CHAPTER - 1

A REVIEW OF ELECTROMAGNETIC MODE ANALYSIS OF PLANAR OPTICAL WAVEGUIDES

1.1 Introduction :

The advent of laser and its extensive applications have created many new fascinating fields like integrated optics. This field is primarily based on the fact that light can be guided and confined in thin-films (with dimensions of the order of wavelength of light) of transparent materials on suitable substrates. The basic concept of integrated optics was first proposed by Anderson in 1965, while Miller coined the term "Integrated Optics" in 1969. It was visualised that thin-films and micro-fabrication technology could be suitably adopted for generation, modulation, switching, multiplexing, processing and such other optical functions in integrated optics form. Since then considerable progress has been made in the thin-film and integrated electronics technology.

Integrated optics is a far reaching attempt to apply thin-films and integrated electronics technology to optical circuits and devices. An integrated optical circuit could include lasers, integrated lenses, switches, interferometers, polarizers, modulators, detectors etc. Such circuits are useful in signal processing and optical communication. These could also be used in systems like optical transmitters, switches, repeaters and receivers. An optical system in the form of an integrated optical circuit is advantageous in reducing the sensitivity to air currents and mechanical vibrations, and in obtaining high efficiency at low driving voltages. The most promising materials for integrated optical circuits are direct band-gap semiconductors composed of Group III-V materials like GaAlAs and InGaAsP because these can be processed to perform almost all necessary operations as lasers, switches, modulators, detectors and so on.

Physically an integrated optical system differs from conventional optical systems in that light waves are propagated as guided waves confined in dielectric thin-films rather than as diffraction limited beams in free space. The propagation of these optical guided waves is very similar to the propagation of microwaves in waveguides, except that optical waveguides are made of dielectric layers of few microns or less in size and that the electric field of the optical guided waves extends both into the air and the substrate. One can conceive devices in the thin-film configuration that will generate, propagate, modulate, demodulate, deflect, switch, divide, combine, and detect these optical guided waves; and one can envision the design of an entire optical communication system (or sub-system) on one piece (or several pieces) of planar substrate material with most of the optical logic and data processing functions performed internally, within the thin-films.

The very high concentration of energy in very small regions in the optical waveguides gives rise to enormous intensities leading to the realization of nonlinear optical effects such as second harmonic generation which are useful to conceive nonlinear optical devices. Thus thin-film waveguides, due to high field intensities associated with it serve as useful tool for research studies in nonlinear optics. The confinement of light energy in small regions of space is also resposible for an efficient interaction of the optical energy with an applied electric field or an acoustic wave. This leads to much more efficient electrooptic and acoustooptic modulators and deflectors requiring very low drive powers.

The specific features that are basic and most important to integrated optics are (a) the fabrication of low-loss waveguides, (b) the control of the modes propagating in the waveguides so that the desired operating characteristics can be obtained, and (c) methods to achieve efficient input and output coupling that will channel an optical beam propagating through free space into guided waves in the thin-films and vice versa. In integrated optics two kinds of waveguides are commonly used (i) **Planar Waveguides** in which light is concentrated near the surface of the substrate with no lateral confinement (ii) **Stripe Waveguides** in which light energy is confined in both the transverse dimensions. This confinement is desirable for the fabrication of devices like amplitude or intensity modulators, directional couplers and optical switches.

In the present work we intend to study the propagation of light only through the planar waveguides. The propagation can be analysed in detail using electromagnetic mode theory although a ray treatment gives a reasonably good idea about the propagation characteristics. In the beginning we introduce electromagnetic theory through Maxwell's equations and briefly outline the propagation of plane electromagnetic waves in unbounded free space and dielectric medium. After summarizing the boundary conditions at an interface separating two dielectrics and giving the main results for reflection and refraction, we will discuss the mode treatment of asymmetric dielectric slab waveguide. This would be considered in view of the fact that almost all waveguides used in integrated optics are asymmetric in nature.

1.2 Maxwell's Equations :

It is well known that Maxwell's equations include the fundamental principles of all large-scale electromagnetic devices such as electronic computers, optical devices, television, microwave radar etc. These equations play vital role in the theories of electromagnetic radiation and relativity.

Maxwell noted that the laws due to Gauss and Faraday were applicable to time-varying electric and magnetic fields without changing their forms. However, he had to correct Ampere's law by introducing the displacement current density. Thus he could give a consistant set of four equations which are applicable to both static and time-varying electromagnetic fields in free space.

$$\left.\begin{aligned}\nabla \cdot E &= 0\\ \nabla \times E &= -\mu_o \frac{\partial H}{\partial t}\\ \nabla \cdot B &= 0\\ \nabla \times B &= \mu_o \epsilon_o \frac{\partial E}{\partial t}\end{aligned}\right\} \text{-------- (1.1) (SI units)}$$

Maxwell could visualise that when a magnetic field is wrapped round a current, a current could equally be wrapped around a magnetic field. This reciprocal relation between the two fields could be expressed mathematically by the curl of a vector field. When the two fields coexist in space, the reaction between them can be expressed as curl curl of a vector field. Combining the four equations appropriately, Maxwell derived the second order differential equations for the two fields.

$$\nabla^2 \psi = \mu_o \epsilon_o \frac{\partial^2 \psi}{\partial t^2} \qquad \text{............ (1.2)}$$

where ψ stands for $\bar{E}$ or $\bar{H}$.

These equations established that electromagnetic disturbances travel in the form of waves even in free space.

1.3 Plane Electromagnetic Waves in Unbounded Media :

a) In free space :

The solutions of the above wave equations can be obtained in the form

$$\psi = A\, e^{i(\omega t - \bar{k}\cdot\bar{r})}$$

where $\bar{k}$ is the wave vector representing the direction of propagation of wave, while $\bar{k}\cdot\bar{r}$ represents the phase factor with $\bar{r}$ as a position vector in space.

This solution represents a plane wave if the field depends only on one space co-ordinate, and time co-ordinate.

A plane wave is the one where the surfaces of the constant phase are planes normal to the direction of propagation of the wave. Such planes are called wavefronts. For a plane wave propagating in z-direction we should have $\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$, $\frac{\partial}{\partial z} \neq 0$ so that we can write $E = E(z,t)$, $H = H(z,t)$. From the divergence Maxwell's equations we can show that E_z and H_z are constant in space while the curl Maxwell's equations lead to the conclusion that these components are also constant in time. Thus they represent static components and consequently, no part of the wave motion. We can therefore choose $E_z = H_z = 0$ so that $E = iE_x + jE_y$, $H = iH_x + jH_y$. It can be shown that in a progerssive plane electromagnetic wave the ratio of amplitudes of E and H fields is given by $(\mu_o/\epsilon_o)^{1/2}$. This is called the **characteristic impedance** or **intrinsic impedance** of the medium. e.g. For free space, the value of this ratio comes out to be roughly 377 ohms. Since the E and H vectors do not have any z-components, both these must be perpendicular to the direction of propagation. This means plane electromagnetic waves are purely transverse in nature. These are termed as TEM waves. If E points in a direction parallel to the x-axis, H will point in a direction parallel to y-axis. Such waves are called **plane polarised**. The plane containing H vector and direction of propagation is termed as the **plane of polarisation** and the plane containing E vector and the direction of propagation is called **plane of vibration**. We often say that the wave is polarised in the direction of its E vector. In general E and

H vectors may lie in x-y plane instead of parallel to x and y-axes. In such a case, each vector can be split up into x and y components. For example, components E_x and E_y for E vector. Since E oscillates in time, the two components also vary in time. If E_x and E_y are not in phase and their magnitudes are unequal, the E vector will describe an ellipse in x-y plane. Such a wave is said to be elliptically polarised. As a special case E_x and E_y may have equal magnitudes and a 90° phase difference. In that case the resultant E vector describes a circle and the wave is said to be circularly polarised.

The phase velocity of the plane electromagnetic wave is defined as $Vp = \frac{1}{\sqrt{\mu_o \epsilon_o}} = 3 \times 10^8$ m/sec in free space. The group velocity is given by $\partial\omega / \partial\beta$ where $\beta^2 = \mu_o \epsilon_o \omega$. As electromagnetic waves propagate through space from their source to distant receiving points, there is a transfer of energy from the source to the receivers. There exists a simple and direct relation between the rate of this energy transfer and the amplitudes of electric and magnetic field strengths of the electromagnetic wave. This relation obtained from Maxwell's equations is given by $S = (E \times H)$ in watts/m^2 and is known as Poynting vector. It is interpreted as the amount of field energy passing through unit area of surface in unit time normal to direction of flow of energy. The time averaged power flow is given as $\frac{1}{2}Re(E \times H^*)$.

b> <u>In Dielectric</u> :

For an isotropic, homogeneous and linear material medium other than vacuum, the set of Maxwell's equations are written as

$$\left.\begin{aligned} \nabla \cdot D &= \rho \\ \nabla \cdot B &= 0 \\ \nabla \times E + \frac{\partial B}{\partial t} &= 0 \\ \nabla \times H - \frac{\partial D}{\partial t} &= J \end{aligned}\right\} \qquad \text{............(1.3)}$$

(SI units)

where the following constitutive relations or material equations are incorporated : $D = \epsilon E$, $H = \dfrac{B}{\mu}$, $J = \sigma E$ with σ = electrical conductivity of the medium. By combining the above equations appropriately the following wave equations can be easily derived.

$$\left.\begin{aligned} \nabla^2 E &= \epsilon\mu \frac{\partial^2 E}{\partial t^2} + \sigma\mu \frac{\partial E}{\partial t} + \nabla \left(\frac{\rho}{\epsilon} \right) \\ \nabla^2 H &= \epsilon\mu \frac{\partial^2 H}{\partial t^2} + \sigma\mu \frac{\partial H}{\partial t} \end{aligned}\right\} \qquad \text{... (1.4)}$$

For a conducting or a non-conducting medium it can be shown that we can set $\rho = 0$ and that both E and H are mutually perpendicular to each other and also transverse to the direction of propagation. The vector product $E \times H$ will point in the direction of propagation. For a non-conducting medium or dielectric $J = 0$ also. Hence the wave equations (1.4) reduce to a following simple form

$$\nabla^2 \psi - \frac{1}{v^2} \frac{\partial^2 \psi}{\partial t^2} = 0$$

with ψ = E or H and $v = \dfrac{1}{\sqrt{\mu\epsilon}}$

By definition the relative permeability and permittivity are given as $\mu_r = \frac{\mu}{\mu_o}$ and $\epsilon_r = \frac{\epsilon}{\epsilon_o}$. Hence the phase velocity can be written as $v_p = \frac{c}{\sqrt{\mu_r \epsilon_r}}$ with $c = 3 \times 10^8$ m/sec in free space. Since μ_r and ϵ_r are greater than unity, we find that $v_p < c$ in a dielectric. Further the refractive index of dielectric is given by $n = \frac{c}{v_p} = \sqrt{\mu_r \epsilon_r}$. For non-magnetic dielectric $\mu_r = 1$ so that $n^2 = \epsilon_r$. This is called Maxwell's relation which is experimentally verified for substances having non-polar molecules e.g. air, hydrogen, benzene, carbon etc. However ϵ_r becomes a function of frequency in case of substances containing polar molecules e.g. water, glass etc. This gives rise to the phenomenon called dispersion. In a dielectric, average Poynting vector is found to be (n/μ_r) times that in free space. Also the total electromagnetic energy density is found to be ϵ_r times that in free space.

1.4 Plane Electromagnetic Waves in Bounded Media :

The characteristics of electromagnetic waves are modified substantially when these waves propagate in a bounded medium. When a plane electromagnetic wave is propagating from a medium with constants (μ_1, ϵ_1) to a medium with constants (μ_2, ϵ_2), the field vectors E, D, B, and H should satisfy the following boundary conditions at the surface of discontinuity separating the two media (Fig.1.1).

$$\left.\begin{aligned}(D_2 - D_1)\cdot n &= \sigma\\ (E_2 - E_1)\cdot n &= 0\\ (B_2 - B_1)\cdot n &= 0\\ (H_2 - H_1)\cdot n &= J\end{aligned}\right\}\quad\ldots\ldots\ldots\ (1.5)\ \text{(SI units)}$$

where n = unit normal vector at the surface, σ = surface charge density and J = surface current density.

These conditions are useful in explaining reflection and refraction of the plane waves at a boundary between two material media. These are also applicable while studying the propagation of waves through waveguides.

With reference to Fig. 1.2, the electromagnetic theory easily verifies the laws of reflection and refraction viz. $\theta_1 = \theta_3$ and $\frac{\sin\theta_2}{\sin\theta_1} = \frac{n_1}{n_2} = (\frac{\epsilon_1}{\epsilon_2})^{1/2}$ where n_1 and n_2 are the refractive indices of the two media. Similarly the expressions for reflection (R) and transmission (T) coefficients for TE and TM polarisations of the propagating waves are derived to be

TE polarisation :

$$\left.\begin{aligned}R_\perp &= \frac{\sin^2(\theta_2 - \theta_1)}{\sin^2(\theta_2 + \theta_1)}\\ T_\perp &= \frac{\sin 2\theta_1 \sin 2\theta_2}{\sin^2(\theta_2 + \theta_1)}\end{aligned}\right\}\ \ldots\ (1.6)$$

TM polarisatiom :

$$\left.\begin{aligned}R_\parallel &= \frac{\tan^2(\theta_1 - \theta_2)}{\tan^2(\theta_1 + \theta_2)}\\ T_\parallel &= \frac{\sin 2\theta_1 \sin 2\theta_2}{\sin^2(\theta_1 + \theta_2)\cos^2(\theta_1 - \theta_2)}\end{aligned}\right\}\ \ldots\ (1.7)$$

In TM case there is a possibility of zero power reflection coefficient for a certain angle of incidence given by

$$\theta_B = \tan^{-1}\left(\frac{n_2}{n_1}\right)$$

This angle is called **Brewster's angle.** Similarly, for some angle of incidence the power transmission coefficient would be zero. This angle is known as the **Critical angle** given by

$$\theta_C = \sin^{-1}\left(\frac{n_2}{n_1}\right)$$

which is true for either type of polarisations at the interface. When a wave in denser medium is incident at an angle exceeding θ_C, it will be totally reflected back into the same medium. It will be accompanied by a surface (evanescent) wave in the less denser medium for which $\sin\theta_2 \geq 1$. This cannot be satisfied for any real value of θ_2, i.e. θ_2 is a complex angle. The physical interpretation of this phenomenon is that the transmitted wave no longer crosses the interface, so that only reflected wave remains.

A phase shift occurs on total internal reflection. The expressions for the same in the case of two types of polarised incident waves are given below

TE polarisation :

$$\delta_\perp = 2\tan^{-1}\left[\frac{(\sin^2\theta_1 - n_2^2 / n_1^2)^{1/2}}{\cos\theta_1}\right]$$

TM polarisation : ... (1.8)

$$\delta_\parallel = 2\tan^{-1}\left[\frac{(\sin^2\theta_1 - n_2^2 / n_1^2)^{1/2}}{(n_2^2 / n_1^2)\cos\theta_1}\right]$$

In the above discussion we have considered reflection and refraction in terms of a single ray incident at a dielectric interface. However in practical situation, a beam of light with well-defined cross-section is incident at the interface. If the axis of this beam is taken as a single ray, the reflected ray is found to be laterally shifted with respect to the incident ray (Fig.1.3). This is called **Goos-Haenchen shift**. It is found that for either type of polarisation the lateral shift becomes infinite at the critical angle and at the angle of grazing incidence. This effect is important in the case of dielectric waveguides.

1.4.1 Propagation of Electromagnetic Waves in Waveguides :

In many actual cases electromagnetic waves are guided along or over conducting or dielectric surfaces. Common examples of guided electromagnetic waves are the waves along ordinary-parallel-wire and coaxial transmission lines, waves in waveguides and waves that are guided along the earth's surface from radio transmitter to the receiving point. The transmission of light via a dielectric waveguide was first proposed and investigated by Hondros and Debye in the beginning of this century. Since then the interest in optical applications has been developed enormously through the achievement of lasing action in semiconductors, development of the heterostructure laser and p-n junction modulator.

In the circular cylindrical waveguide, early work was carried on modes in dielectric rods. This was followed by the suggestion of using optical fibres for long distance communication. Since then a lot of development has been made in reducing the fibre losses and operating the waveguides at longer wavelengths. The most extensively used optical waveguide is the step-index optical fibre which consists of a cylindrical central core cladded by a material of slightly lower refractive index. For a ray entering the fibre core, if the angle of incidence (at the core-cladding interface) ϕ is greater than the critical angle $\phi_C = \sin^{-1}(n_2 / n_1)$, then the ray will undergo total internal reflection at that interface. Here n_1 and n_2 are refractive indices of core and cladding respectively.Due to cylindrical symmetry in the fibre structure, the ray also suffers total internal reflection at the lower interface and will thus be guided through the fibre core by repeated total internal reflections. This is the basic principle of light guidance through the optical fibre. A mathematical analysis of the guided modes in optical fibres is rather complicated. The situation becomes worse in the presence of radiation losses. Therefore the properties of light transmission in dielectric waveguides are difficult to learn in such a study.

However there are simpler dielectric waveguides, whose physical properties are very nearly the same as those of the round optical fibre. Such structures are much easier to analyse. The simplest optical waveguide to analyse is probably the planar waveguide which consists of a thin dielectric film (of refractive index n_1) sandwitched between materials of slightly lower refractive indices, say n_2 and n_s. This structure is useful to study the radiation and mode conversion properties of dielectric waveguides. It resembles a longitudinal cross-section through the cladded optical fibre and can be regarded as its two dimensional analog. Therefore the results obtained with the help of such a waveguide model are usually directly applicable to the round optical fibre. Such planar waveguides are important components in integrated optics. We now present the electromagnetic analysis of an asymmetric dielectric slab waveguide in somewhat detail.

1.4.2 Mode Treatment of Asymmetric Dielectric Slab Waveguide

A typical lossless asymmetric dielectric waveguide is shown in Fig.(1.4). It is considered to have a thickness typically of the order of an optical wavelength supported by a substrate many wavelengths thick. We find the modes guided by this slab waveguide directly from Maxwell's equations, which are written in terms of the refractive indices of the three layers. We also assume the magnetic permeability to be the same as that of free space.

$$\nabla \times H = n_j^2 \epsilon_0 \frac{dE}{dt} \qquad \ldots\ldots\ (1.9)$$

$$\nabla \times E = - \mu_0 \frac{dH}{dt} \qquad \ldots\ldots\ (1.10)$$

$$\nabla \cdot E = 0 \qquad \ldots\ldots\ (1.11)$$

$$\nabla \cdot H = 0 \qquad \ldots\ldots\ (1.12)$$

where n_j = (j = 1,2,3) = refractive index of j^{th} layer.

By applying curl operator to equation (1.10) and simplifying we get

$$\nabla^2 E = \mu_o \epsilon_o n_j^2 \frac{d^2 E}{dt^2}$$

Assuming $E \propto \exp(-i\omega t)$ with $k^2 = \omega^2 \mu_o \epsilon_o$ we obtain

$$\nabla^2 E + k^2 n_j^2 E = 0 \qquad \ldots\ldots (1.13)$$

which is the familiar wave equation for the uniform dielectric with reractive index n_j. For a two dimensional situation $\partial/\partial y = 0$ i.e. the guide extends to infinity in the positive and negative y-directions, so that the waveguide geometry and the field distributions of the modes are uniform throughout the y-direction. This restriction ($\partial/\partial y = 0$) allows us to decompose the field of slab waveguide into TE and TM modes.

<u>Case (a)</u> : In order to simplify the analysis we shall first omit the time and z-dependence factor from the expressions for the fields and shall consider only x-dependence. Then Equation (1.13) may be rewritten for the three regions of the guide as :

Regoin 3 : $$\frac{\partial^2 E_3}{\partial x^2} - r^2 E_3 = 0 \qquad \ldots. (1.14)$$

Regoin 1 : $$\frac{\partial^2 E_1}{\partial x^2} + q^2 E_1 = 0 \qquad \ldots. (1.15)$$

Regoin 2 : $$\frac{\partial^2 E_2}{\partial x^2} - p^2 E_2 = 0 \qquad \ldots. (1.16)$$

where $q^2 = n_1^2 k^2 - \beta^2$, $p^2 = \beta^2 - n_2^2 k^2$, $r^2 = \beta^2 - n_3^2 k^2$

Similar forms of the wave equation in the three regions may be derived for the magnetic field H from Maxwell's equations.

TE Guided Modes :

The assumption $\partial/\partial y = 0$ when applied to Eqn.(1.9)-(1.12) leads to the result that the only non-zero field components for TE modes are E_y , H_x and H_z. The Eqn.(1.10) yields

$$\nabla \times E = -\mu_o \frac{dH}{dt}$$

L.H.S.=

$$\nabla \times E = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & 0 & \frac{\partial}{\partial z} \\ 0 & E_y & 0 \end{vmatrix} = i\left[-\frac{\partial E_y}{\partial z}\right] + k\left[\frac{\partial E_y}{\partial x}\right]$$

$$= i\,\beta E_y + k\left[\frac{\partial E_y}{\partial x}\right]$$

$$R.H.S. = -\mu_o \frac{\partial}{\partial t}\left[i\,H_x + k\,H_z\right] = i(i\mu_o\omega)H_x + k\,(i\,\mu_o\omega)H_z$$

Simplifying further we get

$$H_x = -\frac{\beta}{\omega\,\mu_o} E_y \qquad \ldots\ldots (1.17)$$

$$H_z = -\frac{i}{\omega\,\mu_o} \frac{\partial E_y}{\partial x} \qquad \ldots\ldots (1.18)$$

These express the two nonzero magnetic field components in terms of the single nonzero electric field component E_y , which itself is given by the solution of wave Eqns.(1.14)-(1.16) in each region.

The other requirement to be satisfied by these field components is that the tangential components E_y and H_z should be continuous at the interfaces 1-2 and 1-3 between dielectric layers. Let us choose the origin of the x-axis at the 1-3 interface, so that the 1-2 interface is at x = -2a.

For guided modes we require that the power be confined largely to the central layer of the guide i.e. region 1. The form of Eqns.(1.14)-(1.16) then implise that this requirement will be satisfied for an oscillatory solution in region 1 ($q^2 \geq 0$) with evanescent tails in the cladding regions 2 and 3 ($p^2, r^2 \geq 0$). Combining these conditions on β we have

$$n_1 k \geq \beta \geq n_2 k \geq n_3 k$$

From the above consideration we may write down the solutions for E_y in the three regions for a guided mode.

$$E_y = \begin{cases} Ae^{-rx} & x > 0 \\ A\cos(qx) + B\sin(qx) & 0 \geq x \geq -2a \quad \ldots (1.19) \\ (A\cos 2aq - B\sin 2aq)e^{p(x+2a)} & -2a \geq x \end{cases}$$

The form of Eqns.(1.19) has been so chosen that the requirement of continuity of E_y at $x = 0$ and $x = -2a$ is satisfied explicitly. To complete the boundary requirement, the continuity of H_z is to be ensured. This component is given from Eqn.(1.18) as

$$H_z = \frac{-i}{\omega \mu_0} \begin{cases} -rAe^{-rx} & x > 0 \\ q(-A\sin qx + B\cos qx) & 0 \geq x \geq -2a \quad ..(1.20) \\ p(A\cos 2aq - B\sin 2aq)e^{p(x+2a)} & -2a \geq x \end{cases}$$

The continuity condition yields

$$\tan(2aq) = \frac{q(p + r)}{q^2 - p\,r} \qquad \ldots\ldots (1.21)$$

which is the eigenvalue equation for TE modes.

For TM guided modes the similar eigenvalue equation can be obtained as

$$\tan(2aq) = \frac{(n_3^2 p + n_2^2 r) n_1^2 q}{n_2^2 \, n_3^2 \, q^2 - n_1^4 pr} \quad \ldots\ldots (1.22)$$

Mode Numbers and Cut-offs :

We use the notation TE_N (and similarly TM_N) for a mode possessing N nodes in the field distribution. The value of N is obtained by taking the argument of the tangent in the eigenvalue Eqn.(1.21) or (1.22) to be $(2aq - N\pi)$ and using the cut-off condition $\beta = k\, n_2$ which corresponds in the modal analysis to loss of optical confinement and field spreading throughout the region 2.

Using the above definitions we may find expressions for cut-off frequencies for TE and TM guided modes of the slab waveguide. Substituting the expressions for p, q, r at cut-off with the definition of normalized frequency v $(= a\, k\, (n_1^2 - n_2^2)^{1/2})$ we get the cut-off value v_c as

$$\text{TE} : \quad v_c = \frac{1}{2} \tan^{-1}\left[\left(\frac{n_2^2 - n_3^2}{n_1^2 - n_2^2} \right)^{1/2} \right] + \frac{N\pi}{2} \quad \ldots\ldots (1.23)$$

where $\tan^{-1}$ is restricted to the range $0 - \pi/2$. This relation may also be used as a method of counting the number of guided TE modes. Thus for the first mode N = 0. Then for a normalized frequency v, Eq.(1.23) gives the number (M) of guided modes as below.

$$\text{TE} : \quad M = \left\{ \frac{1}{\pi} \left(2v - \tan^{-1}\left[\left(\frac{n_2^2 - n_3^2}{n_1^2 - n_2^2} \right)^{1/2} \right] \right) \right\}_{int} \quad \ldots (1.24)$$

where the subscript `int'indicates the next largest integer.

The corresponding results for TM modes are as follows :

$$v_c = \frac{1}{2}\tan^{-1}\left[\left(\frac{n_1^2}{n_3^2}\right)^2\left(\frac{n_2^2 - n_3^2}{n_1^2 - n_2^2}\right)^{1/2}\right] + \frac{N\pi}{2} \quad \ldots. (1.25)$$

$$M = \left\{\frac{1}{\pi}\left(2v - \tan^{-1}\left[\left(\frac{n_1^2}{n_3^2}\right)^2\left(\frac{n_2^2 - n_3^2}{n_1^2 - n_2^2}\right)^{1/2}\right]\right)\right\}_{int} \quad \ldots.. (1.26)$$

<u>**Normalization in terms of Power flow :**</u>

The time averaged power flow in the guide is given as:

$$P = \int_{-\infty}^{\infty} S_z\, dx = \frac{1}{2}\int_{-\infty}^{\infty} Re(E \times H^*)_z\, dx \quad \ldots.. (1.27)$$

For TE modes, Poynting vector S_z is given by

$$S_z = -\frac{1}{2}E_y H_x^* = \frac{\beta}{2\omega\mu_o}|E_y|^2$$

The integral in Eq.(1.27) is split into three parts corresponding to the three layers and the power in each region is obtained.

$$\left.\begin{aligned} P_3 &= \left(\frac{\beta}{2\omega\mu_o}\right)\frac{A^2}{2r} \\ P_1 &= \left(\frac{\beta}{2\omega\mu_o}\right)\frac{A^2}{2}\left(\frac{q^2 + r^2}{q^2}\right)\left[2a + \frac{p}{q^2 + p^2} + \frac{r}{q^2 + r^2}\right] \\ P_2 &= \left(\frac{\beta}{2\omega\mu_o}\right)\frac{A^2}{2p}\left[\frac{q^2 + r^2}{p^2 + q^2}\right] \end{aligned}\right\} \quad \ldots (1.28)$$

The total power P is given by

$$\begin{aligned} P &= P_1 + P_2 + P_3 \\ &= \left(\frac{\beta}{2\omega\mu_o}\right)\frac{A^2}{2}\left(\frac{q^2 + r^2}{p^2}\right)\left[2a + \frac{1}{p} + \frac{1}{r}\right] \end{aligned} \quad \ldots.. (1.29)$$

Furthermore we get

$$\frac{P_{core}}{P} = \frac{2a + \frac{p}{(q^2 + p^2)} + \frac{r}{(q^2 + r^2)}}{2a + \frac{1}{p} + \frac{1}{r}} \quad \ldots (1.30)$$

$$\frac{P_{clad}}{P} = \frac{\frac{q^2}{p(q^2 + p^2)} + \frac{q^2}{r(q^2 + r^2)}}{2a + \frac{1}{p} + \frac{1}{r}} \quad \ldots (1.31)$$

where $P_{core} = P_1$ and $P_{clad} = P_1 + P_2$

Similar experssions hold good for TM modes.

Losses:

Consider the case of a dielectric slab waveguide composed of lossy medium which is characterised by a complex refractive index $n_j + iK_j$ where n_j is real refractive index and K_j is termed as the extinction coefficient. It is related to the attenuation coefficient as

$$K_j = - \frac{\alpha_j}{2k}$$

where k is the wavenumber. We assume that the attenuation α_j is due to (i) absorption in core region 1 (ii) transmission into medium 2 on reflection at 1-2 interface (iii) transmission into medium 3 on reflection at 1-3 interface.

The losses (ii) and (iii) occur because total internal reflection is strictly only possible at an interface between lossless dielectrics.

The result for attenuation is especially simple if we consider the case of weakly-guiding dielectric slab which is subject to the condition that $(n_1 - n_j) \ll n_1$ $(j = 2,3)$. In this case Eq.(1.21) reduces to

$$\tan(2aq) = \frac{2p\,q}{q^2 - p^2}$$

As cut-off is approached, the power spreads out into the cladding regions until guidance is lost completely at cut-off. The modal attenuation coefficient α can be expressed with power ratios as :

$$\alpha = \alpha_1 \frac{P_{core}}{P} + \alpha_2 \frac{P_{clad}}{P} \qquad \ldots\ldots (1.32)$$

where α_1 , α_2 are the losses in core and cladding respectively. Alternatively we can define a "Radiation confinement factor" as $\Gamma = P_{core} / P$ so that Eq.(1.32) becomes

$$\alpha = \alpha_1 \Gamma + \alpha_2 (1 - \Gamma) \qquad \ldots\ldots (1.33)$$

This result is of great practical importance in calculating threshold currents for semiconductor injection lasers.

Finally a useful relationship between phase velocity, group velocity and power confinement factor, neglecting material dispersion effects is given by

$$\frac{c^2}{v_p\, v_g} = n_1^2\, \Gamma + n_2^2 (1 - \Gamma) \qquad \ldots.. (1.34)$$

<u>Case (b)</u> : Now we include the z-dependence of the fields i.e. $e^{-j\beta_m z}$ and present the analysis without considering time dependence once again.

<u>TE Modes</u> : For simplicity we take Region 3 of Fig.(1.4) as air and t as the film thickness (instead of 2a). The new geometry of the slab waveguide is shown in Fig.(1.5). Then the wave equations for the three regions are written as :

$$\left[\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2} \right] E_y(x,z) + \omega^2 \epsilon(x) \mu_o E_y(x,z) = 0 \quad \ldots (1.35)$$

with

$$\left.\begin{aligned} \epsilon(x) &= \epsilon_o && \text{in air region} \\ &= n_1^2 \epsilon_o && \text{in film region} \\ &= n_2^2 \epsilon_o && \text{in substrate region} \end{aligned}\right\} \quad \ldots (1.36)$$

where n_1 = refractive index of film

n_2 = refractive index of substrate

and

$$H_x = - \frac{j}{\omega \mu_o} \frac{\partial E_y}{\partial z}$$

$$H_z = \frac{j}{\omega \mu_o} \frac{\partial E_y}{\partial x} \quad \ldots\ldots (1.37)$$

Other symbols have their usual meanings. The solutions of Eq.(1.35) for m^{th} mode guided in the three regions are as below :

1> <u>air</u>

$$E_m = A_m \sin (h_m t + \phi_m) \exp [-p_m(x - t)] \exp [-j\beta_m z]$$

2> <u>film</u>

$$E_m = A_m \sin (h_m x + \phi_m) \exp [-j\beta_m z] \quad \ldots\ldots. (1.38)$$

3> <u>substrate</u>

$$E_m = A_m \sin (\phi_m) \exp [q_m x] \exp [-j\beta_m z]$$

with $k = \omega \sqrt{\epsilon_o \mu_o} = \frac{2\pi}{\lambda}$

The above solutions are respectively subjected to the following conditions :

$$(\beta_m / k)^2 - (p_m / k)^2 = 1 \quad \ldots\ldots (1.39a)$$

$$(\beta_m / k)^2 + (h_m / k)^2 = n_1^2 \quad \ldots\ldots (1.39b)$$

$$(\beta_m / k)^2 - (q_m / k)^2 = n_2^2 \quad \ldots\ldots (1.39c)$$

with $\tan (\phi_m) = h_m / q_m \quad \ldots\ldots (1.39d)$

Here h_m , q_m and p_m are the m^{th} set of real roots of transcendental equation.

$$\tan [(h_m / k) k t + \phi_m] = - h_m / p_m \quad .... (1.40)$$

The values of (h / k), (q / k) and (p / k) are dependent on refractive indices. For a given normalized thickness (kt) there is only a finite number of roots leading to a descrete set of real values for h, q and p. Thus the guided modes are also called as **descrete modes** and are labelled by the subscript m which takes values 0,1,2,... so that $h_o < h_1 < h_2 \ldots$. From Eq.(1.39) it is obvious that

$$n_2 < | \beta_m / k | < n_1$$

From Eq.(1.40) we have

$$\cot \left[(h_m / k) k t + q_m\right] = - p_m / h_m \quad .. (1.41)$$

Using the trigonometric relation

$$\tan \left[\left(\frac{2m + 1}{2}\right)\pi - \theta \right] = \cot \theta \text{ with } m = 0,1,2, \ldots$$

we can rewrite Eq.(1.41) as

$$\tan \left\{\left(\frac{2m + 1}{2}\right)\pi - [(h_m / k)k t + \phi_m]\right\} = - p_m / h_m$$

which after simplification gives

$$kt = \left\{\left(\frac{2m + 1}{2}\right)\pi + \tan^{-1}(p_m / h_m) - \tan^{-1}(h_m / q_m)\right\} / (h_m/k) \quad ...... (1.42)$$

From Eqs.(1.39a)-(1.39c) we can obtain the values of (p_m/k), (h_m/k) and (q_m/k) which yield the ratios.

$$(p_m/h_m) = \left[(\beta_m/k)^2 - 1\right]^{1/2} / \left[n_1^2 - (\beta_m/k)^2\right]^{1/2} \quad ....(1.43)$$

and

$$(h_m/q_m) = \left[n_1^2 - (\beta_m/k)^2\right]^{1/2} / \left[(\beta_m/k)^2 - n_2^2\right]^{1/2} \quad ...(1.44)$$

At cut-off of m^{th} mode we have (q_m=0), so that Eq.(1.39c) gives $(\beta_m/k) = n_2$. Using this we have

$$(p_m/h_m) = (n_2^2 - 1)^{1/2} / (n_1^2 - n_2^2)^{1/2} \qquad \ldots\ldots (1.45)$$

and

$$(h_m/k) = (n_1^2 - n_2^2)^{1/2} \qquad \ldots\ldots (1.46)$$

Applying $q_m = 0$ and using Eqs.(1.45) and (1.46) in Eq.(1.42) we get the expression for the minimum film thickness

$$k\, t_m = \left\{ m\,\pi + \tan^{-1}\left[\frac{(n_2^2 - 1)}{(n_1^2 - n_2^2)} \right]^{1/2} \right\} (n_1^2 - n_2^2)^{-1/2} \ldots (1.47)$$

where m = 0,1,2,3, ...

TM Modes :

The guided wave mode treatment for this class is similar to TE case described above. The main results are given here. The wave equation for n^{th} mode is

$$\left[\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2} \right] H_y(x,z) + \omega^2 \epsilon(x) \mu_o H_y(x,z) = 0 \quad \ldots (1.48)$$

$$E_x = \frac{j}{\omega\,\epsilon} \frac{\partial H_y}{\partial z} \quad , \qquad E_z = - \frac{j}{\omega\,\epsilon} \frac{\partial H_y}{\partial x} \qquad \ldots\ldots (1.49)$$

where $\epsilon(x)$ is given by Eqs.(1.36). The solutions in different regions are given below.

1> air

$$H_n = B_n' \sin\,(h_n t + \phi_n')\, \exp\,[-p_n(x - t)]\, \exp\,[-j\beta_n z]$$

2> film

$$H_n = B_n' \sin\,(h_n x + \phi_n')\, \exp\,[-j\beta_n z]$$

3> substrate

$$H_n = B_n' \sin\,(\phi_n')\, \exp\,[q_n\, x\,]\, \exp\,[-j\beta_n z]$$

These solutions obey the conditions (1.39a)-(1.39c) except for

$$\tan \phi_n' = (n_2/n_1)^2 \; h_n/q_n \quad \ldots\ldots (1.50)$$

Here h_n , q_n and β_n are the n^{th} set of real roots of the transcendental equation.

$$\tan [(h_n / k) \; k \; t + \phi_n'] = - h_n / (n_1^2 \; p_n) \quad \ldots\ldots (1.51)$$

where $n = 0,1,2, \ldots$ so that $h_o < h_1 < h_2 \ldots$.

The minimum normalized thickness is given by

$$kt_n = \left\{ n \pi + \tan^{-1}\left\{ n_1^2 \left[\frac{(n_2^2 - 1)}{(n_1^2 - n_2^2)}\right]^{1/2} \right\}\right\}(n_1^2 - n_2^2)^{-1/2} \ldots (1.52)$$

Comparing the cut-off thickness t_m and t_n it is found that t_n needed for n^{th} order TM mode is always larger than t_m for the m^{th} order TE mode. From this it is possible to design the thickness of the guide with appropriate n_1 and n_2 for the desired mode to propagate in that structure.

1.5 Summary :

At the outset the importance of the field of integrated optics in various applications including nonlinear optics is brought out. This is followed by a brief outline of electromagnetic mode theory which is essential for analysing the propagation of light waves through the thin-film optical waveguides. Maxwell's wave equation is given and its solutions as plane electromagnetic waves in unbounded, free space and dielectric media are discussed. Subsequently the propagation of plane waves in bounded media is explained with reference to both TE and TM polarisations. The mode treatment of an asymmetric dielectric slab

waveguide is discussed in detail by both excluding and including the z-dependence of electric and magnetic fields in the case of TE and TM guided modes. Also the expressions for the minimum film thicknesses at cut-off are obtained in each case.

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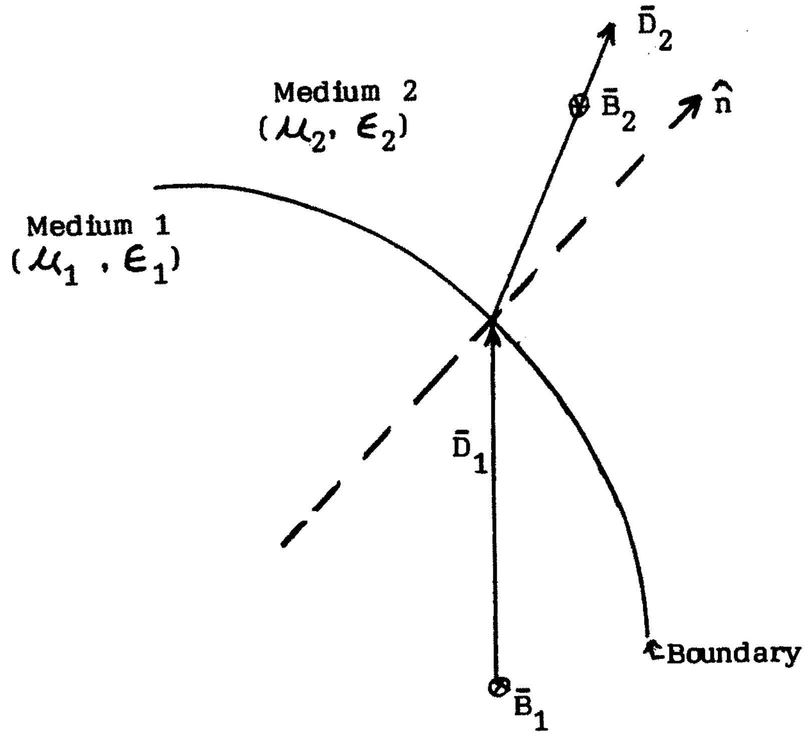

Fig. 1.1 : Propagation of Wave Across A Boundary Surface

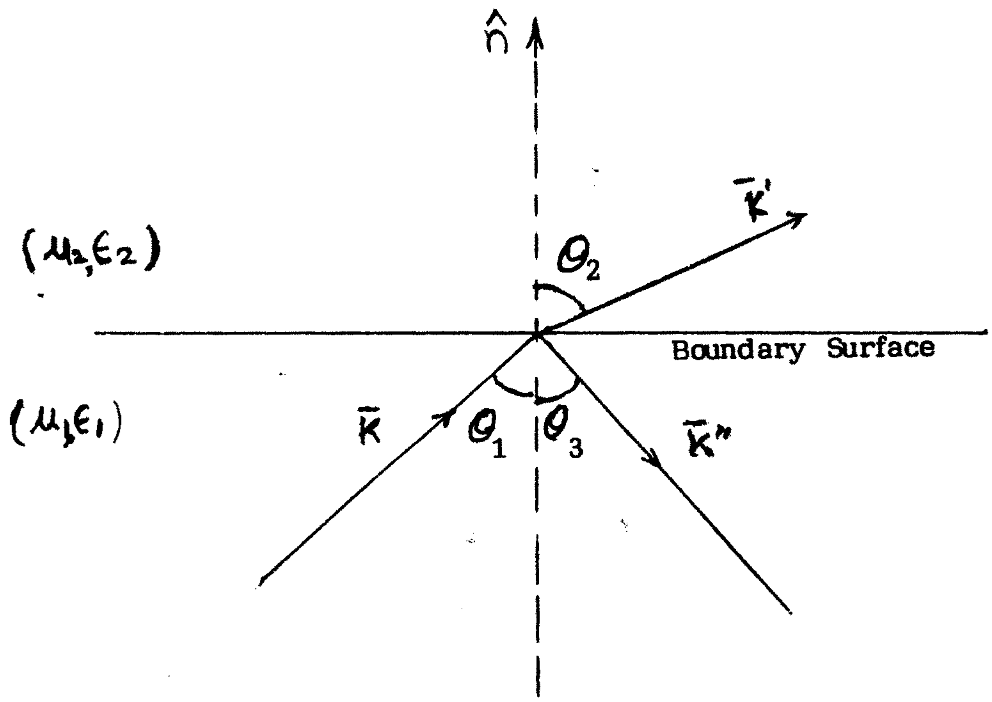

Fig.1.2 : Reflection and Refraction at a Boundary Surface

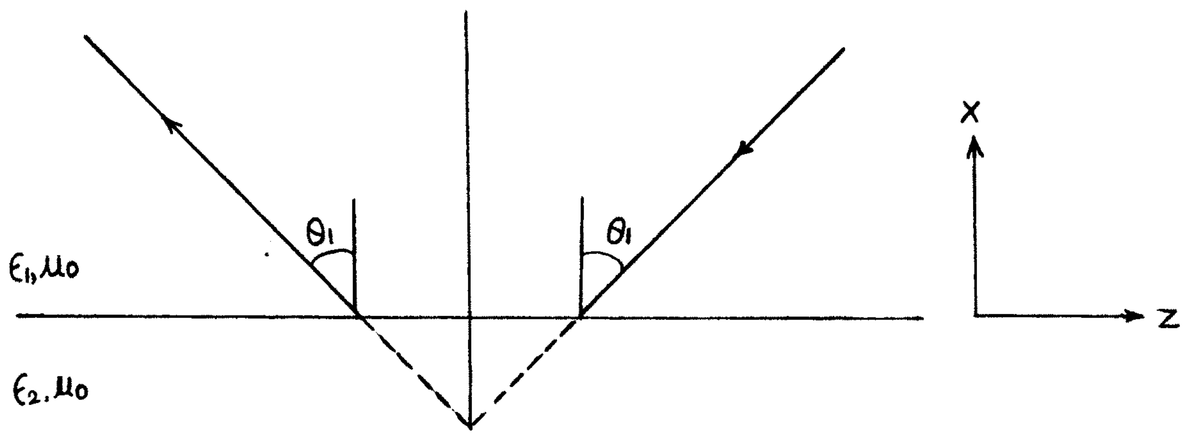

Fig. 1.3 : Goos -Haenchen Shift on Reflection

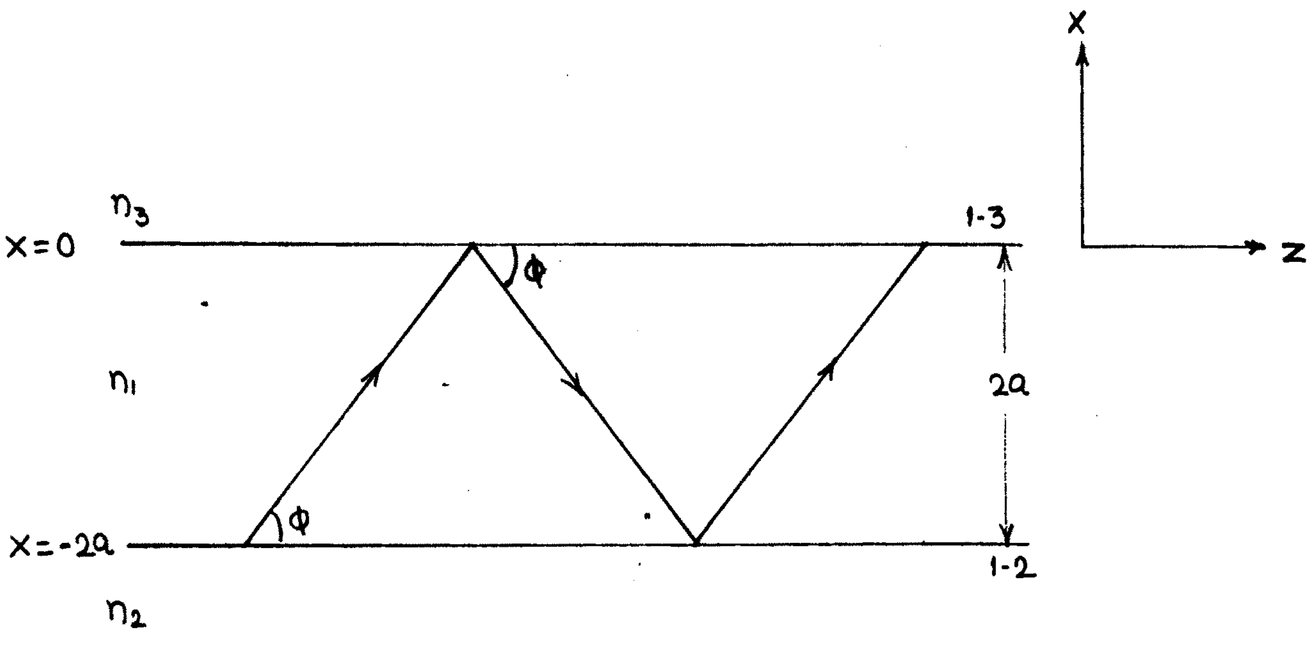

Fig. 1.4 : Propagation of a Ray in Dielkectric Slab Waveguide

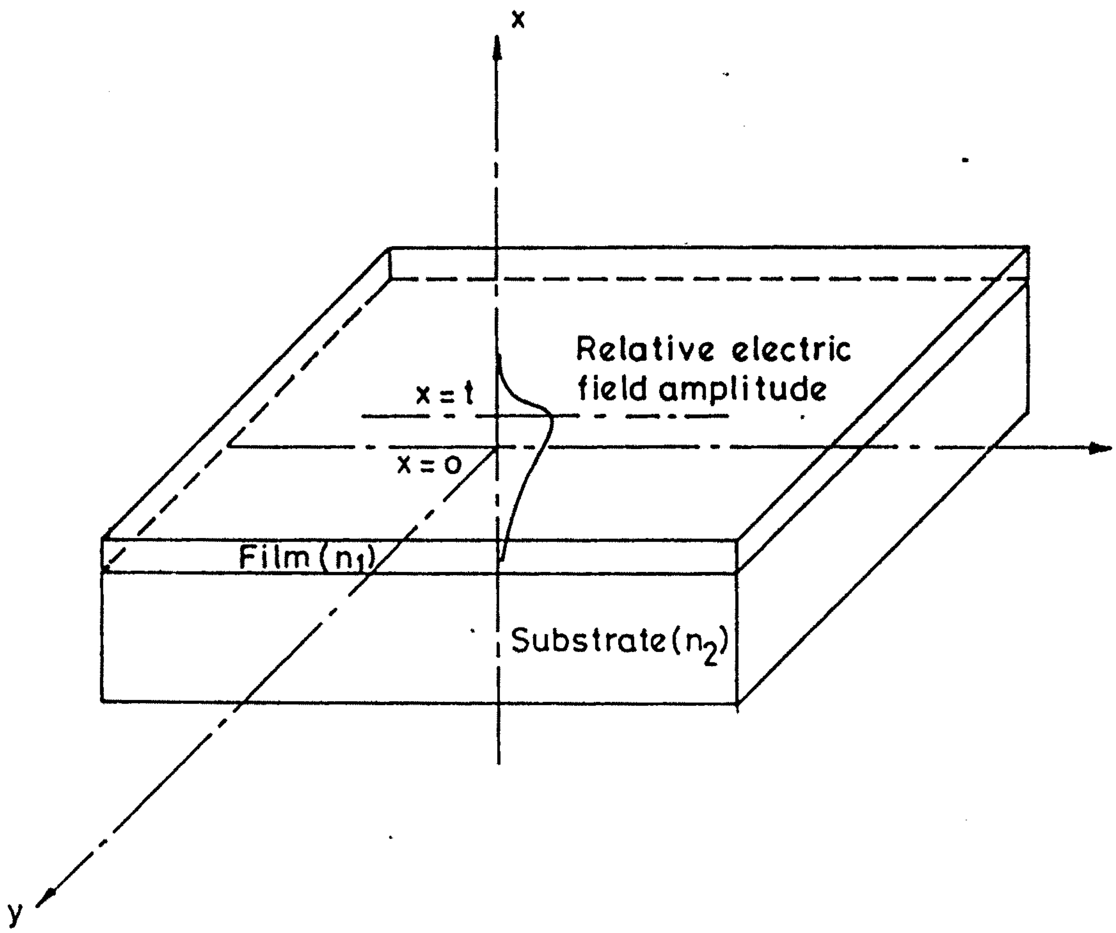

Fig.1.5 : Electric Field Pattern of A Typical TE_0 Guided Wave Mode