CHAPTER - 2

PROPAGATION CHARACTERISTICS OF MULTILAYER SLAB WAVEGUIDES

2.1 Introduction :

In the previous chapter we have introduced electromagnetic theory and discussed its application to a three layer asymmetric slab waveguide. In the early 60's the theory of this waveguide was applied to fabricate the semiconductor injection laser. At that time the device consisted of a GaAs film incorporating a p-n junction. By applying a forward bias, the laser radiation at a wavelength of 0.9 μm could be achieved. In order to reduce considerably the current density required for the lasing action, heterojunctions were later introduced in place of a single p-n junction. This was possible due to a liquid phase epitaxy technique, in which a epitaxial growth of successive layers of specified material could be achieved on the n-type GaAs substrate. These layers were of the alloy $Al_xGa_{1-x}As$. which could help to improve the performance of heterostructure laser in two ways : by confining the carriers to the active layer and simultaneously confining the electromagnetic radiation in this region. A typical double heterostructure is as given below :

Superstrate : p^{+}-$Al_xGa_{1-x}As$
Guiding region : p-GaAs
Substrate : n-$Al_xGa_{1-x}As$

In order to achieve large optical confinement regions for high power output a four-layer structure was proposed. This structure differs from the conventional double heterostructure laser, in that the p-n junction is displaced from one of the heterointerfaces into the central region. Such a four layer structure exhibits gain only in the p-layer of the region between the heterojunctions. A typical four-layer structure is given below :

Superstrate : n-$Al_xGa_{1-x}As$

Guiding region : p-GaAs

Additional layer : n-GaAs

Substrate : p-$Al_xGa_{1-x}As$

Four-layer structures are useful to fabricate optical waveguide lens, tapper couplers and metal-clad waveguides.

Later on five-layer structures were introduced which were found to be useful in devising the so-called 'W' guide (having a refractive index distribution resembling the shape of the letter 'W') and separate confinement heterostructure (SCH) lasers. The W-guide is seen to have interesting properties regarding mode cut-offs, confinement factor and mode filter characteristics. It offers an increased range of single mode operation as compared to symmetric three-layer slab. In SCH laser, the layers are fabricated so asto confine the electron-hole recombination process to the central layer. The resulting confinement of radiation to the recombination region can be made superior to that for the conventional double heterostructure laser. As a result, there is more efficient pumping of the optical field by the recombination process. A further advantage offered by the SCH structure is that it may be used in devices where it is desired to prevent current carriers from reaching a region where nonradiative recombination may occur, while permitting the optical field to penetrate this region. e.g. the use of SCH structure has solved this problem in the distributed feedback (DFB) laser and resulted in room temperature c.w. DFB semiconductor laser.

Multilayer structured waveguides have been widely used recently in many optical devices, such as modulators, switches, directional couplers, Bragg deflectors, spectrum analysers, and semiconductor lasers. A three-layer slab waveguide is the simplest optical waveguide that has been well studied and documented[1-5]. We have already studied its theory in Chapter 1 as applied to a three-layer asymmetric slab. Waveguides with more than three-layers have been theoretically studied by many authors.[6-10] The eigenvalue equations for the four-layer structure have been derived by the wave theory and the ray theory.[7-11] The five-layer symmetrical guide with anisotropic dielectric permittivity has been considered by Nelson and Mckenna.[12] Ruschin and Marom[14] have obtained the explicit eigenvalue equations of the symmetrical seven-layer waveguide for both even and odd modes by using matrix treatment. Multilayer waveguides with periodic index distributions have also been studied.[15-17] An explicit eigenvalue equation of a periodic stratified waveguide has been obtained by Yeh et al.[17] By using the matrix method, Walpita[18] and Revelli[19] have studied the general N-layer waveguide, but their results involved complex matrices.[18-19] More recently Li and Lit[20] have obtained general formulae describing field distributions and eigenvalue equations for both TE and TM modes in a multilayer slab waveguide. Their results show that additional multilayers can produce useful effects such as increasing the cut-off values and the confinement factors of guided modes.

In principle, a waveguide which is perfectly plane and uniform along its length can permit various modes to propagate without attenuation. However all practical waveguides and waveguide devices are imperfect. The important imperfections are found to be losses of the dielectric material, departures from perfect straightness, inhomogeneities of the dielectric material, and departures of the core cladding interface from a perfect plane. These induce some energy loss in the propagation so that electromagnetic waves are attenuated. Four-layer structures have been used to model a variety of low-loss planar optical waveguides formed on silicon substrates with a silicon dioxide cladding layer[21-24] so that the attenuation is reduced considerably. The attenuation properties of five-layer planar waveguides have been investigated using perturbation approach and other techniques.[6,12,25,27-29]

After introducing the subject of multilayer planar structures and taking a brief literature survey, we shall first give the standard analyses of four-layer and five-layer slab waveguides. At the end we shall present an account of mode treatment of inhomogeneous planar waveguides.

2.2 Four-layer Asymmetric Slab Theory :

Consider the geometry and dielectric distribution of the general four-layer waveguide as shown in Fig.(2.1). We take Region 1 as the primary region of electromagnetic confinement although for some cases region 2 can also be chosen. The structure is regarded as a real guide with refractive index distribution as $n_1 \geq n_2 \geq n_3 \geq n_4$.

For guided modes there are two cases to be considered as :

<u>case (a)</u> : $kn_2 \geq \beta \geq kn_3$

For this case the TE and TM modal fields may be expressed as :

$$E_y, H_y = \begin{cases} (-A \sin 2ah_1 + B \cos 2ah_1)\, e^{h_4(x + 2a)} & x \leq -2a \\ A \sin h_1 x + B \cos h_1 x & -2a \leq x \leq 0 \\ \dfrac{B \cos (h_2 x + \varkappa)}{\cos \varkappa} & 0 \leq x \leq 2d \\ \dfrac{B \cos (2dh_2 + \varkappa)}{\cos \varkappa}\, e^{h_3(2d - x)} & x \geq 2d \end{cases} \quad \ldots\ldots (2.1)$$

$$\text{where} \quad \left.\begin{aligned} h_1^2 &= k^2 n_1^2 - \beta^2 \\ h_2^2 &= k^2 n_2^2 - \beta^2 \\ h_3^2 &= \beta^2 - k^2 n_3^2 \\ h_4^2 &= \beta^2 - k^2 n_4^2 \end{aligned}\right\} \quad \ldots\ldots\ldots (2.2)$$

Applying the boundary conditions at the three dielectric interfaces and simplifying we get the eigenvalue equation for the asymmetric four-layer structure as :

$$2ah_1 = N\pi + \tan^{-1}\left(\eta_{14}\frac{h_4}{h_1}\right)$$

$$+ \tan^{-1}\left\{\eta_{12}\frac{h_2}{h_1} \tan \left[\tan^{-1}\left(\eta_{23}\frac{h_3}{h_2} - 2dh_2\right)\right]\right\}$$

...... (2.3)

where (N = 0,1,2, ...), and

$$\eta_{ij} = \begin{cases} 1 & \text{, for TE modes} \\ n_i^2 / n_j^2 & \text{, for TM modes} \end{cases}$$

<u>Case (b)</u> : $kn_1 \geq \beta \geq kn_2$

For this the modal fields may be written by analogy with Eq.(2.1) as :

$$E_y, H_y = \begin{cases} (-A \sin 2ah_1 + B \cos 2ah_1)\, e^{h_4(x + 2a)} & x \leq -2a \\ A \sin h_1 x + B \cos h_1 x & -2a \leq x \leq 0 \\ \dfrac{B \cosh (h_2'' x + \varkappa)}{\cosh \varkappa} & 0 \leq x \leq 2d \\ \dfrac{B \cosh (2dh_2'' + \varkappa)}{\cosh \varkappa}\, e^{h_3(2d - x)} & x \geq 2d \end{cases}$$

...... (2.4)

where h_1 , h_3 , h_4 are defined as in Eq.(2.2) and $h_2''^2 = \beta^2 - k^2 n_2^2$. Applying the boundary conditions at the three dielectric interfaces and simplifying, the eigenvalue equation is found to be

$$2ah_1 = N\pi + \tan^{-1}\left(\eta_{14}\frac{h_4}{h_1}\right)$$

$$+ \tan^{-1}\left\{\eta_{12}\frac{h_2''}{h_1} \tanh \left[\tanh^{-1}\left(\eta_{23}\frac{h_3}{h_2''} + 2dh_2''\right)\right]\right\}$$

...... (2.5)

Numerical solutions of the eigenvalue Eqs.(2.3) and (2.5) for real four-layer dielectric waveguides are easily obtained.

2.3 Five-layer Symmetric Slab Theory[6] :

The five-layer symmetric slab waveguide is shown schematically as in Fig.(2.2). We take the refractive index distribution as $n_1 \geq n_2 \geq n_3$. The structure is symmetric about the midpoint of the core region which is designated as $x=0$. As in the case of asymmetric four-layer guide, there are two cases for the guided modes. For simplicity we consider only even-order modes so that zero-order mode is also included.

Case (a) : $kn_2 \geq \beta \geq kn_3$

The field distributions for even-order modes are written as:

$$E_y, H_y = \begin{cases} A \cos h_1 x & x \leq |x| \leq a \\ \dfrac{A \cos h_1 a \cos (h_2 |x| + \varkappa)}{\cos (h_2 a + \varkappa)} & a \leq |x| \leq b \\ \dfrac{A \cos h_1 a \cos (h_2 b + \varkappa)}{\cos (h_2 a + \varkappa)} e^{h_3 (b - |x|)} & |x| \geq b \end{cases} \qquad \ldots\ldots (2.6)$$

where

$$\left.\begin{aligned} h_1^2 &= k^2 n_1^2 - \beta^2 \\ h_2^2 &= k^2 n_2^2 - \beta^2 \\ h_3^2 &= \beta - k^2 n_3^2 \end{aligned}\right\} \qquad \ldots\ldots\ldots (2.7)$$

Applying the boundary conditions and eliminating $\varkappa$ we obtain :

$$h_1 a = M\pi + \tan^{-1}\left\{\eta_{12}\frac{h_2}{h_1} \tan \left[\tan^{-1}\left(\eta_{23}\frac{h_3}{h_2}\right) - h_2 (b - a)\right]\right\} \qquad \ldots\ldots (2.8)$$

where $(M = 0,1,2, \ldots)$ and

$$\eta_{ij} = \begin{cases} 1 & , \quad \text{for TE modes} \\ n_i^2 / n_j^2 & , \quad \text{for TM modes} \end{cases}$$

Note that the index M in the eigenvalue Eq.(2.8) gives only the even-order modes i.e. $N = 2M$ in the notation previously used.

<u>Case (b)</u> : $kn_1 \geq \beta \geq kn_2$

In this case the even-order field distribution becomes :

$$E_y, H_y = \begin{cases} A \cos h_1 x & 0 \leq |x| \leq a \\ \dfrac{A \cos h_1 a \cosh (h_2''|x| + \varkappa)}{\cosh (h_2'' a + \varkappa)} & 0 \leq |x| \leq b \\ \dfrac{A \cos h_1 a \cosh (h_2'' b + \varkappa)}{\cosh (h_2'' a + \varkappa)} e^{h_3 (b - |x|)} & |x| \geq b \end{cases} \qquad \ldots\ldots (2.9)$$

where h_1 , h_3 are defined as in Eq.(2.7) and $h_2'' = \beta^2 - k^2 n_2^2$.
The corresponding eigenvalue equation is :

$$h_1 a = M\pi + \tan^{-1}\left\{\eta_{12} \frac{h_2''}{h_1} \tanh \left[\tanh^{-1}\left(\eta_{23} \frac{h_3}{h_2''}\right) + h_2''(b - a)\right]\right\} \qquad \ldots\ldots (2.10)$$

where $M = 0,1,2, \ldots$

The eigenvalue Eqs.(2.8) and (2.10) may be conveniently re-written in terms of normalised variables (for the weakly-guiding situation $n_{ij} \simeq 1$) which are defined as :

$$\left.\begin{aligned} v^2 &= a^2k^2(n_1^2 - n_2^2) \\ u^2 &= a^2h_1^2 \\ w^2 &= v^2 - u^2 = a^2h_3^2 \\ t^2 &= u^2 - v^2c^2 = a^2h_2^2 \\ t''^2 &= v^2c^2 - u^2 = a^2h_2''^2 \end{aligned}\right\} \quad \ldots\ldots \quad (2.11)$$

where $c^2 = \dfrac{n_1^2 - n_2^2}{n_1^2 - n_3^2} \simeq \dfrac{n_1 - n_2}{n_1 - n_3}$

Case (a) : $c \le u/v \le 1$

$$u = M\pi + \tan^{-1}\left\{\frac{t}{u} \tan\left[\tan^{-1}\left(\frac{\omega}{t}\right) - t\left(b/a - 1\right)\right]\right\} \quad .. \quad (2.12)$$

Case (b) : $0 \le u/v \le c$

$$u = M\pi + \tan^{-1}\left\{\frac{t''}{u} \tanh\left[\tanh^{-1}\left(\frac{\omega}{t''}\right) + t''\left(b/a - 1\right)\right]\right\} \quad ..(2.13)$$

These equations may easily be solved numerically to yield, say, u as a function v for various values of the two parameters c and b/a.

We have noted in Sec.(2.1) that a simple application of five-layer symmetric slab is found in SCH lasers. This structure is advantageous regarding the confinement of radiation to the recombination region. This is measured by the power confinement factor $\Gamma = P_{core}/\ P$. Using the field distributions and the normalised variables we can obtain expressions for Γ factor for the two cases discussed above. Fig.(2.3) shows Γ versus v for the five-layer structure. For comparison such a plot for the equivalent three-layer slab is also given.

From the figure it is seen that for small v, the values of Γ obtained with the five-layer structure are larger than those for the equivalent three-layer slab. Higher Γ values correspond to lower threshold current density in a semiconductor laser. Thus SCH laser has a lower threshold current density as compared with the equivalent DH device.

2.4 Two-Dimensional Parabolic-index Media[6] :

In this section we deal with waveguides whose refractive index is inhomogeneous in the direction(s) normal to the waveguide axis, i.e. graded index guides. This grading occurs as a result of the fabrication process, e.g. diffusion or ion implantation in planar guides, and may produce waveguides with characteristics unique to the specific variation of refractive index obtained. We consider the simplest form of symmetric refractive index variation n(x), that obeys the parabolic law :

$$n^2(x) = n_1^2 \left[1 - 2\Delta \left(\frac{x}{a}\right)^2 \right] \qquad \ldots\ldots (2.14)$$

where x is the distance normal to the axis of propagation (i.e. z-axis); n_1 = refractive index on axis, i.e. at the guide centre; 2a = width of guiding layer while Δ is a parameter governing the index variation which is defined as:

$$2\Delta = \frac{n_1^2 - n_2^2}{n_1^2} \qquad \ldots\ldots (2.15)$$

For the weakly-guiding approximation $n_1 - n_2 \ll n_1$, n_2. Eq.(2.15) reduces to

$$\Delta \simeq \frac{n_1 - n_2}{n_1}$$

which is the relative core-cladding index difference of a slab waveguide.

A particular case which is often considered is that of a graded-index guiding layer ($|x| \le a$) sandwitched between a substrate and a superstrate each with refractive index n_2. The index distribution is as shown in Fig.(2.4) for the case of no index discontinuity at the boundary $|x| = a$ i.e. $n(a) = n_2$.

2.4.1 <u>Electromagnetic Mode Treatment of Parabolic-index Media</u>[6] :

For a general graded index medium with refractive index n(x), Maxwell's equations can be written as :

$$\nabla \times H = n^2(x)\epsilon_o \frac{dE}{dt} \quad \text{...... (2.16a)}$$

$$\nabla \times E = -\mu_o \frac{dH}{dt} \quad \text{...... (2.16b)}$$

$$\nabla \cdot (n^2(x)\epsilon_o E) = 0 \quad \text{...... (2.16c)}$$

$$\nabla \cdot H = 0 \quad \text{...... (2.16d)}$$

Applying curl operator to Eq.(2.16b) and simplifying we get

$$\nabla^2 E + \nabla\left[\frac{E \cdot \nabla n^2(x)}{n^2(x)}\right] + k^2 n^2(x)\, E = 0 \quad \text{...... (2.17)}$$

with $k^2 = \omega^2 \mu_o \epsilon_o$. Eq.(2.17) is the vector wave equation for the electric field E. Similar expression may be obtained for magnetic field H.

Since n(x) is a function of transverse coordinate x only, the Eq.(2.17) can be expressed as scalar wave equation in the component E_x and is written as :

$$\frac{d^2E_x}{dx^2} + \frac{d}{dx}\left[\frac{E}{n^2(x)}\frac{d^2n^2(x)}{dx^2}\right] + (k^2n^2(x) - \beta^2)E_x = 0 \qquad \ldots\ldots (2.18)$$

In order to eliminate the (dE_x / dx) term in (2.18) we introduce the transformation as :

$$E_x = \frac{\psi}{n(x)} \qquad \ldots\ldots (2.18a)$$

Putting in Eq.(2.18) we get scalar wave equation for ψ :

$$\frac{d^2\psi}{dx^2} + \left[\frac{1}{2n^2(x)}\frac{d^2n^2(x)}{dx^2} - \frac{3}{4n^4(x)}\frac{d^2n^2(x)}{dx^2} + k^2n^2(x) - \beta^2\right]\psi = 0 \qquad \ldots\ldots (2.19)$$

a) <u>TE solutions :</u>

For the TE modes of the general graded-index medium, we take $E_x = 0$ and assume H_x is a solution of equation with $\partial/\partial y = 0$

$$\frac{d^2H_x}{dx^2} + (k^2n^2(x) - \beta^2)\, H_x = 0 \qquad \ldots\ldots (2.20)$$

The other field components are given from Eqs.(2.16a) and (2.16b) as :

$$E_x = H_y = 0$$

$$E_y = -\frac{\omega\,\mu_o}{\beta}H_x$$

$$H_z = \frac{i}{\beta}\frac{dH_x}{dx}$$

We consider the special case of the parabolic-index medium with $n^2(x)$ given by (2.14) Consequently Eq.(2.20) becomes :

$$\frac{d^2H_x}{dx^2} + \left[k^2n_1^2\left(1 - 2\Delta\left(\frac{x}{a}\right)^2 - \beta^2\right]H_x = 0 \qquad \ldots\ldots (2.21)$$

Let its trial solution be

$$H_x = X(x)\,\exp\left[-\frac{x^2}{w_o^2}\right] \qquad \ldots\ldots (2.22)$$

so that we get

$$\frac{d^2X}{dx^2} - \frac{4x}{w_o^2}\frac{dX}{dx} - \frac{2}{w_o^2}\left[1 - \frac{2x^2}{w_o^2}\right]X + \left[k^2n_1^2\left\{1 - 2\Delta\left(\frac{x}{a}\right)^2 - \beta^2\right]X = 0 \quad \ldots(2.23)$$

We eliminate the x^2-terms by appropriately defining the 'beam-waist', w_o as :

$$w_o^2 = \frac{2a}{kn_1(2\Delta)^{1/2}} = \frac{2a^2}{V} \qquad \ldots\ldots (2.24)$$

Using this definition alongwith the transformation

$$x' = \frac{x\sqrt{2}}{w_o}$$

Eq.(2.19) is written in the form given below :

$$\frac{d^2X}{dx'^2} - 2x'\frac{dX}{dx'} + \left[(k^2n_1^2 - \beta^2)\frac{w_o^2}{2} - 1\right]X = 0 \quad \ldots (2.25)$$

The solutions of Eq.(2.25) are found to be the Hermite polynomials $H_N(x')$, where N and β are related by

$$2N = (k^2n_1^2 - \beta^2)\frac{w_o^2}{2} - 1 \quad (N = 0,1,2,\ldots) \quad \ldots (2.26)$$

Thus solution for the TE modes of the parabolic medium is given as Hermite-Gaussian functions :

$$H_x = \frac{2^{1/4}}{\pi^{1/4}(2^N\, N!\, w_o)^{1/2}} H_N\left[\frac{x\sqrt{2}}{w_o}\right]\exp\left[-\frac{x^2}{w_o^2}\right]$$

$$(N = 0,1,2,\ \ldots) \qquad \ldots\ldots (2.27)$$

where the constant has been so chosen that the modes are normalized according to

$$\int_{-\infty}^{\infty} |H_x|^2 \, dx = 1$$

The Eq.(2.27) may be expressed in terms of v rather than w_o by using Eq.(2.24)

$$H_x = \frac{v^{1/4}}{\pi^{1/4}(2^N \, N! \, a)^{1/2}} \, H_N \left[\frac{v^{1/2} x}{a}\right] \exp\left[-\frac{v \, x^2}{2 \, a^2}\right]$$

$$(N = 0,1,2, \ldots) \qquad \ldots\ldots (2.28)$$

The eigenvalue equation for these modes given by Eq.(2.26) may be re-written in terms of normalized variables and propagation constant which are respectively defined as :

$$u^2 = v \, (2N + 1) \quad (N = 0,1,2, \ldots) \quad \ldots (2.29)$$

$$b = 1 - \left[\frac{2N + 1}{v}\right] \quad (N = 0,1,2, \ldots) \quad \ldots (2.30)$$

Plots of b versus v calculated from Eq.(2.30) for a few low-order modes are given in Fig.(2.5). It is to be noted that the condition b = 0 ($\beta = kn_2$) occurs for values of v given by

$$v_c = 2N + 1 \qquad (N = 0,1,2, \ldots) \quad \ldots (2.31)$$

But this does not correspond to a definition of waveguide cut-off in the usual sense. At values of v given by Eq.(2.31) the modal fields will not possess the characteristics of modes at cut-off as in, say, the three-layer slab guide, discussed in Sec.(1.4.2) In the lattercase field distributions become constant in the cladding layers so that power is distributed uniformly throughout all space. For the Hermite-Gaussian modes the fields will still possess the characteristics of guided modes at all v values, since the refractive index distribution of Eq.(2.14) is unbounded in the x-direction. Hence the results of parabolic-index medium are only a approximation to those of a real cladded waveguide for modes that are tightly confined in a region near the axis $x = 0$.

b) TM Solutions :

In this case we assume $H_x = 0$ and suppose that E_x can be a solution of Eq. (2.19) under the transformation $E_x = \psi / n(x)$. The remaining field components are given from Eqs.(2.16a) and (2.16b) as :

$$E_y = H_z = 0$$
$$E_z = \frac{i}{\beta n^2(x)} \frac{d}{dx} (n^2(x) E_x)$$
$$H_y = \frac{\omega\, n^2(x) \epsilon_0}{\beta} E_x$$

In the case of parabolic-index media, Eq.(2.19) may be solved by expanding terms in $n^2(x)$ as a power series and neglecting $\Delta^2(x/a)^4$ and higher order terms. The equation then reduces to :

$$\frac{d^2\psi}{dx^2} + \left[\left(k^2 n_1^2 - \beta^2 - \frac{2\Delta}{a^2}\right) - \left(\frac{k^2 n_1^2\, 2\Delta}{a^2} + \frac{16\Delta^2}{a^4}\right) x^2 \right] \psi = 0 \qquad \text{...... (2.32)}$$

This equation has a form similar to that of Eq.(2.21). Hence its solution would be of the form

$$E_x = \frac{\psi}{n(x)} = \frac{2^{1/4}}{n(x)\pi^{1/4}(2^N N!\, w_0)^{1/2}} H_N\left(\frac{x\sqrt{2}}{w_0}\right) \exp\left(-\frac{x^2}{w_0^2}\right)$$

$$(N = 0,1,2, \ldots) \qquad \text{..... (2.33)}$$

where the 'beam-waist' w_0 is now given by

$$w_0^2 = 2 \Big/ \left[\frac{k^2 n_1^2\, 2\Delta}{a^2} + \frac{16\Delta^2}{a^4}\right]^{1/2} = \frac{2a^2}{V} \quad \ldots. \; (2.34)$$

The corresponding eigenvalue equation is

$$\beta^2 = k^2 n_1^2 - \frac{2\Delta}{a^2} - (2N + 1)\left[\frac{k^2 n_1^2\, 2\Delta}{a^2} + \frac{16\Delta^2}{a^4}\right]^{1/2}$$

$$(N = 0,1,2, \ldots) \quad \ldots.. \; (2.35)$$

Note that for $2\Delta \ll k^2 n_1^2 a^2$ the TM and TE solutions become similar.

c) Group Delay and Radiation Confinement Factor :

The expressions for propagation constant and hence group delay can be obtained for TE and TM modes from the expressions derived above. For this purpose we expand Eqs.(2.26) and (2.35) in ascending powers of $\Delta^{1/2}$. Retaining terms upto order Δ, Eq.(2.26) yields :

$$\text{TE:} \quad \beta \simeq kn_1 - \frac{(2\Delta)^{1/2}}{a}(N + \tfrac{1}{2}) - \frac{\Delta}{a^2 kn_1}(N + \tfrac{1}{2})^2 \quad ..(2.36)$$

and Eq.(2.35) gives

$$\text{TM:} \quad \beta \simeq kn_1 - \frac{(2\Delta)^{1/2}}{a}(N + \tfrac{1}{2}) - \frac{\Delta}{a^2 kn_1}\left[(N + \tfrac{1}{2})^2 + 1\right]$$

$$\ldots\ldots \; (2.37)$$

Defferentiating each of these equations, we find the group delay τ as :

$$\text{TE :} \quad \tau \simeq \frac{Ln_1}{c}\left[1 + \frac{\Delta}{a^2 k^2 n_1^2}(N + \tfrac{1}{2})^2\right] \quad \ldots\ldots \; (2.38)$$

$$\text{TM :} \quad \tau \simeq \frac{Ln_1}{c}\left\{1 + \frac{\Delta}{a^2 k^2 n_1^2}\left[(N + \tfrac{1}{2})^2 + 1\right]\right\} \quad .. \; (2.39)$$

The following points are noteworthy from these equations :(i) Except for a few low-order modes, the group delay is very similar for TE and TM modes, (ii) For small $(\Delta/a^2k^2n_1^2)$, the TE and TM modes are quasi-degenerate, (iii) For small $(\Delta/a^2k^2n_1^2)$, the group delay is the same for all modes independent of mode number.

The result (iii) can be re-interpreted for the weakly-guiding case (quasi-degeneracy of TE and TM modes) by using the normalized Eq.(2.30). From this equation the normalized dispersion parameter d(vb)/dv is given by

$$\frac{d(vb)}{dv} = 1 \quad \text{(all N)}$$

i.e. to this level of approximation the group delay is the same for all modes. Also in this limit of small Δ, the radiation confinement factor Γ, is given by

$$\Gamma = \frac{v^{1/2}}{\pi^{1/2}\, 2^N\, N!\, a} \int_{-a}^{a} \left[H_N\left(\frac{v^{1/2}x}{a}\right)\right]^2 \exp\left(-\frac{vx^2}{a^2}\right) dx \qquad \ldots\ldots. (2.40)$$

The values of Γ may always be evaluated in terms of the error function $erf(v^{1/2})$. e.g. For the two lowest-order modes we find.

$$\left.\begin{array}{ll} N = 0 : & \Gamma = erf\,(v^{1/2}) \\ N = 1 : & \Gamma = erf\,(v^{1/2}) - \dfrac{2v^{1/2}}{\pi^{1/2}}\, e^{-v} \end{array}\right\} \text{for } v > v_o \qquad \ldots.. (2.41)$$

Since $v > v_o$ is an artificial cut-off condition, the confinement factor does not go to zero at $v > v_o$ and great care should be taken in applying these results to real waveguides of parabolic-index variation.

The above discussed mode treatment applies to a guiding layer having infinitely extended parabolic-index profile defined by Eq.(2.14). In reality this profile has a discontinuity at $|x| = a$ due to the presence of substrate and superstrate. Such guiding layers are required to be treated by approximation methods like peturbation technique and WKB approximation.

2.5 Summary :

At the outset of this chapter we have given the practical applications of three-layer and four-layer asymmetric slab waveguides. The need for introducing a fifth layer is also mentioned. This is followed by a brief survey of the theoretical studies of multilayer structured waveguides. The references on the studies of attenuation properties of four-layer structures are also noted. Further we have given outlines of the theoretical analyses of four-layer asymmetric and five-layer symmetric slab structures. Finally we have introduced the inhomogeneous planar waveguide and have discussed the electromagnetic mode treatment of parabolic-index media by obtaining both TE and TM solutions. At the end the importance of group delay and the radiation confinement factor has been explained.

References :

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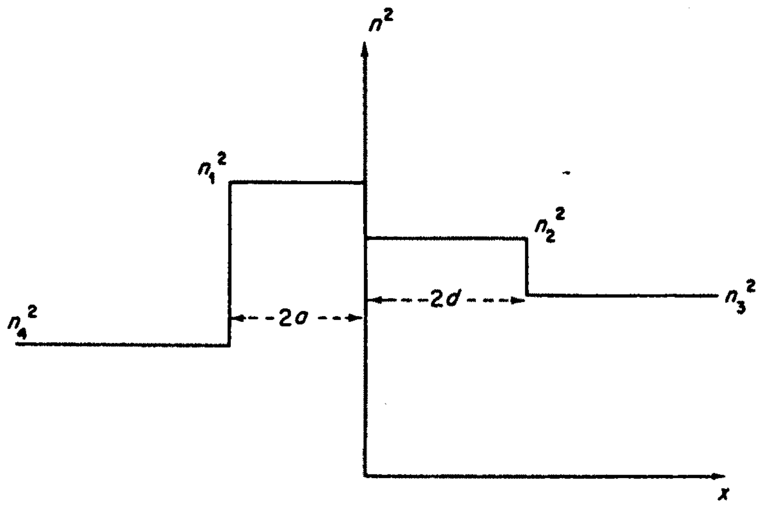

Fig.2.1 : The Four-layer Asymmetric ?Slab Structure

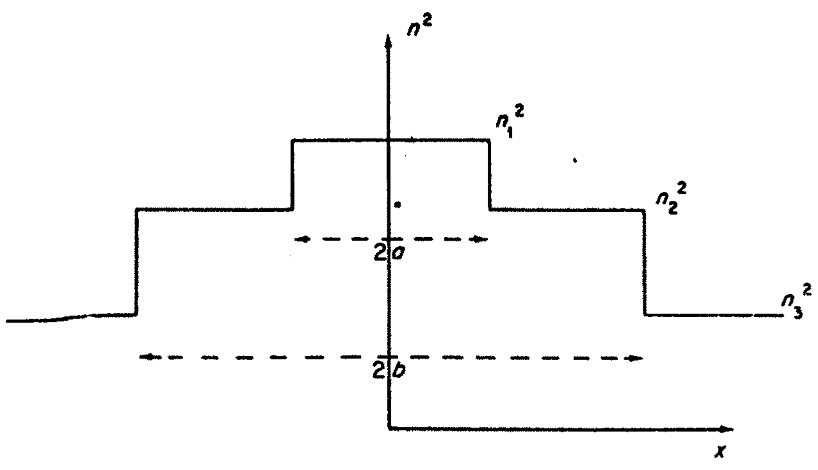

Fig.2.2 : The Five-layer Symmetric Slab Structure

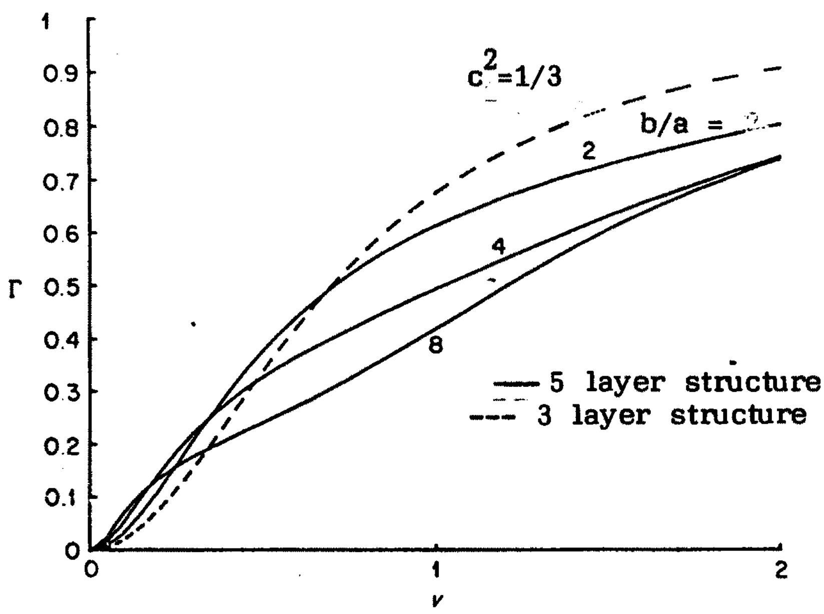

Fig.2.3 : Confinenent factor versus v for TE_{01} mode

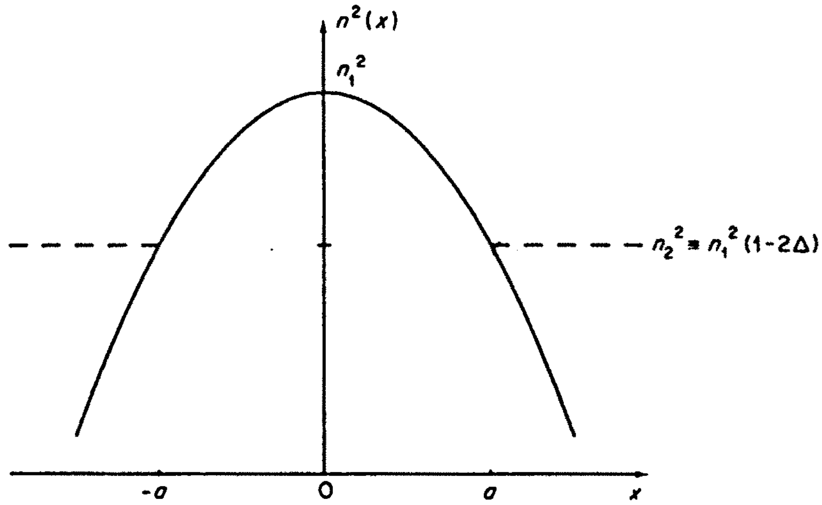

Fig.2.4 : Parabolic Refractive Index Profile

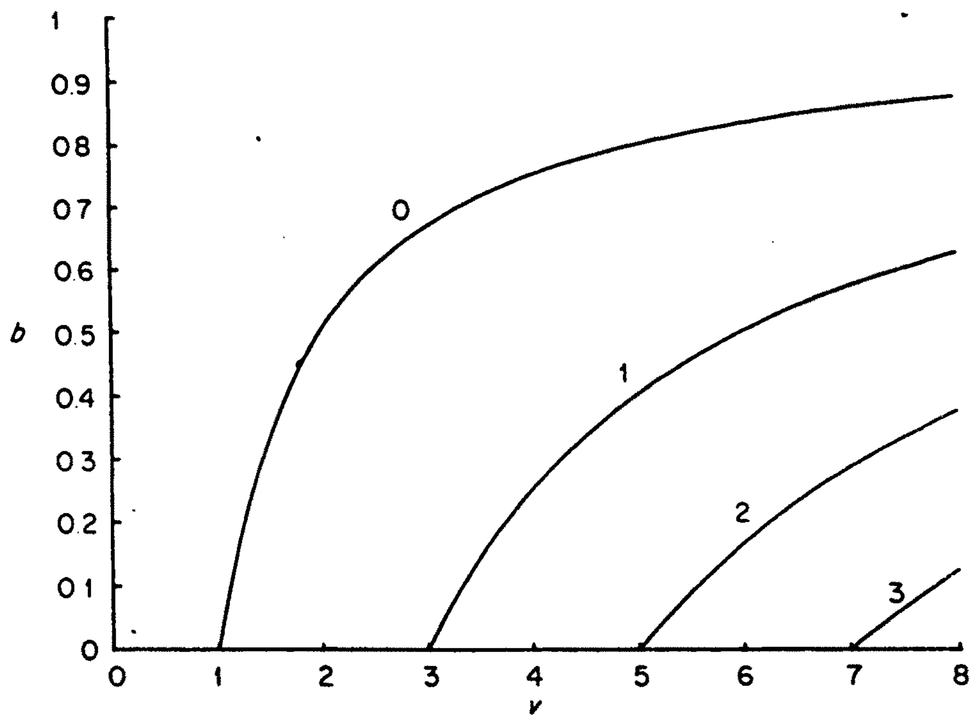

Fig.2.5 : b-v Curves for Parabolic Index Medium