CHAPTER : 0

INTRODUCTION

Let $\mathcal{F} = \{F_\theta \ , \ \theta \in \Theta\}$ be the family of distributions under consideration and θ be the parameter of interest which is to be estimated based on a sequence of observations. This problem of estimation of θ is equivalent to identify the distribution from which the observations have been drawn.

If $\Theta = \{\theta_0\}$, that is if the distribution from which the observations have come is known and it is F_{θ_0}, then the solution is trivial. If the support of the distribution P_{θ_1} and P_{θ_2} for $\theta_1 \neq \theta_2$, $\theta_1, \theta_2 \in \Theta$ are disjoint then based on single observation the parameter can be identified without error.

Let $\Theta = \{\theta_1, \theta_2\}$ and S_1, S_2 be supports of the distributions P_{θ_1} and P_{θ_2} respectively and $P_{\theta_1}(S_1 - S_2)$ and $P_{\theta_2}(S_2 - S_1)$ be positive. Let $X_1, X_2, \ldots X_n$ be a sequence of independent identicaly distributed (i.i.d.) random variables (r.v.'s) from F_θ.

Then for any n fixed $P_{\theta_i}\{(X_1, X_2, \ldots X_n) \in S_1^n \cap S_2^n\} > 0$ $i = 1,2$ and hence based on $X_1, X_2, \ldots X_n$ it is not possible to estimate the θ without error. But if $P_\theta\{X_1, X_2, \ldots X_n \in S_1^k \cap S_2^k$ for some $K > n\} = 0$, then based on at least K observations one can take a decision without any error. Roughly speaking in the long run the decision coincides with true value of the parameter.

Let $T_n(X_1, X_2, \ldots X_n)$ be an estimator of θ based on observations $X_1, X_2, \ldots X_n$. A desirable property of T_n is that in a sense T_n should approach to θ, $\theta \in \Theta$. When the mode of convergence is in probability then the above property of T_n is known as Weak Consistency.

Chapter one deals with the definition of Weak Consisteney and some properties of consistent estimators. It further deals with uniformly consistent estimator and Fisher consistent estimator. These concepts are aexplai ed with the help of examples.

Chapter two deals with discussion on the various methods of obtaining consistent estimators, necessary conditions of consistent estimators and related theorems. Consistency of maximum likelihood estimators and examples of inconsistency of maximum likelihood estimators (m.l.e.) have been also discussed. Besides the discussion on the order of consistent estimators is also included in it.

Third chapter is concerned with asymptotic properties of consistent estimator, effective standard deviation and asymptotic relative efficiency. Further discussion on consistency of testing procedures and related theorem on it have been included in it.