Chapter-1.

SOME BASIC CONCEPTS

1.1 Scope And Meaning Of Reliability:

Due to technological advances, the modern products are becoming more and more improved. Hence these products/processes are advantages over the previous (old) in the sence of life and well quality. In military applications, namely radars, missiles and some modern wapens have been invented. Also many complex equipments have been researched essentially for the purpose of space study. Latter for commercial uses, computers namely PCAT,PCXT etc. are designed and developed with different fascilities.

In general, each piece of equipment (system) is made-up of large number of elementary (very small) components or/& subsystems. These components or/and subsystems are designed to perform a specific function or veriety of functions. As a consequence (result), attention has been focused on evaluation of performance of a system. That is, measures to examine wheather the system is performing its intended (applied) function succesfully or not and also the quality of the performance have been developed. While a unit is put in use some data evolves naturally, this data itself can be used to study the performance of the unit.

One of the branch which deals the evaluation of performance of the equipments/systems is *Reliability Analysis*. There are various measures to evalute the performance of the system.

1.2 Different Reliability Measures:

Since different devices are developed and applied for different purposes, different measures of reliability are essential at appropriate situation. For example, in case of nuclear power reactor components, it is most important to study failure rate of these components, hence failure rate itself can be considerd as a reliability measure. On the other hand, in case of power supply for an unmanned exploratory space craft transmitting information about its environment, we are expecting its functioning without failure for its entire task. So here reliability measure of interest is probability of survival for its task. Also, during life time of a device, it might have different probabilities of succesfully performing its required function under the stated conditions at different times.

Quiet often we apply the term *Reliability* to mean the probability that a piece of equipment (component, subsystem, or system), under designed conditions, performs its specified work (function) for a given period of time

For example, if the variable T denotes the time to failure of a given 60 watt electric bulb, then reliability of the bulb at time t, say R(t), symbolically given by

$$R(t) = P(T>t).$$

Where $P(T>t)$ denotes the probability that time to failure exceeds t. In other words, reliability of bulb at time t is nothing but probability that it does not fail before time t.

It is very difficult to understand the physics of the failure process, and it is quiet complex too. Mathematical discription is also very difficult. To provide statistical summary account of length of life of a device, failure distribution is to be used. The choice of distribution is an art as well the science. One may attempts to use failure test data, to distinguish amoung various symmetrical distributions, but still problem arises. The reason is that, assymmetric distributions are importantly different in their tail regions.But due to small sample sizes, actual observations are thinly scattered, perticularly in the right hand tail region.

It has been recognised that these difficulties can be overcome by appealing to a concept which permits different distributions to be distinguished on the basis of physical considerations. Such a concept is expressed as a hazard rate and the closely related concept is that of failure rate.

Here we discuss following reliabolity measures.

1. Reliability (Survival) Function.
2. Mean Time To Failure.
3. Failure And Hazard Rate.
4. Probability Of Success For Assigned Task.
5. Reliable Life.
6. System Effectiveness.

1. Reliability (Survival) Function :

Define a random variable (r.v.) T to denote time required to failure of a component (or subsystem or system). Let $f_T(t)$ be probability density function (p.d.f.) of T. Thus probability of failure as a function of time, t, is defined as

$$F_T(t) = P(T \le t) = \int_0^t f_T(\tau).d\tau. \quad t>0 \qquad ...(1.1).$$

where. $F_T(t)$ denote the probability that the device will fail by time t. Some times time t is replaced by cycles, stress, etc. at appropriate situations. In such situations we speak of cycles to failure or stress to failure and so on.

Now, if we define reliability as the probability of success, that is, the probability that the device will perform its assigned function for at least a period of time t (.or at least c cycles or at least for s unit of stress), then is

$$R(t) = P(T \ge t) = \int_t^{\infty} f_T(x).dx = 1 - F_T(t) \qquad ...(1.2).$$

where. R(t) is called a reliability or survival function.

2>Mean Time To Failure(MTTF):

If r.v. T denotes the life time of the system then expected life time during which system performs succesfully is called MTTF. It is given by

$$E(T) = \int_0^{\infty} t.f_T(t).dt \qquad ...(1.3).$$

Generally E(T) is referred to as the expected life. If $\lim_{t\to\infty} t*R(t) = 0$ then another convinient method for determining the MTTF is given by

$$E(T) = \int_0^{\infty} R_T(t).dt \qquad ...(1.4).$$

We can also define the mean time to failure of a device that has survived to time t, namely

$$M(t) = \frac{1}{R(t)} \int_0^{\infty} \tau . f_T(t+\tau) . d\tau \qquad ...(1.5).$$

called mean residual life.Thus M(0) = E(T).

If device under consideration is renewed through maintainance and repair, E(T) is also known as *Mean Time Between Failures*(MTBF).Mostly renewal theory assumes that repaired systems are as good as new from failure stand point. But situations occur where perfect debugging is not possible.In such cases, it is convinient to consider the term like "the mean time to the j^{th} failure".

3>Failure And Hazard Rates:

Failure rate in the interval $[t_1,t_2]$ is nothing but the rate at which failure occurs durong that interval.It is defined as the probability that a failure per unit time occurs in the interval $[t_1,t_2]$,given that failure has not occured prior to the beginning. of the interval t_1. Mathematically, interval failure rate in the interval $[t_1,t_2]$ is given by

$$FR[t_1,t_2] = \left[\frac{R(t_1)-R(t_2)}{R(t_1)}\right]\left[\frac{1}{t_2 - t_1}\right] \qquad ...(1.6).$$

where first bracket stands for conditional probability of failure during $[t_1,t_2]$ given survived to time t_1, and second factor is dimentional characteristic used to express the conditional probality on a per unit time basis. In above, rate is expressed as failure per unit time, ferquently time units might be "cycles","distance", "stress", etc. as previously mentioned.

The hazard rate(or hazard rate function or, simply hazard function) is defined as the limit of failure rate. Hazard rate h(t) is defined as

$$h(t) = \lim_{\Delta_t \to 0} \left[\frac{R(t)-R(t+\Delta_t)}{R(t)\Delta_t} \right]$$

$$=1/R(t) * \left[- \frac{d}{dt} R(t) \right]$$

$$= - \frac{d}{dt} log(R(t)).$$

$$= \frac{f_T(t)}{R(t)} \qquad \ldots(1.7).$$

(since $- \frac{d}{dt} log(R(t)) = f_T(t)$,p.d.f. of failure life time.). Note that h(t)dt denotes the probability that a device which has survived to time 't' will fail in the small interval of time t to $t+\Delta_t$. Hence h(t) is the rate of change of the conditional probability of failure given survived to time t. Importance of the hazard rate is that it indicates the change in the failure rate over the life time of a device.

If h(t) is decreasing [increasing] in $t \geq 0$, then $f_T(t)$ is said to be an decreasing failure rate(DFR), [increasing failure rate(IFR)],distribution.

For Example : Let T has Weibull distribution with following p.d.f. given by

$$f_T(t:\theta,\alpha,\beta) = (\beta/\alpha).[(t-\theta)/\alpha]^{\beta-1}.\exp\{-[(t-\theta)/\alpha]^{\beta}\}; \ t > \theta \qquad ...(1.8).$$

and the parameters θ, α, β are positiv constants.

Note that, hazard rate for this distribution is given by

$$h(t) = (\beta/\alpha).[(t-\theta)/\alpha]^{\beta-1}. \qquad ...(1.9).$$

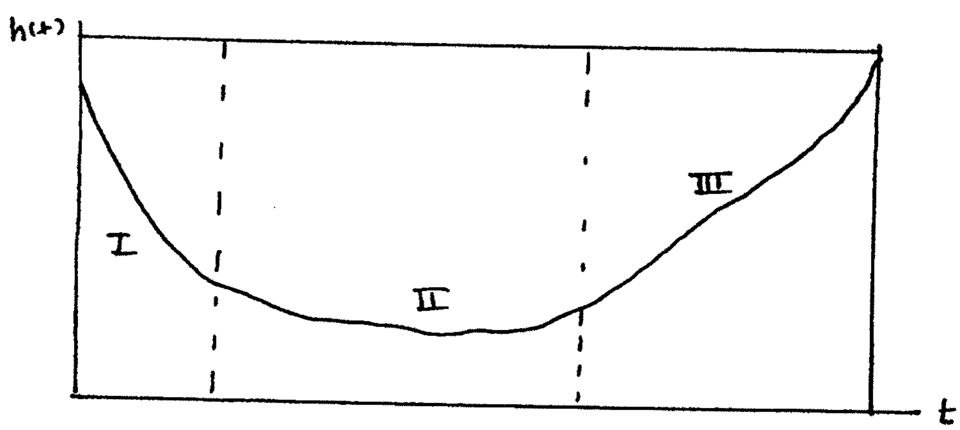
Now observe that-

i> For $\beta > 1$, h(t) is increasing in t, therefore Weibull distribution belongs to IFR distribution's class.

ii> For $\beta < 1$, h(t) is decreasing in t, therefore Weibull distribution belongs to DFR distribution's class.

iii> For $\beta = 1$, h(t) is neither increasing nor decreasing in t. It is constant with value $(1/\alpha)$ for all t, which is same as the hazrad rate of exponential distribution with parameter α.

A typical hazard rate generally has the so called bath tub shape (Dummer and Winton (1986)) shown below



Three distinct regions are shown.The first region is called initial failure region, is characterised by decreasing failure rate(DFR). It represents early failures due to material, or manufacturing defects. To remove many substandered devices it is useful to apply good quality controlwd. Also burn-in product testing removes such faults. In this dissertation chapter-4 is devoted to optimum criteria related to burn-in. Thus these two (namely, good quality control & burn-in testing) are useful to avoid this high initial failure rate.

The second one is characterized by near a constant failure rate. This region is called chance or random failure region. It represents chance failures are caused by sudden stress, unusual break-offs and unpredictable operating conditions, and so on. To avoid such things, environment has to be improved.

The third portion is called wear-out failure region, shown by increasing failure rate. This might be due to wearout and/or effect of accumulated shocks.

An important mathematical relationship between hazard rate and the reliability function :

We have,

$$h(x) = \frac{f(x)}{R(x)}$$

Integrating both side with respect to (w.r.t.) x over 0 to t,

assuming R(0) = 1,

$$\int_0^t h(x).dx = \int_0^t \frac{f(x)}{R(x)}.dx$$

$$= -\int_0^t \frac{d/dt(R(x))}{R(x)}.dx$$

$$= -\log(R(t)) \text{ gives}$$

$$R(t) = \exp\{-\int_0^t h(x).dx\} \quad ...(1.10).$$

$$f(t) = h(t)* \exp\{-\int_0^t h(x).dx\} \quad ...(1.11).$$

(combining (1.6) & (1.7)).

Thus h(t),R(t) & f(t) are all related and any one uniquely determine other two.

Barlow and Proschan (1975) define $H(t) = 1/t* \int_0^t h(x).dx$ as the failure (hazard) rate average. Thus $H(t) = -1/t*\log_0(R(t))$ .If H(t) is decreasing [increasing] in $t \geq 0$, then f(t) is said to be a decreasing failure rate average (DFRA) [increasing failure rate average (IFRA)] distribution.

4>. Probability Of Success For Assigned Task :

In case of attribute test data, where we need to see only the success or failure of a device. In such situations the probability of success is used as a measure of reliability of the device.

For example, suppose we have two medicines, say A and B, applicable for the dignosis of the same disease. Depending upon the strength (physical and/or mental) of patient these medicines may act in order to cure that disease. Now, we prefer medicine 'A' if it cures more proportion of people than that of medicine 'B', based on previous report.

5>Reliable Life:

It is sometimes necessary to consider the time t_R for which the reliability will be R. This time is known as reliable life.

Reliable life is one of the membar of reliability of a device. It is also a measure of quality a device, depending on value of t_R. If t_R is the reliable life, then it can be thought of as at time t_R, 100*R% of the population will servive. Also note that t_R is nothing but (1-R)*100th percentile of the life time distribution. We judge the goodness of the unit depending on the value of t_R for fixed given value of R. Note that, for large value of R (nearest to 1) if still t_R is larger then it indicates goodness of the unit.

6>System Effectiveness:

From consumers point of view, the term "reliability" usually has broder meaning that provided by the reliability function.For example, if consumer has the choice amongst the following two systems, namely

i>System is highly reliable but very easy to repair; &

ii>System is less reliable but very easy to repair.

Then he may choose "system ii>", the less reliable system.Hence system effectiveness can be defined as "The probability that the system can succesfully meet an operational demand within a given time period when operated under specified conditions is called *System Effectiveness*".

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