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# INTRODUCTION AND SUMMARY

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### 0.1 INTRODUCTION :

The term 'Reliability' has various meanings in various fields. For ages, things and people are said to be reliable if they had lived upto certain expectations and unreliable otherwise. A person is said to be reliable if he would never fail to deliver what he had promised or a watch is said to be reliable if it keeps time day after day. This reliability is judged by the performance of certain components or systems or some function or duty. When a system is working we say that its reliability is high if it has repeatedly performed its work with success and low if it fails frequently. The classical definition of reliability was given as fallows "Reliability is the probability of a device or system performing its functions adequately for the period of time intended, under the operating conditions intended."

To study reliability, life time models must be considered. To understand better which life distributions are important in reliability, we first consider a notion of aging. Aging is conveniently studied in terms of the failure rate function of distribution function with support on $[0, \infty)$. In the simplest case where no aging is present, we obtain a constant failure rate which correspond to the exponeitial distribution and it is the only distribution having constant failure rate. The exponential distribution is in several senses the most fundamental distribution in reliability theory. Epstein (1958) shows that, the exponential distribution plays an important role in life testing experiments. The property which makes exponential distribution important in reliability theory and applications, is that the remaining life of a used component is independent of its initial age ( the " memoryless" property).

There have been a few parametric models extensively examined for applications in reliability, these include the exponential distribution of Epsteinsobel (1953), the weibull distribution given by Proschan, F (1963) and fatigue model of Birnbaum-Sauders (1969). The one most widely used for electronic components has been the exponential model. For a class of failures which are exponential that is have a constant failure rate the theory and methods for estimation are now well known and are appropriate when life time has a constant failure rate.

What is undertaken in this dissertation is an exposition of the theory and method of parametric estimation in the increasingly important situation namely when failure rate decreases with age; not that each component necesserily improves with age but each component in a group may have a different but constant failure rate in service and so for older component is becomes more likely that its failure rate is small. Proschan (1963) was the first to discover that the mixtures of exponeitially distributed random variables have a decreasing failure rate. Thus any two groups of components with constant, but different failure rates would if mixed and sampled at random, exihibits a decreasing failure rate.

In this dissertation we examine one such mixed exponential model namely the Generalized Pareto Distribution (GPD) given by Davis and Feldstein (1979). Naturally two parameter Pareto distribution given by Myhre (1983) is a particular case of the GPD. Lomax (1954) termed the two parameter Pareto distribution as a generalization of a Pareto distribution. After a reparameterization the three parameter GPD given by Davis and Feldstein (1979) was studied by Saunders (1983). His paper is also discussed in this dissertation.

## 0.2 CHAPTERWISE SUMMARY :

This dissertation contains three chapters. A brief account of which is presented below.

In Chapter I the probabilistic study of DFR systems is discussed. Section 1.1 contains Basic concepts of reliability, such as life time distribution, survival function, failure rate function, their properties. In Section 1.2 the concept of burn-in period is introduced. The optimum burn-in period for given values of estimates, costs and benefits are computed for three parameters GPD. Also concept of cessoring is discussed.

Chapter II deals with the generalized Pareto distribution and its properties. In section 2.1 we introduce the generalized Pareto distribution given by Davis and Feldstein (1979). Section 2.2 gives sectionwise summary. In Section 2.3 we give definitions of three parameter and two parameter generalized Pareto distribution. The mean and variance for both type of generalized Pareto distribution is computed. For three parameter GPD the exact solution of mean and variance is not obtained by solving proper integrals, so it becomes a problem of numerical integration. Thus mean and variance are obtained by using 5 point Gauss-Laguerre integration method. A computer program in Basic is written. Also this section contains characterization of GPD in terms of burn-in period. section 2.4 contain failure rate function and some of the properties of GPD through failure rate function. These properties are stated and proved in appropriate theorems. We also discuss the behaviour of failure rate function depending upon their parameters. Section 2.5 deals with simulation of observations from both type of GPD. Also we present some tables of random number from GPD type III and GPD type II for given values of their parameters respectively, which is prepared on computer by using a program written in BASIC.

Chapter III contains the estimation of the parameters of both types of GPD. In section 3.1 we give introduction of the chapter. In section 3.2 we discuss in detail the method to obtain maximum likelihood estimates, given by Myhre (1983) under progressive censored case. Also conditions for existence of m.l.e. is given. Estimate of reliability is obtained. But we observe that the likelihood obtained in Section 3.2.1 can not be solved analytically. In Section 3.2.4 we use an appropriate method that is Newton-Raphson iterative method to obtain m.l.e.

Further the estimators of $\alpha$ , $\beta$ and R(t) is obtained by using real data sets given in Chapter II. A computer prcgram in PASCAL is written to obtain it. Section 3.3 contains the method of obtaining maximum likelihood estimates for the parameters given by Saunder and Myhre (1983) for type III GPD by reparameterizing parameters as $\theta = \alpha\beta\gamma$ , $\lambda = \alpha$ and $\Phi = 1/\beta$ . These estimates are obtained under progressive censored case. Also estimate of realiability is given. In this dissertation we denote notation $\uparrow$ for non-decreasing and $\downarrow$ for non-increasing.