CHAPTER - 0

INTRODUCTION

Information theory is a branch of mathematical statistics and probability. It is used in any probabilistic or statistical system of observations. It plays important role in communication theory. The communication system can be viewed as a probabilistic one. In every day life we get information by some system such as radio television, telegraph etc.

Following are the some of the views :

Tuller (1950) says "statistical theory of communication is often called information theory". Rothstein (1951) has defined information theory as 'abstract mathematics dealing with measurable sets, with choices from alternatives of an unspecified nature.' Pierce (1956) says that communication theory and information theory have same meaning. Gilbert (1958) remarks, 'Information will be a measure of time or cost of a sort which is of particular use to the engineer in his role of designer of an experiment'.

The concept of information plays central role in the life of a person and society as well as all sorts of

scientific research. According to Renyi (1960), one can say that modern foundation of civilization is based on the concept of 'information'. The concept of information is compared with concept of energy. This is very interesting to compare these concepts. The great culture of our civilization and discoveries are connected with the conversion, storage and transfer of the energy as, e.g. discovery of fire, use of water and wind power (sailing) in ancient age, the steam engine, the electric energy in modern time, the reactive motors and atomic energy recently. The new inventions and discoveries connected with storage and transmission of information are speech, writing, printing, telegraph, telephone, radio, phonographs, films, television and computer etc. There is another analogy between energy and information, which took a long time until the abstract notion of energy to be developed i.e. known as the mechanical energy, heat, chemical energy, electricity, atomic energy and so on, are compared and measured with same units. The energy concept is as old as century, but the concept of information is fairly new.

The notion of entropy is also used in thermodynamics. Hartley (1928) first introduced logarithmic measure of information to measure storage capacity of some instrument. According to him 'to

characterize an element of a set of size N, we need $\log_2 N$ units of information', the unit of information is called 'bit'. The notion of entropy first introduced into information theory by C.E. Shannon (1958). He pointed out that the formula used by Hartley is valid for any case, i.e. for Unequal Probabilities. If all events are equiprobable, the Shannon's entropy is same as the Hartley's entropy.

Now, we consider probabilistic notion of entropy. Any probability distribution may be considered as showing how the probability of certainty is distributed among the possible outcomes (possible event) of a random experiment. Every probability distribution can be considered as a measure in the space of possible events.

Let X be a random variable and the distribution X is non-degenerate i.e. X is not surely equal to constant C. Thus, there is uncertainty concerning the values of X. The probabilistic notion of entropy gives answer to the question, 'What amount of uncertainty is contained in random variable X ?' Thus, the entropy can be viewed as measure of uncertainty.

Let X be discrete random variable with values $x_1, x_2, \ldots, x_n$ with probabilities $P_1, P_2, \ldots, P_n$ where,

$P\ (X = x_k) = P_k$

$P_k > 0$ and $\sum_{i=1}^{n} P_k = 1$

The entropy is given as

$$H_o(X) = -\lambda \sum_{k=1}^{n} P_k \log P_k.$$

Here, λ is chosen arbitrary if

$$\lambda = 1/\log 2.$$

$H_o(X)$ is given as

$$H_o(X) = -\sum_{k=1}^{n} P_k \log_2 P_k .$$

It is the formula of the entropy given by C.E. Shannon.

The Fisher measure of information, given by data about unknown parameter is the first measure of information in mathematical statistics. And it is used in theory of estimation.

This dissertation contains four chapters. The Chapter O itself is introduction in which general review of dissertation is taken.

The Chapter one deals with 'Meaning and Scope of the information'. This chapter contains four sections, the section 1.1 deals with basic concepts of information. The

concepts are given through communication model and measure of information is developed as the Shannon's entropy. The section 1.2 deals with the Shannon's entropy & its properties. The section 1.3 deals with principle of maximum information wherein the distribution which maximizes entropy is found under given contraints. Some logarithmic functions which satisfy certain properties of entropy are also considered as measure of information. In the section 1.4 we introduce other information measures such as the Hartley's entropy, the Renyi's entropy, the generalised entropies, the Kullaback-Leibler information measure and the Fishers' information measure.

The Chapter two deals with Sufficiency and Information. This chapter contains three sections. In section 2.1 the sufficiency of estimators is given based on the information theory. The section 2.2 deals with minimum discrimination function in which the density function which minimizes the discrimination function under certain condition is found out. The section 2.3 deals with a variant of Neyman pearson Lemma (Renyi 1966).

Chapter three deals with Sufficiency of Experiments. The section 3.1 deals with sufficient experiments in which definition of sufficient experiment is given and some related results are established.

Note that section 0 in each chapter is introductory section all over the dissertation.