.11)$$

Eqs.(4.6), (4.9a & b), (4.10) and (4.11) are the five conditions which relate six parameters namely k_o, k_x, k_{1z}, k_{2z}, ψ_c and z_o involved in the field distribution function. All these parameters can be expressed in terms of a single independent parameter. This is chosen as a quantity D defined by

$$\epsilon_2 E_{2o}^2 = 2|\Delta\epsilon| (1 + D) \qquad \ldots\ldots (4.12)$$

Using this definition Eqs.(4.6) and (4.9) are rewritten as explained below.

Subtracting Eq.(4.9a) from Eq.(4.6) and using Eq.(4.9b) we obtain

$$k_{1z}^2 = k_o^2 \psi_c^2 \left[\frac{\epsilon_2 E_{2o}^2}{2 |\Delta\epsilon|} - 1 \right]$$

From Eq.(4.12) we deduce

$$\frac{\epsilon_2 E_{2o}^2}{2 |\Delta\epsilon|} = 1 + D$$

Using this in the above equation we get

$$k_{1z} = k_o \psi_c \sqrt{D} \qquad \ldots\ldots (4.13)$$

Again subtracting Eq.(4.9a) from (4.6) and using Eq.(4.13) we obtain

$$k_{2z} = k_o \psi_c \sqrt{1 + D} \qquad \ldots\ldots (4.14)$$

Similarly using Eq.(4.13) in Eq.(4.6) we get

$$k_x = k_o \psi_c \sqrt{\psi_c^{-2} + D} \qquad \ldots\ldots (4.15)$$

Further dividing Eq.(4.10) by Eq.(4.11) we deduce

$$\tanh \left[k_{2z}\ z_o\right] = \frac{k_{1z}}{k_{2z}} = \frac{\sqrt{D}}{\sqrt{1 + D}} \qquad \ldots\ldots (4.15a)$$

so that $\quad \operatorname{sech} [\ldots] = \dfrac{1}{\sqrt{1 + D}}$

where we have utilised the conditions given by Eqs.(4.13) and (4.14).

Putting this in Eq.(4.10), squaring and multiplying by ϵ_2 we get

$$\epsilon_2\ E_{1o}^2 = 2\ |\Delta\epsilon| \qquad \ldots\ldots (4.16)$$

From Eq.(4.10) we can deduce

$$k_{2z}\ z_o = \cosh^{-1}(X)$$

i.e.

$$z_o = \frac{\ln \left[X + \sqrt{X^2 - 1} \right]}{k_{2z}} \qquad \ldots\ldots (4.17)$$

with $X = E_{2o} / E_{1o}$

Now from Eqs.(4.12) and (4.16) we have

$$\frac{E_{2o}}{E_{1o}} = \sqrt{1 + D}$$

Hence Eq.(4.17) is rewritten as

$$z_o = \frac{\ln \left[\sqrt{1 + D} + \sqrt{D} \right]}{k_o \psi_c \sqrt{1 + D}} \qquad \ldots\ldots (4.18)$$

This parameter gives the position of the peak intensity of the surface wave. It has a maximum value of 0.663/($k_o \psi_o$) which is obtainable at D = 2.277. In order that the surface wave should exist, the three parameters k_{1z}, k_{2z} and k_x must be real. This means from Eq.(4.13), the condition D > 0 should be satisfied. As a result Eq.(4.12) implies that the field-induced refractive index change $(\epsilon_2 E^2_{2o})$ at z_o should be at least twice the zero-field index difference $|\Delta\epsilon|$. This minimum value of $(\epsilon_2 E^2_{2o})$ is equal to the value of $(\epsilon_2 E^2_{1o})$ given by Eq.(4.16). The latter equation means that, in the presence of the surface wave with critical power P, the field-induced index-change at the interface is always exactly twice the zero-field index difference, independent of the value of D.

4.2.2 Critical Power of the Surface Wave : Correction of Tomlinson's Expression :

The critical power carried in the surface wave per unit distance along y-direction is given by

$$P = k_x \int_{-\infty}^{\infty} |E|^2 \, dz \qquad \ldots\ldots (4.19)$$

The integral in this equation should be solved for the three regions.

i) linear medium ($z = -\infty$ to 0)

ii) nonlinear medium ($z = 0$ to z_o)

iii) nonlinear medium ($z = z_o$ to ∞)

Accordingly we write

$$P = P_L + P_{NLI} + P_{NLII} \qquad \ldots\ldots (4.20)$$

with $$P_L = k_x \int_{-\infty}^{o} |E|^2 \, dz \qquad \text{..... (4.20a)}$$

$$P_{NLI} = k_x \int_{o}^{z_o} |E|^2 \, dz \qquad \text{..... (4.20b)}$$

$$P_{NLII} = k_x \int_{z_o}^{\infty} |E|^2 \, dz \qquad \text{..... (4.20c)}$$

The integrals are evaluated as explained below :

We first divide Eq.(4.16) by (4.13) and then Eq.(4.12) by (4.14) and deduce the following ratios :

$$\frac{E_{1o}^2}{k_{1z}} = \frac{2\ |\Delta\epsilon|}{\epsilon_2 k_o \psi_c} \frac{1}{\sqrt{D}} \qquad \text{..... (4.20d)}$$

$$\frac{E_{2o}^2}{k_{2z}} = \frac{2\ |\Delta\epsilon|}{\epsilon_2 k_o \psi_c} \sqrt{1 + D} \qquad \text{..... (4.20e)}$$

Using Eq.(4.3) in Eq.(4.20a) we write

$$P_L = k_x \int_{-\infty}^{o} |E_{1o} \exp(ik_x\ x) \exp(k_{1z}\ z)|^2 \, dz$$

$$= k_x E_{1o}^2 \left[\frac{1}{2k_{1z}} \exp\ (2k_{1z}\ z) \right]_{-\infty}^{o}$$

$$= \frac{k_x E_{1o}^2}{2k_{1z}}$$

$$= \frac{k_x |\Delta\epsilon|}{k_o\ \psi_c\ \epsilon_2\ \sqrt{D}} \qquad \text{...... (4.21a)}$$

Here we have used Eqs.(4.13) and (4.16). We employ Eq.(4.4) to evaluate the other integrals.

$$P_{NLI} = k_x \int_{o}^{z_o} |E_{2o} \exp(ik_x\ x) \operatorname{sech} [\ k_{2z}(z - z_o)\ ]|^2 \, dz$$

$$= k_x E_{2o}^2 \int_{o}^{z_o} \left[\operatorname{sech} [\ k_{2z}(z - z_o)\ ] \right]^2 dz$$

By change of variable $X = z - z_o$ we get

$$P_{NLI} = k_x E_{2o}^2 \int_{-z_o}^{o} \left[\operatorname{sech}\ (\ k_{2z}\ X\ ) \right]^2 dX$$

$$= \frac{k_x}{k_{2z}} E_{2o}^2 \tanh (k_{2z} z_o)$$

$$= k_x \left(\frac{E_{2o}^2}{k_{2z}} \right) \left[\frac{\sqrt{D}}{\sqrt{1 + D}} \right]$$

Hence

$$P_{NLI} = \frac{k_x \ 2|\Delta\epsilon|}{\epsilon_2 k_o \psi_c} \sqrt{D} \qquad \ldots\ldots (4.21b)$$

Similarly

$$P_{NLII} = k_x \int_{z_o}^{\infty} |E_{2o} \exp(ik_x x) \operatorname{sech} [k_{2z}(z - z_o)]|^2 dz$$

$$= k_x E_{2o}^2 \int_{z_o}^{\infty} \left[\operatorname{sech} [k_{2z}(z - z_o)] \right]^2 dz$$

$$= k_x \left(\frac{E_{2o}^2}{k_{2z}} \right)$$

$$= \frac{k_x \ 2|\Delta\epsilon|}{\epsilon_2 k_o \psi_c} \sqrt{1 + D} \qquad \ldots\ldots (4.21c)$$

In getting Eqs.(4.21b) and (4.21c) we have utilised Eq.(4.20e).

Substituting Eqs.(4.21) in Eq.(4.20) we get

$$P = k_x \frac{2 |\Delta\epsilon|}{\epsilon_2 k_o \psi_c} \left[\frac{1}{2\sqrt{D}} + \sqrt{D} + \sqrt{1 + D} \right]$$

or $$P_c = 2 \frac{\epsilon_o \psi_c^2}{\epsilon_2} \sqrt{\psi_c^{-2} + D} \left[\frac{1}{2\sqrt{D}} + \sqrt{D} + \sqrt{1 + D} \right] \quad ..(4.22)$$ ✓

where Eq.(4.15) for k_x is utilised.

We henceforth use the notation P_c for the total critical power in the surface due to the reason given in Sec.(4.2.3) to be followed.

<u>Critical Power of Self Trapped Wave</u> :

We have made use of 'sech' distribution function in deriving Eq.(4.22) because this function satisfies Maxwell's equations in the nonlinear medium. It is obvious that it would still be a solution if the location of the interface is shifted to $z = -\infty$. In that case the nonlinear medium would extend from $z = +\infty$ to $z = -\infty$. In such a medium we will have to consider a freely propagating self-trapped wave. We now obtain expression for the critical power of the self-trapped wave.

For this purpose we use Eq.(4.4). Thus

$$P_{ST} = k_x \int_{-\infty}^{\infty} |E_{2o} \exp(ik_x x) \operatorname{sech} [k_{2z}(z - z_o)] |^2 dz$$

$$= k_x E_{2o}^2 \int_{-\infty}^{\infty} |\operatorname{sech} [k_{2z}(z - z_o)] |^2 dz$$

The integral is solved again by changing the variable as $X = z - z_o$. Hence we obtain

$$P_{ST} = k_x \left(\frac{E_{2o}^2}{k_{2z}} \right) \left[\tanh (k_{2z} X) \right]_{-\infty}^{\infty}$$

$$= 2 k_x \frac{E_{2o}^2}{k_{2z}}$$

By using Eqs.(4.7c), (4.15) and (4.20e) the above result can be rewritten as

$$P_{ST} = 2 \left[\frac{2\epsilon_o \psi_c^2}{\epsilon_2} \sqrt{\psi_c^{-2} + D} \quad \sqrt{1 + D} \right] \quad .... (4.23)$$

<u>**Comparison of Total Critical Power (P_c) with Power (P_{ST}) of Self-Trapped Wave :**</u>

Dividing Eq.(4.22) by Eq.(4.23) we get

$$\frac{P_c}{P_{ST}} = \frac{\left(\frac{1}{2\sqrt{D}} + \sqrt{D} + \sqrt{1 + D} \right)}{2 \sqrt{1 + D}}$$

i.e. $$P_c = \frac{(\sqrt{1 + D} + \sqrt{D})^2}{4 \sqrt{D} \sqrt{1 + D}} P_{ST} \quad .... (4.24)$$

On the other hand the division of Tomlinson's Eq.(17) by (4.23) gives us

$$P = \frac{(\sqrt{1+D} + \sqrt{D})}{2\sqrt{D}} P_{ST} \qquad \ldots\ldots (4.25)$$

Comparison of Peak-Field Amplitudes for the Surface Wave and Self-Trapped Wave :

From Sec.(4.2.2) we list the following expressions again.

$$P_L = \frac{k_x E_{10}^2}{2k_{1z}} , \quad P_{NLI} = k_x \left(\frac{E_{20}^2}{k_{2z}} \right) \left[\frac{\sqrt{D}}{\sqrt{1+D}} \right]$$

$$P_{NLII} = k_x \left(\frac{E_{20}^2}{k_{2z}} \right)$$

Hence

$$P_c = k_x \left[\frac{E_{10}^2}{2k_{1z}} + \frac{E_{20}^2}{k_{2z}} \left(\frac{\sqrt{D}}{\sqrt{1+D}} + 1 \right) \right] \ldots (4.26)$$

From Eqs.(4.20d) and (4.20e) we can easily deduce

$$\frac{E_{10}^2}{k_{1z}} = \frac{E_{20}^2}{k_{2z}} \frac{\sqrt{D}}{\sqrt{1+D}}$$

Hence Eq.(4.26) becomes

$$P_c = k_x \frac{E_{20}^2}{k_{2z}} \left[\frac{3\sqrt{D} + 2\sqrt{1+D}}{2\sqrt{1+D}} \right]$$

Consequently the amplitude of the surface wave is written as

$$A_{SW} = \left\{ \frac{\epsilon_2 k_{2z}}{k_x} \frac{P_c}{\left[\frac{3\sqrt{D} + 2\sqrt{1+D}}{2\sqrt{1+D}} \right]} \right\}^{1/2} \qquad \ldots\ldots (4.27)$$

Similarly from Eq.(4.23) we obtain amplitude for the self-trapped wave.

$$A_{ST} = \left\{ \frac{\epsilon_2 k_{2z}}{k_x} \frac{P_{ST}}{2} \right\}^{1/2} \qquad \ldots\ldots (4.28)$$

Comparing Eq.(4.27) and (4.28) for $P_c = P_{ST}$ we have

$$A_{ST} = \left[\frac{3\sqrt{D} + 2\sqrt{1+D}}{4\sqrt{1+D}} \right]^{1/2} A_{SW} \qquad \ldots\ldots (4.29)$$

Behaviour of Field Parameters as Functions of D :

We have plotted the various parameters of the field distribution as functions of D in Fig.(4.2). Also field amplitude as a function of z for several values of D has been illustrated in Fig.(4.3). For this purpose the above given expressions for the field parameters are rewritten as given below :

We have chosen the set $\sqrt{\epsilon_o} = 1.5$ and $\Delta\epsilon = -\ 0.06$. By definition $k_o = \sqrt{\epsilon_o}\frac{2\pi}{\lambda} = \frac{3\pi}{\lambda}$. As a result Eqs.(4.13)-(4.15) and Eq.(4.18) take the following forms.

$$k_{1z}\,\lambda = 3\pi\ \psi_c\ \sqrt{D}$$

$$k_{2z}\,\lambda = 3\pi\ \psi_o\ \sqrt{1 + D}$$

$$k_x\,\lambda = 3\pi\sqrt{1 + D\ \psi_c^2} \qquad \ldots.(4.30)$$

and

$$\frac{z_o}{\lambda} = \frac{\ln\ [\ \sqrt{1 + D} + \sqrt{D}\ ]}{3\pi\ \psi_c\ \sqrt{1 + D}}$$

The graphs of these can be seen in Fig.(4.2). For $D < 1$, $k_x \approx k_o$, so we have also included in this figure a plot of $10^2(k_x - k_o)\lambda$ in order to show the degree to which the velocity of the wave reduces on account of the interaction with the nonlinear medium.

Further Eq.(4.22) is utilised to study the variation of $\epsilon_z P_c$ versus D. This graph is also included in Fig.(4.2).

In order to plot the graphs given in Fig.(4.3) we have used the reformed expressions given below :

For $z < 0$

$$|E(x,y,z)| = E_{1o}\ \exp(k_{1z}z)$$

Using Eqs.(4.13) and (4.16) we simplify the above expression as

$$\epsilon_2^{1/2} \, |E(z)| = \sqrt{2|\Delta\epsilon|} \exp \left[3\pi \, \psi_o \sqrt{D} \left(\frac{z}{\lambda} \right) \right] \quad \ldots. \ (4.31)$$

For $z > 0$

$$|E(x,y,z)| = E_{2o} \operatorname{sech} [k_{2z}(z - z_o)]$$

Using Eqs.(4.12) and (4.14) we write

$$\epsilon_2^{1/2} \, |E(z)| = \sqrt{2|\Delta\epsilon|(1 + D)} \operatorname{sech} y$$

Using $\operatorname{sech} y = \dfrac{2e^y}{e^{2y} + 1}$, the expression finally becomes

$$\epsilon_2^{1/2} \, |E(z)| = 2 \sqrt{2|\Delta\epsilon|(1 + D)} \, \frac{2e^y}{e^{2y} + 1} \quad \ldots.(4.32)$$

where $y = 3\pi \, \psi_o \sqrt{1 + D} \left(\frac{z}{\lambda} - \frac{z_o}{\lambda} \right)$.

4.2.3 Results and Discussion :

When we compare Eq.(4.22) with Eq.(17) of Tomlinson, it is seen that the first term contribution should actually be one half of that reported by Tomlinson. In other words, Tomlinson[2] has determined the critical power (P_L) in the linear medium two times its actual value. Consequently his Eq.(17) is in error and the correct total critical power should be given by Eq.(4.22) of the present work. That is why we adopt the notation P_c for the corrected total critical power to distinguish it from Tomlinson's erroneous P value given by his Eq.(17).

Comparing the bracketted expression with the R.H.S. of Eq.(4.21c) we note that, it represents the critical power in the surface wave between z_o and $+ \infty$. Hence we have

$$P_{ST} = 2\,P_{NLII} \qquad \ldots\ldots (4.33)$$

This is in agreement with Tomlinson's conclusion that the critical power in the self-trapped wave is twice the critical power in the surface wave between $z = z_o$ and $z = +\infty$.

__Comparison of P_c with P_{ST} :__

The critical powers P_c and P_{ST} are compared by numerical calculations presented in Table (4.1). It is seen that the corrected power P_c is larger than P_{ST} for the same peak-field amplitude because the decay constant k_{1z} in the linear medium is always smaller than the decay constant k_{2z} in the nonlinear medium (as seen by comparing Eqs.(4.13) and (4.14)). However this is true only for the lower values of D in the range 10^{-2} to 1 and not for all D values upto 10^{2} as concluded by Tomlinson. In fact for D $\geq$ 10 the powers under consideration are practically the same. This can be noted by comparing the numerical values of P_c / P_{ST} and P/P_{ST} given in Table (4.1).

If we consider the case in which $\Delta\epsilon > 0$ and $\epsilon_2 > 0$ and follow the above given treatment the following expressions are obtained for the decay constants.

$$k_{1z} = k_o\, \nu_c \sqrt{D + 2}$$

and

$$k_{2z} = k_o\, \nu_c \sqrt{D + 1} \qquad \ldots\ldots (4.34)$$

Since $k_{1z} > k_{2z}$ for all D values in the range 10^{-2} to 10^{2}, it is obvious that P_c would be lower than P_{ST} . This means the surface wave will not be supported by the interface.

Comparison of A_{SW} with A_{ST} :

By varying the value of D in the range 10^{-3} to 10^{3} at a regular step of 10, we have numerically estimated values of the multiplying factor of Eq.(4.29). It is found to be less than unity for $0 < D < 1$, while greater than unity for $D > 1$. This means $A_{SW} < A_{ST}$ for $D > 1$, whereas $A_{SW} > A_{ST}$ for $D < 1$. However Tomlinson has concluded that the surface wave has a smaller peak-field amplitude for all D values. This conclusion need to be modified in the light of above discussion. Thus the surface wave has a total stored energy smaller than that for the freely propagating wave only if $D > 1$.

Behaviour of Field Parameters :

From the Figs.(4.2) and (4.3) the following features are noted :

1) The threshold for the existance of the surface wave is given by $D = 0$ so that $k_{1z} = z_o = 0$. This means the surface wave travels with uniform amplitude $(2|\Delta\epsilon| / \epsilon_2)^{1/2}$ throughout the linear medium. However it decreases monotonically into the nonlinear medium.

As D increases the field distribution is more and more sharply peaked in the nonlinear medium i.e. the wave becomes more and more intense at the peak. However the field amplitude at the interface always remains equal to $(2|\Delta\epsilon| / \epsilon_2)^{1/2}$. Further with the increase in D the peak shifts into the nonlinear medium and reaches a maximum distance for $D = 2.277$. For larger D values the peak shifts back towards the interface.

2) At $D = 0$ the critical power in the surface wave becomes infinite as the wave extends to $-\infty$ in the linear medium. This extent decreases in proportion to $D^{-1/2}$ as D value is increased above the threshold. Simultaneously the power of the wave decreases upto a minimum value at $D \sim 1$ beyond which it increases again. This increase for low D values is approximately proportional to $D^{1/2}$, while it approaches $(4\epsilon_o \, \nu_c^2 \, D \,/\, \epsilon_2)$ for very large D values.

4.3 Nonlinear Propagation of TE Waves in a Planar Optical Waveguide with high-index Guiding layer :(Present Work)

In this section we consider the effect of intensity-dependent refractive index on the propagation of TE modes of electromagnetic wave in a three-layer planar optical waveguide with a geometry[9] as shown in Fig.(1.5).

We make use of Eqs.(1.35) and (1.36) of Chapter 1 and assume that the effective refractive index of the guiding layer is given as :

$$n_1 \simeq n_L + n_{NL}$$

where n_L = field-independent refractive index.

n_{NL} = Intensity-dependent part of refractive index.

Usually we take $n_{NL} = n_2' \langle E^2 \rangle$

with n_2' = nonlinear coefficient $\sim 10^{-12}$(esu) or $10^{-9}(m^2/v^2)$

Hence

$$n_1 = n_L + n_2' \langle E^2 \rangle$$

By squaring both sides

$$n_1^2 \simeq n_L^2 + 2\, n_L n_2' \langle E^2 \rangle$$

where the small term containing $n_2'^2$ is neglected.

By definition $\epsilon = n^2$ so that above expression can be written as

$$\epsilon(x) = \epsilon_L + \epsilon_2' \langle E^2 \rangle$$

$$\text{with } \epsilon_2' = 2n_L n_2'$$

Using this in Eq.(1.36) for the guiding layer we get

$$\epsilon(x) \simeq \epsilon_L\epsilon_o + \epsilon_o\epsilon_2' \langle E^2 \rangle$$

Assuming $E = E_y\, e^{i\omega t}$ we can estimate

$$\langle E^2 \rangle = \frac{1}{2} E_y^2$$

$$\therefore \quad \epsilon(x) = \epsilon_L\epsilon_o + \frac{1}{2}\epsilon_L\epsilon_2' E^2 \qquad \text{....(4.35)}$$

4.3.1 <u>Intensity Effect on Field variation</u> :

Substituting this in Eq.(1.35) we obtain

$$\left[\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2}\right] E_y + P\, E_y\, Q\, E_y^3 = 0 \qquad \text{...... (4.36)}$$

where $P = k^2\epsilon_L$, $Q = \frac{1}{2} k^2\epsilon_2'$ and $k^2 = \omega^2\mu_o\epsilon_o$

The wave equation is written in a more compact form by adopting the change of variable

$$s^2 = \frac{1}{2}(x^2 + z^2)$$

so that $\quad \dfrac{\partial^2}{\partial x^2} + \dfrac{\partial^2}{\partial z^2} = \dfrac{\partial^2}{\partial s^2}$

Hence Eq.(4.36) becomes

$$\frac{\partial^2 E_y}{\partial s^2} + P\, E_y + Q\, E_y^3 = 0 \qquad \text{..... (4.37)}$$

This is a nonlinear equation whose solution can be obtained by employing the integration method due to Dufing.

If $\dfrac{\partial E_y}{\partial s} = R$, Eq.(4.37) becomes

$$\frac{\partial R}{\partial s} = -[\, PE_y(s) + QE_y^3(s)\,]$$

Now

$$\frac{\partial R}{\partial E_y} = \frac{\partial R}{\partial s} \cdot \frac{\partial s}{\partial E_y} = - \frac{[\, PE_y(s) + QE_y^3(s)\,]}{R}$$

i.e. $\quad R\,\partial R = -\,[\, PE_y(s) + QE_y^3(s)\,]\,\partial E_y$

Integrating

$$R^2 = -\,(PE_y^2 + \frac{Q}{2} E_y^4\,) + c$$

By initial condition $E_y = 0$ at $s = 0$ so that $c = 0$

Consequently the above equation takes the form

$$\frac{\partial E_y}{\partial s} = i\,(PE_y^2 + T\,E_y^4\,)^{1/2}$$

with $T = Q/2$

This differential equation is solved by separating the variables

$$\frac{1}{2\sqrt{P}} \ln \left| \frac{\sqrt{P + T\,E_y^2} - \sqrt{P}}{\sqrt{P + T\,E_y^2} + \sqrt{P}} \right| = is + c'$$

This solution is valid under the condition that $P > 0$ which holds good in the present study. By the same initial condition we have $c' = 0$. As a result the solution takes the form

$$\left| \frac{\sqrt{P + T\,E_y^2} - \sqrt{P}}{\sqrt{P + T\,E_y^2} + \sqrt{P}} \right| = \mathrm{Re}\,\exp\,(i\;2\sqrt{P}\;s)$$

$$= \cos\,(2\sqrt{P}\;s)$$

Upon rationalising and simplifying we can finally obtain

$$E_y = 2\sqrt{\frac{2P}{Q}} \left[\frac{\sqrt{\cos\,(2\sqrt{P}\;s)}}{(1 - \cos\,(2\sqrt{P}\;s))} \right] \qquad \ldots\ldots (4.38)$$

For simplicity, the variation of E_y can be studied either with respect to x or z co-ordinate. In the present work we assume $z = 0$ so that $s = x/\sqrt{2}$.

$$\sqrt{P} = k\,\epsilon_2^{1/2} = \frac{2\pi}{\lambda}\,n_L$$

and $\quad \sqrt{Q} = \frac{1}{\sqrt{2}}\,k\,\epsilon_2'^{1/2} = \frac{2\pi}{\lambda}\sqrt{n_L n_2'}$

Hence we obtain

$$2\sqrt{P}\, s = \sqrt{2}\, n_L\, d \qquad \text{with } d = \frac{2\pi}{\lambda} x$$

and $\sqrt{P/Q} = \sqrt{n_L / n_2'}$

Hence Eq.(4.38) is rewritten as

$$E_y = 2\sqrt{2}\sqrt{\frac{n_L}{n_2'}}\left[\frac{\sqrt{\cos(\sqrt{2}\, n_L\, d)}}{1 - \cos(\sqrt{2}\, n_L\, d)}\right] \qquad \ldots.(4.39)$$

4.3.2 Results and Discussion :

Eq.(4.39) has been used to examine the field variation in the x-direction. Fig.(4.4). For the numerical calculations we have chosen $n_L = 2.29$, $n_2' = 10^{-8}$ (m^2/v^2) and varied d in the range 0.01 to 0.5. The nature of the graph shows that the field profile is sharply confined across the nonlinear layer, this indicates that under the intensity effect the guiding layer exhibits the self-focusing of the propagating wave.

4.4 Nonlinear Propagation of TE Waves in a Planar Optical Waveguide with low-index Guiding layer : (Present Work)

Holland[8] has investigated the optical wave-guiding properties of a three-layered asymmetric structure. In his numerical calculations it has been shown that in such a structure guided waves exist with symmetric field profiles. These are considered as the nonlinear analog of conventional guided modes. For such a nonlinear propagation he has established a scalar wave equation. Although he has not given the solution of this equation, he has reported the intensity profiles of the symmetric guided waves for three different N values. In the present work we intend to solve this nonlinear equation by Dufing's method as explained in the earlier Sec.(4.3).

With slight changes in notations, we write the equation as:

$$\frac{\partial^2 E_y}{\partial x^2} + k^2 \left[\epsilon_L + \frac{\alpha}{2} E_y^2 - N^2 \right] E_y = 0 \quad \ldots (4.40)$$

where E_y = Electric field along y-direction.

ϵ_L = field free dielectric constant of the guiding layer.

α = nonlinear coefficient.

N = mode index.

In writing the above equation we have taken the x-axis normal to the guiding layer while z-axis is parallel to it.

If $P = k^2(\epsilon_L - N^2) = k^2\epsilon$ and $Q = \frac{1}{2} k^2\alpha$

The above equation can be re-written as :

$$\frac{d^2 E_y}{dx^2} = P\, E_y - Q\, E_y^3 \quad \ldots\ldots (4.41)$$

The first R.H.S. term is positive because $P < 0$ for the low-index guiding layer.

Put $s = \frac{dE_y}{dx}$ so that $\frac{\partial s}{\partial E_y} = \frac{PE_y - QE_y^3}{s}$

Separating the variables and integrating, as we have done in the previous section, we obtain

$$s = \frac{dE_y}{dx} = \left[P\, E_y^2 - T\, E_y^4 \right]^{1/2}$$

where T = Q/2

Integrating again we get

$$\frac{1}{\sqrt{T}} \left[-\frac{1}{u} \ln \left| \frac{u + \sqrt{u^2 - E_y^2}}{E_y} \right| \right] = x$$

In solving the above integrals the constants of integration are found to be zero. A further simplification leads to the expression for the electric field

$$E_y = \frac{2\,u\,\exp(-\sqrt{P}\,x)}{1 + \exp(-2\sqrt{P}\,x)} \qquad \ldots\ldots (4.42)$$

with $\quad u = \sqrt{\frac{P}{T}} = 2\sqrt{(\epsilon_L - N^2)/\alpha}$

4.4.1 Results and Discussion :

Eq.(4.42) is employed to examine the field variation as a function of propagation distance across the guiding layer for different group index values. For this purpose the following data of various constants have been chosen, $\epsilon_L = 2.24$, $\lambda = 0.5\ \mu m$, thickness of guiding layer = 1 μm , $\alpha = 10^{-8}$, N = 1.5035 , 1.5170 , and 1.5434. It is to be noted that we have substituted only $|P|$ values in Eq.(4.42) because the condition $P < 0$ is already taken care of in Eq.(4.41). The results are presented graphically in Fig.(4.5). The nature of field plots is the same as that for intensity plots of Holland given in his Fig.(5). From Fig.(4.5) we find that

i) For a given mode index the field profile has a peak value at the location of axis of the guiding layer.

ii) With the increase in mode index value the peak becomes more and more sharper with a simultaneous decrease in the extent of field profile across the layer. Physically it means the guided wave (hence the energy) is more and more tightly confined across the guiding medium.

4.5 Summary and Conclusions :

In the beginning of this chapter we have revisited Tomlinson's method of analysing surface wave at a nonlinear interface. Expressions for various field parameters in both linear and nonlinear media are obtained. In deriving the expression for critical power of the surface wave, however, we have noted a small correction in Tomlinson's expression. The critical power of the surface wave in the linear medium should be actually one half of that reported by Tomlinson. This has led to a corrected expression for total critical power. Numerical calculations have been carried out and critical powers for the surface wave and self-trapped wave have been compared. A similar comparison has been done for the peak-field amplitudes of these waves. The behaviour of different field parameters in the two media has been discussed at the end.

Next we have examined the nonlinear propagation of TE waves in a three-layer planar optical waveguide. The expression for effective dielectric constant of the guiding layer (having higher refractive index than that for substrate) is established and the same is employed in the scalar wave equation. The resulting nonlinear equation is solved by Dufing's method. The behaviour of electric field across the guiding layer is graphically presented and discussed.

At the end of this chapter we have studied the nonlinear propagation of TE waves in a planar waveguide with low-index guiding layer. Holland's nonlinear equation for this problem has been solved by Dufing's method and the field variation across the guiding medium has been examined by presenting the results graphically.

Conclusions :

From our study reported in this chapter the following observations are noted :

1) Tomlinson's expression for total critical power of the surface wave is found to be erroneous in that the critical power in the linear medium is reported to be two times its actual value. This expression has been corrected in the present work.

Numerical estimates based upon the corrected expression indicate that the total power in the surface wave is larger than that of self-trapped wave only for lower values of parameter D in the range 10^{-3} to 1, while for D $\geq$ 1) the two powers under consideration are practically the same. Further the peak-field amplitude for the surface wave has a smaller value than that for self-trapped wave only for D > 1 and not for all D values as concluded by Tomlinson. This means the surface wave has a smaller total stored energy only if D > 1.

2) In the study of nonlinear propagation of TE waves through a planar waveguide having high-index guiding layer, it is noted that the field profile is sharply confined across the guiding layer which favours for the self-focusing of the wave on account of intensity effect.

3) In examining the nonlinear propagation of TE waves in a planar waveguide with low-index guiding layer, we have found that the nature of field profile is the same as that for intensity plots reported by Holland. With the increase in mode-index the profile exhibits more and more sharper peak close to the axis of the guiding layer. Physically it implies that the energy of the guided wave is more and more tightly confined across the guiding layer.

References :

1. Kaplan A.E., IEEE J. Quantum Electron, QE-17, 3, 336, (1979).
2. Tomlinson W.J., Opt. Lett. (USA), 5, 323, (1980).
3. Akhmediev N.N., Sov.Phys. JETP (USA), 56, 299, (1982).
4. Seaton C.T., ValeraJ.D., Shoemaker R.L., and Stegeman G.I., Appl.Phys.Lett., 45(11), 1162, (1984).
5. Ariyasu J., Seaton C.T., and Stegeman G.I., Appl.Phys.Lett., 47(4), 355, (1985).
6. Ariyasu J., Seaton C.T., and Stegeman G.I., Opt.Lett., 11(5), 315, (1986).
7. Leine L., Wachter C., Labgbein U., and Lederer F., Opt.Lett., 11(9), 590, (1986).
8. Holland W.R., Opt.Soc.Am. B, 3(11), 1529, (1986).
9. Chang W.S.C., Muller M.W., "Integrated Optics" in "Laser Applications", vol.2, (Academic Press N.Y. 1974).

Table 4.1 Comparison of Critical Powers and Peak-field Amplitudes for Surface and Self-Trapped Waves.

D	P_c/P_{ST}	P/P_{ST}	Multiplying factor of Eq.(4.29)
0.001	8.417547	16.31929	0.723674
0.01	3.037344	5.524937	0.758042
0.1	1.404534	2.158312	0.852134
1	1.030330	1.207106	1.015051
10	1.000567	1.024404	1.102314
100	1.000006	1.002493	1.116368
1000	1.000000	1.000249	1.117866

Note : P_c/P_{ST} ——> Present work

P/P_{ST} ——> Tomlinson's equation

Multiplying factor of Eq.(4.30) ——> Present work

Fig. 4.1 Interface between Nonlinear and Linear Dielectrics.

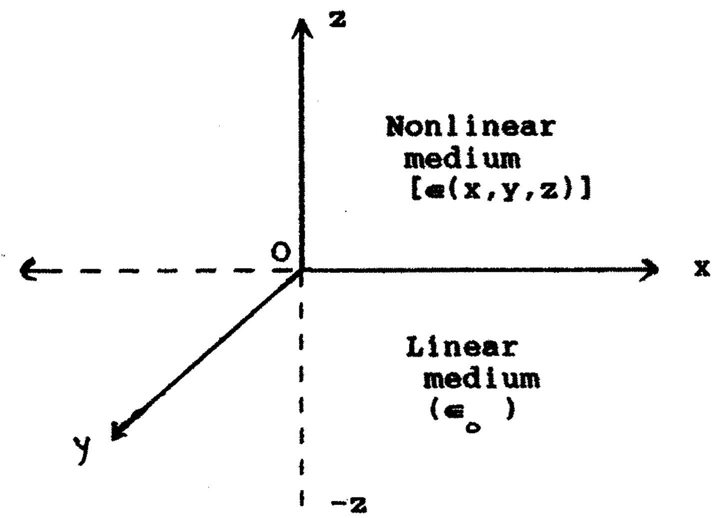

Fig. 4.2 : Field parameters of the surface wave as functions of D.

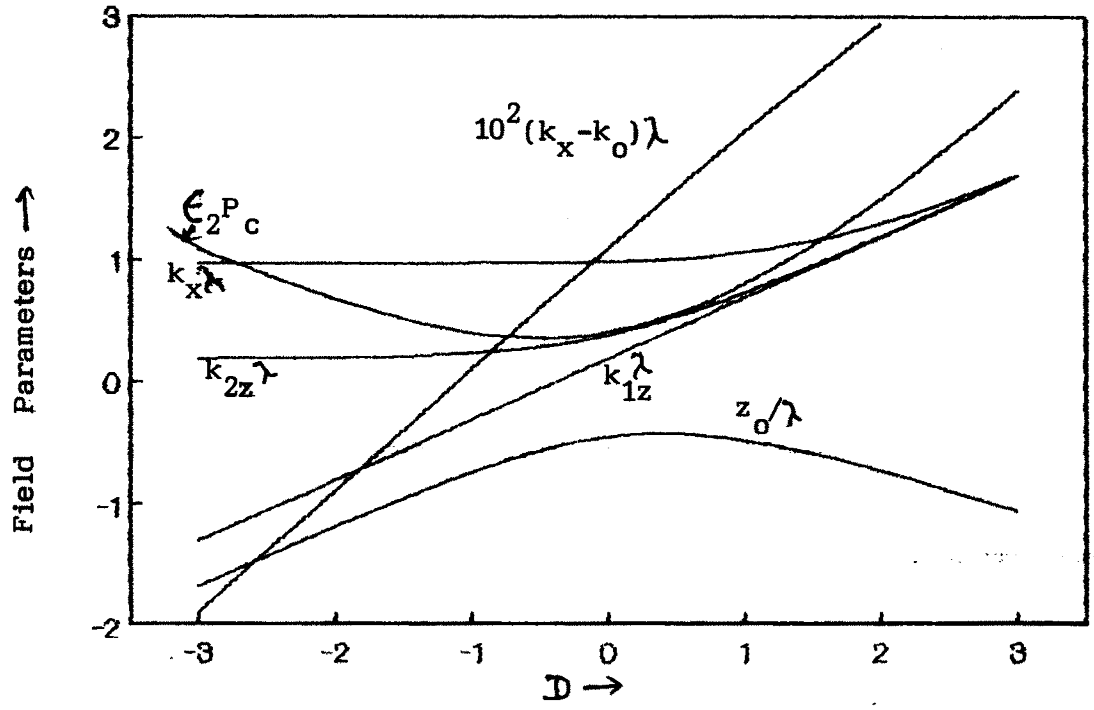

Fig. 4.3 : Field amplitude of the surface wave as a function of z/λ.

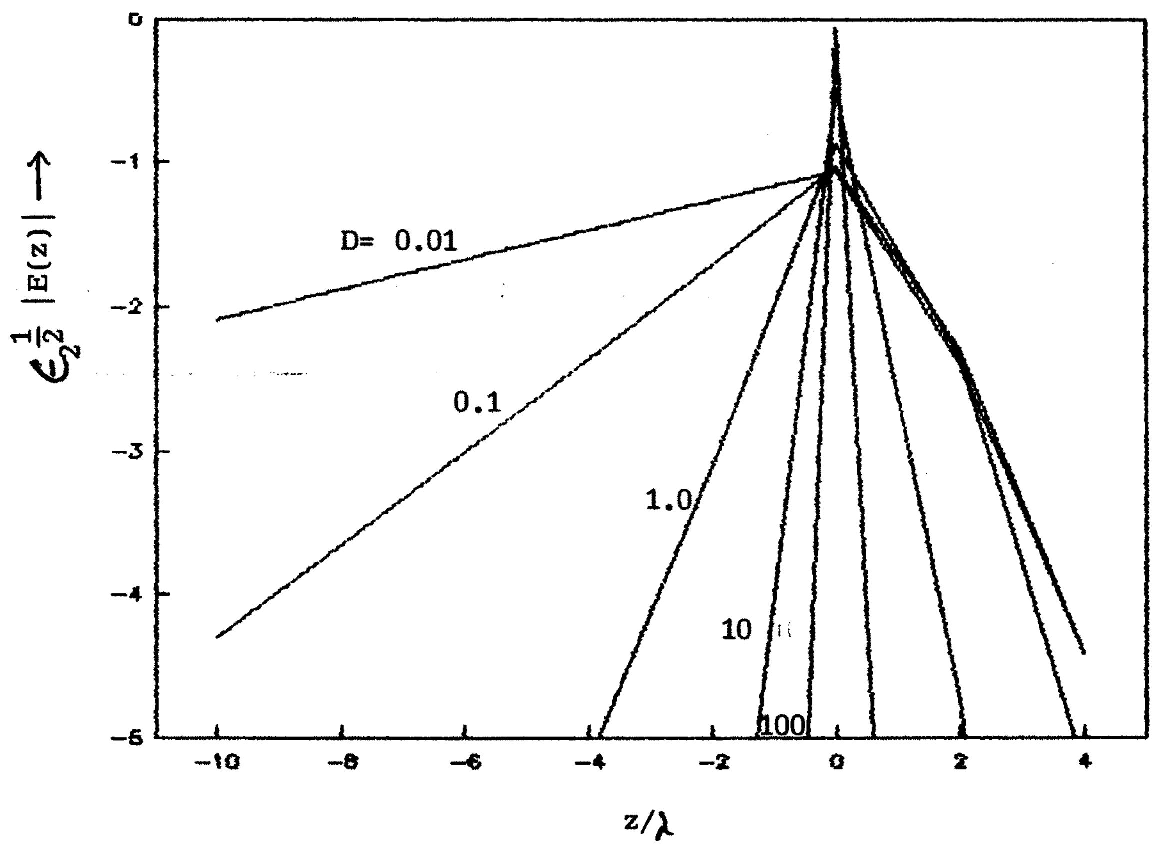

Fig. 4.4 : Intensity effect on field variation for TE guided wave.

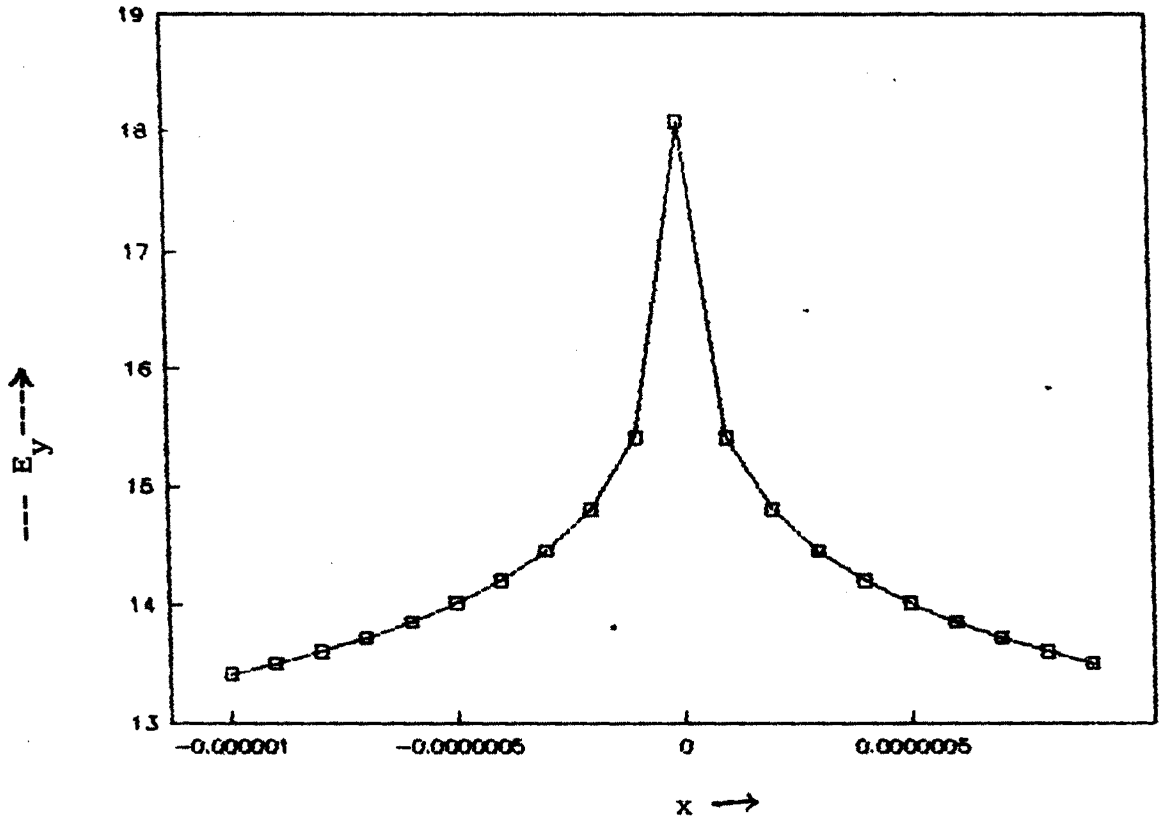

Fig. 4.5 : Field profiles of the nonlinear guided TE wave.