C H A P T E R : I

I N T R O D U C T I O N

The dictionary meaning of the word reliability is "trustworthiness", that is a thing, on which one can depend. We often see in advertisements "a reliable T.V.","a reliable bulb'" and so on.

Here the word reliable referes to that the T.V. or the bulb has enough large or a reasonable amount of life time.

In general the term system is used for a devise like a T.V., a bulb or anything of this kind.

A system is made up of various components and the performance of the system very much depends upon the performance of the components. For example T.V. will function "good" if the picture tube, integreted circuit and other revelent component of it are at "good" level.

We note that a certain set of components of a particular system themselves may form a systems, called a subsystems.

Many a times we use the word 'goodness'. Naturally a question arises goodness to what extent?.

To beginwith one can assume that goodness of a system as well as of components means that they are working (and badness means that they are in failed state). That is we distingwish only two levels one the functioning and the failed. This is called the binary approch. The theory of binary system of binary components is well developped (Ref. Barlow and Proschan (1975)).

In section 1 of chapter 1 we have taken a quick review of binary system designed from binary components. Some properties of these systems are studied under the following models.

1. Deterministic
2. Stochastic (Probabilistic)
3. Dynamic

We have assumed that the system is either in failed state or in working state. However in many real-life situations the system and their components are capable of working at various levels. The levels M(>1) and 0 of performance indicate perfect functioning and complete failure state respectively. Such systems are refered to as multistate systems.

It is but natural to assume that the system level is nonincreasing function of component levels. A multistate system with this property alongwith the property that its performance is bounded below by performance the series structure function and above by performance of the parallel structure function is called a multistate monotone system (mms).

In the section 2 of chapter 1 we consider a system of n components. For every component and also for the system we distingwish among M+1 states representing successive level of performance ranging from M (perfect functioning level) to 0 (a failed state). Various definitions of multistate systems are presented (Barlow and Wu (1978), El-Neweihi, Proschan and Sethuraman (EPS) (1978), Griffith (1980), Natwig (1982)) and the interrelation among these definitions have been studied wherever possible. Some examples to describe the difference / relations among these definitions, have also been given.

Definiton of Barlow and Wu (1978) essentially extends the domain and range of usual binary coherent system {0,1} to {0,1,...,M}

The coherency criteria given by EPS (1978) demands for every fixed level of a component, there is a situation in which the level of the component determines the level of the system. This is strongest criteria of relevency. But the class of structure functions given by EPS is a superclass of Barlow and Wu's class, which is proved in theorem 1.2.1 and an example 1.2.7 is given to show that the converse is not true.

Later in 1980 Griffith gave a formal definition of mms.

Definition 0.1 : Let Ψ be a mapping from $\{0,1,\ldots,M\}^n$ to $\{0,\ldots,M\}$ where M and n are positive integers then Ψ is said to be mms if (i) $\Psi(x)$ is nondecreasing and
(ii) Performance of Ψ is bounded below by the performance of the series structure function and above by the performance of the parallel structure function.

The coherency criteria given by Griffith (1980) is, for any component and state j there is a situation in which level of structure function is lesser when the component is at level (j-1) than the one when the component is at level j, $j \geq 1$.

This coherency criteria is weaker than that of given by EPS (1978). Hence it forms a superclass of EPS class. Example 1.2.8 describes a system belonging to Griffith class but not in EPS class.

Griffith also gave another coherency criteria, called weak coherency, which is weaker than his first criteria of coherency. For any component there is a situation in which the level of the structure function when the component is at level 0 is lesser than the one when the component is at level M (Perfect functioning level). Example 1.2.9 shows that weak coherancy does not imply coherency.

Natwig (1982) gave two definitions of multistate coherent systems (mcs) namely mcs of type 1 and mcs of type 2.

A structure function Ψ is said to be an mcs of type 1 if for any component and state $j \geq 1$ there is a situation in which level of the system is greater than or equal to j when level of the component is at j and the level of the system is lesser than or equal to j-1 when the level of the component is j-1. Remark (1.2.2) says that mcs suggested by EPS is a particular case of mcs of type 1 however the converse is not true, and is shown in example 1.2.10.

A structure function Ψ is said to be mcs of type 2 if every level $j \geq 1$, there exits a binary coherent structure function Ψ_j such that $\Psi(x) \geq j \Leftrightarrow \Psi_j(I_j(x)) = 1$, $\forall$ j and $\forall$ x, where

$$I_j(x_i) = \begin{cases} 1 & \text{if } x_i \geq j \\ 0 & \text{otherewise} \end{cases}$$

and $I_j(x) = (I_j(x_1),\ldots,I_j(x_n))$
Theorem (1.2.7) states that mcs of type 2 is also an mcs of type 1, however the converse is not true, and is illustrated in example 1.2.12. It is shown that neither EPS class contains mcs of type 1 class nor converse.

Fig 1.1.2 illustrates the relationship among various definitions of mcs.

Ebrahimi (1984) has weakned the relevency criteria given by EPS (1978) and Natwig (1982) (mcs of type 1), which is instead of assuming the relevency of the component at each level. He assumes that there is a level of each component which is relevent.

Chapter 2 deals with properties of mms. In section 1 of chapter 2 we assume that the performance of the system depends deterministically on the performance of each components. That is given x, the state vector of components, we can determine $\varphi(x)$, the state of the system.

Lemma 2.1.1 essentially states that a structure function of n components can be expressed as structure of (n-1) components. In theorem 2.1.1 it is proved that dual of an mms is also an mms. The dual possesses the same type of coherency as φ, the original one is shown in theorem 2.1.3.

In section 2 of chapter 2 we study the relationship between probabilistic performance of the system and that of its components.

First a model for multistate system cosidered by Ross (1978) is presented. Proposition 2.2.1. and Theorem 2.2.2. indicate that the reliability (expected level of performance) of the mms increases as reliabilities of the components increase.

Theorem 2.2.5 gives bounds on system performance function and system performance distribution.

Also upper critical connection vectors are used to obtain bounds on the system performance distribution and system performance function.

Griffith (1980) used concept of utility (assining weighes to various level of the system) function to describe system perfomance and generalizes Birnbaum's reliability importance to the multistate system.

Importance of an i^{th} component interms of the improvement vector is obtained in proposition 2.2.4. Example 2.1.1. is given to illustrate the importance of component 1 over component 2 under various types of utility function.

Subsequently stochastic performance of an mms of type 1 and type 2 alongwith bounds on system reliability function are studied.

In section 3 of chapter 2 dynamic models for multistate systems are studied. That is we consider models in which state of both, system as well as components vary with time.

At the time of begining system as well as components are in state M (Perfect level) and as time passes, the state of each component and also of system deteriorates to a lower level and altimately level 0 (failed state) is reached.

IFRA (NBU) stochastic processes are define in definition 2.3.1 (2.3.2) and closure theorem of IFRA (NBU), that is if component processes are IFRA (NBU) then system process is also IFRA (NBU) is shown in theorem 2.3.1.(2.3.2).

In section 1 of chapter 3 for the sake of completeness idea of cotinuous strucure function is introduced in brief. There are so many real-life examples in which system deteriorates cotinuously for example a power generator.

Note that in a sense a continuous structure function can also be visualised as limit of an mms structure function. Finally a case studied by Natwig et.ai.(1986) is studied.

In section 2 of chapter 3 by using optimization technique given by Puri and Singh (1986) We independently develop results related to optimal repair/replacement policy for mms when N(t), state of the sytem is a homogenious death process.

Optimal time of repair of such systems have been found which minimizes the expected cost perunit time.

Further optimal states of repair and optimal number of repairs have been found, when the repairs are not perfect, which minimizes the expected cost per unit time.

Further extension/improvements of these results is in progress.