CHAPTER II

A COMPOSITE SAMPLING PROCEDURE FOR BULK MATERIAL

2.1 Introduction :-

In this chapter we are going to study the Duncan's (1961) model for the analysis of composite sampling procedure for bulk material such as fertilizer, coal, wool.

If population is very large (bulk) and if one wants to estimate mean and variance of some charactristics of a lot for fixed cost and time (minimum), Then this procedure is better than conventional sampling methods. In this procedure, Populations which are in segmented form or which can be made into segmented form are considered. The population such as bales of wool, bags of fertilizer are in segmented form while the population such as coal, mangoes can be made in segmented form.

2.2. Composite Sampling Procedure :-

Duncan considered composite sampling procedure with parameters $(N, n, 1, r, 1, 1)$ as described in chapter I. FOr completness the procedure is described below again.

Consider a population consisting of N segment and select "n" segments by any one of the conventional sampling methods. Then from each selected segment, take one increment. Combine these increments to form r independent composite samples. Reduce each composite sample to one laboratory sample and take one test on each laboratory sample.

This composite sampling procedure is also known as the bulk sampling. The main advantage of this procedure is that it is economical and requires less time.

2.3 Examples of Composite Sampling Procedure :-

2.3.1 Fertilizers : -

Fertilizers are always in bags. Hence first select a few bags from population. Then from each selected bags, draw an increment (by using a slotted tube with a solid and unbiased cone on tip at one end) and mix these increments to form composite samples. Reduce each composite sample to size suitable for laboratory sample, and select one test unit from each laboratory sample for analysis.

A similar procedure can be used to obtain a test unit from bales of wool, drums of oil, bales of cotton etc.

2.3.2 Coal :-

Coal is not found in segmented form. So select increments at the time of loading, unloading or while crushing the coal. Suppose that coal was crushed and moved on a conver belt, then select increments at constant time interval with random start. Mix these selected increments to form sample to size suitable for the laboratory sample and take one test unit from each laboratory sample for analysis.

A similar procedure can be used to obtained a test unit from water, sugar, etc.

2.4 Formation of theoretical model : -

If an estimate of the mean of a lot is to be meanigful, its sampling variance should also be compared. The main problem is to find the most efficient procedure that will have minimum variance for minimum cost and time. This problem an appropriate model to the stocastic properties of sampling device and method of reduction employed , and their manner of use as well as the systematic and stochastic character of bulk material.

We consider two example. One is sampling of fertilizers and the other is sampling of coal.

2.4.1 Sampling of Fetilizers : -

Let us suppose that we have to estimate the percentage of nitrogen in a fertilizer which is packed in to bags. Thus population of fertilizer is segmented into "N" bags. Select "n" bags from "N" bags and take small quantity of fertilizer from each selected bags. Mix these incremets to form "r" composite samples. Reduce it to a suitable size for laboratory sample and take a test unit from each laboratory sample and take measurement on it.

The mean of test measurement gives an estimate of percentage of nitrogen in the lot.

To form a mathematical model, make following assumptions.

i) There are N bags in the lot and "n" bags are selected by sampling method.

ii) The fertilizers in bags are so mixed that any increments taken by unbiased sampling tube gives random sample.

iii) The variance of super-subpopulation of incremetnts is the same for each bag.

iv) The analytical errors are randomly distributes with mean zero and variance 6^2_e.

v) The sampling tube is unbiased; and

vi) The process of mixing and reduction gives random subsample for analysis.

Noation :

N = Number of bags in the lot.

n = Number of bags selected from the lot.

$\bar{Y}$ = Mean charactristic of selected bags.

$\bar{Y}_{ci}$ = Mean of the ith composite sample.

$\bar{Y}_{li}$ = Mean of the ith laboratory sample.

Y_i = Measurment on i^{th} test unit $i = 1,2,3,....r$

ei = Analytical error associated with yi

By using these notations and assumptions, a single observatiopn on a test unit may be written as

$$Yi = \bar{Y} + C + di + ri + ti + ei \qquad i = 1,2,..r \qquad [2.4.1]$$

Where,

C- deviation of mean of n increments from mean of lot.
di- deviation of mean of ith composite from Y + C
ri- deviation of mean of ith laboratory sample from Y+C+di
ti- deviation of ith test unit from Y+C+di+ri { i=1,2,..r}

Here we assume that C, di, ri and ti are independent random variables. Also we assume that the composite samples are equal size so that

$$\sum_{i=1}^{r} di = \emptyset$$

that is

$$d_1 + d_2 + \ldots + d_r = \bar{Y}c_1 - \bar{Y} - C + \bar{Y}c_2 - \bar{Y} - c \ldots + \bar{Y}c_r - \bar{Y} - C$$

$$= \bar{Y}c_1 - \bar{Y} - (\bar{Y} - \bar{Y}c_1) + \bar{Y}c_2 - \bar{Y} - (\bar{Y} - \bar{Y}c_2) \ldots + \bar{Y}c_r - \bar{Y} - (\bar{Y} - \bar{Y}c_r)$$

$$= 2\,[\,\bar{Y}c_1 + \bar{Y}c_2 + \ldots + \bar{Y}c_r] - 2\,r\bar{Y}$$

$$= 2\,[\,\bar{Y}c_1 + \bar{Y}\,] - 2r[\,\bar{Y}c_1 + \bar{Y}c_2 + \ldots \bar{Y}c_r]/r$$

$$Edi = \emptyset \qquad [2.4.2]$$

with this assumption mean of r measurment will be

$$\bar{Y} = \bar{Y} + c + [\,\Sigma ri + \Sigma ti + \Sigma ei\,]/r \qquad [2.4.3]$$

and its variance is given by

$$\mathrm{Var}(\bar{Y}) = \mathrm{Var}[\bar{Y} + c + 1/r\ [\Sigma ri + \Sigma ti + \Sigma ei]]$$

$$= \mathrm{var}(\bar{Y}) + \mathrm{var}(c) + [\Sigma \mathrm{var}(ri) + \Sigma \mathrm{var}(ti) + \Sigma \mathrm{var}(ei)]/r^2$$

$$= \frac{N-n}{Nn} S^2\beta + \frac{6^2 I}{n} + \frac{1}{r^2}[r\ \mathrm{var}(ri) + r6^2 t + r6^2 e]$$

$$= \frac{N-n}{Nn} S^2\beta + \frac{6^2 I}{n} + \frac{1}{r}\mathrm{var}(ri) + (6^2 t + 6^2 e)/r$$

[2.4.4]

Now if we select each increment of size "a" then the composite sample is of size "na" and if we reduce it to a laboratory sample of size "L" then proporation sample is

$$k = L/an$$

so that reduction variance is

$$6^2 r = \frac{an - L}{an.L} 6^2$$

$$= \frac{1 - k}{k} \frac{6^2}{an}$$

let us suppose that r composite samples of equal size are to be formed. Reduce each composite samnple to a laboratory sample of size 'L'. Then proporation of each r compositte samples to form 'r' laboratory samples is L/(an/r) which is Lr/an = rk.
Hence reduction variance becomes

$$\frac{(an/r) - L}{(an/r)\ L} 6^2$$

$$= \frac{1 - rL/an}{L} 6^2$$

$$= \frac{1 - rL/an}{L/an} \frac{6^2}{an}$$

$$= \frac{1 - rk}{k} \frac{6^2}{an}$$

$$= \frac{1-rk}{k} \frac{1-k}{1-k} \frac{6^2}{an}$$

$$= \frac{1 - rk}{1 - k} \frac{1-k}{an} 6^2$$

$$= \frac{1 - rk}{1 - k} 6^2 r \qquad [2.4.5]$$

Hence equation (2.4.4) becomes

$$\mathrm{var}(\overline{Y}) = \frac{N-n}{Nn} S^2B + \frac{6^2I}{n} + \frac{1-rk}{r(1-k)} 6^2r + \frac{6^2t+6^2e}{r}$$

[2.4.6]

Where

S^2B = variance between bags.

6^2I = variance of increments obtainable from bags of fertilizer with the given sampling tube.

6^2r = Reduction variance (the variance of one laboratory sample obtained from one composite sample)

6^2t = measurment variance

6^2e = error variance.

2.5 Comparison of one composite sample procedure with 'r' composite sample procedure

If we form only one composite sample then the variance of sample mean becomes

$$var(\bar{y}) = \frac{N-n}{Nn} S^2\beta + \frac{6^2I}{n} + 6^2r + \frac{6^2t+6^2e}{2}$$

[2.5.1]

For comparison , let us consider some fixed value of the constants as,

$S\beta = 0.20$, $6I = 0.10$, $(6^2t + 6^2e)^{1/2} = 0.07$
$6r = 0.12$, $N = 200$, $n = 10$

a) For one composite sample

$$var(\bar{Y}) = \frac{200-10}{200x100} (0.20)^2 + \frac{(0.1)^2}{10} + (.12)^2 + \frac{(.07)^2}{2}$$

$$= 0.0038 + 0.001 + 0.0144 + 0.00245$$

$$= 0.02165$$

b) For two composite sample procedure r = 2 and k = Ø.25

$$var(\bar{Y}) = \frac{2ØØ-1Ø}{2ØØ}\frac{(Ø.2Ø)^2}{1Ø} + \frac{(Ø.1Ø)^2}{1Ø}(1-Ø.5) + \frac{}{2(1-Ø.25)}(Ø.12)^2 + (.Ø7)^2/2$$

$$= Ø.ØØ38 + Ø.ØØ1 + Ø.ØØ48 + Ø.ØØ245$$

$$= Ø.Ø12Ø5$$

c) For three composite sample procedure (r = 3)

$$var(\bar{y}) = \frac{2ØØ-1Ø}{2ØØ}\frac{(.2)^2}{1Ø} + \frac{(Ø.1Ø)^2}{1Ø}\frac{(1-Ø.75)}{3(1-.25)}(.12)^2 + \frac{(.Ø7)^2}{3}$$

$$= Ø.ØØ38 + Ø.ØØ1 + Ø.ØØ16 + Ø.ØØ16$$

$$= Ø.ØØ8Ø33$$

and so on.

Here we see that by using two composite sample procedure, We can reduce 44% variance and by using three composite sample procedure, we can reduce 62% variance and so on. That is as number of composite samples increases the variance reduces.

2.6 <u>An overall optimum procedure</u> : -

Let us assume that inspection authorities place an acceptance limit on a mean estimate equal to nomial value minus Ø6y so that acceptance limit will vary directly with 6y.

Let us assume that gain(G) by using sampling procedure is given by

$$G = a - b6\bar{y} \qquad [2.6.1]$$

Where a and b are constants.

Let us fix the size of laboratory sample is "L" and size of increment is "a". If "n" increments are selected and p equal sized composite samples are formed then

$k = L/na$

and if $a/L = f$ then $k = 1/(fn)$

To obtain net again by using sampling method, let us assume that the cost of inspection at various stages is as follows.

C_1 = cost of taking a increment

C_2 = cost of blending increment to form a composite sample.

C_3 = cost of taking laboratory sample

C_4 = cost of grinding and preparing laboratory sample.

C_5 = cost of taking and measuring test unit

with these costs the total cost of composite sampling procedure is

$$nc_1 + p(c_2 + c_3 + c_4 + c_5)$$

$$= nc_1 + pc_7 \qquad [2.6.2]$$

Hence net gain by using composite sampling procedure is

$$g = G - nc_1 - pc_7 \qquad [2.6.3]$$

$$= a - b6y - nc_1 - pc_7$$

$$= a - b[((N-n)/Nn)S^2\beta + (6^2I)/n +$$

$$((1-(p/fn))/(p(1-(1/fn)))6^2r + (6^2t + 6^2e)/p]^{1/2} -$$

$$nc_1 - pc_7 \qquad [2.6.4]$$

Here we have to find the best composite sampling procedure which maximizes net gain by proper selection on n & p, Which can be done by solving two equations:-

$$\frac{dg}{dn} = 0 \quad \text{and} \quad \frac{dg}{dp} = 0$$

Now,

$$\frac{dg}{dn} = 0 \text{ gives } \frac{b}{26y}\left[\frac{S^2\beta}{n^2} + \frac{6^2 I}{n^2} + \frac{f(p-1)}{p(fn-1)^2}6^2 r\right] - c_1 = 0$$

[2.6.5]

and

$$\frac{dg}{dp} = 0 \text{ gives } \frac{b}{26y}\left[\frac{fn}{fn-1}\frac{6^2 r}{p^2} + \frac{6^2 t + 6^2 e}{p^2}\right] - c_7 = 0$$

[2.6.6]

It is difficult to get analytical solution for above equations, so let us solve above equations for perticular values.

$N = 200$, $f = 1$, $b = 1000$, $a = 2000$ $c_1 = 1$, $c_7 = 7$

$S\beta = 0.2$ $6I = 0.1$ $6r = 0.12$ $(6^2 t + 6^2 e)^{1/2} = 0.07$

let

$$A = \frac{S^2}{n^2} + \frac{6^2 I}{n^2} - \frac{f(p-1)}{p(fn-1)^2} 6^2 r$$

$$= \frac{0.04}{n^2} + \frac{0.01}{n^2} - \frac{p-1}{p(n-1)^2}(0.0144)$$

$$= \frac{0.05}{n^2} - \frac{0.0144(p-1)}{p(n-1)^2}$$

$$= \frac{1}{pn^2(n-1)^2} \quad [\ p(n-1)^2\ .05 - 0.0144(p-1)n^2]$$

$$= \frac{1}{pn^2(n-1)^2} \quad [0.0144n^2 + p(0.0356n^2 - 0.1n + 0.05]$$

[2.6.7]

and let

$$B = \frac{fn}{fn-1}\ \frac{6^2r}{p^2} + \frac{6^2t+6^2e}{p^2}$$

$$= \frac{n}{(n-1)}\ \frac{0.0144}{p^2} + \frac{0.0049}{p^2}$$

$$= \frac{1}{(n-1)p^2}\ [\ 0.0144n - 0.0049\ (n-1)]$$

$$= \frac{1}{(n-1)p^2}\ [\ 0.0193n - 0.0049]$$

[2.6.8]

Using these values of A and B equations (2.6.5) and (2.6.8) becomes

$$\frac{b}{26y} A - c_1 = 0$$

that is

$$\frac{b}{26y} A = c_1 \qquad [2.6.9]$$

and

$$\frac{b}{26y} B - c_7 = 0$$

that is

$$\frac{b}{26y} B = c_7 \qquad [2.6.10]$$

solving (2.6.9) and (2.6.10) we get

$$Bc_1 = Ac_7$$

and using values of A & B from equations (2.6.7), (2.6.8) we get

$$\frac{1}{(n-1)p^2}[0.0193n+0.0049] = \frac{7}{n^2(n-1)^2p}[.0049n^2+p(.0356n^2-.1n+.05)]$$

$$(n-1)n^2[.0193n+.0049] = 7[p^2(.0356n^2-.1n+.05)+.0049n^2p]$$

$$p^2[.0356n^2 -.1n+.05] + p(.0049n^2)-$$

$$n^2(n-1)(0.0193n + 0.0049)/7 = 0$$

$$p^2x + py - z = 0$$

$$p = \{-y \pm SQR[y^2+4xz]\}/2x$$

for different values of 'n' , the values of p and corrousponding values of net gain are given in the following table.

n	p	net gain
4	1.33	1811.23
5	1.59	1848.14
6	1.85	1852.15
7	2.1285	1854.93
8	2.4015	1856.87
9	2.6771	1867.09
10	2.9515	1868.91
11	3.2276	1871.62
12	3.5041	1871.71
13	3.7895	1872.17
14	4.058	1872.22
15	4.3352	1871.43
16	4.6126	1871.31
17	4.8901	1871.10
18	5.1677	1870.10
19	5.4453	1870.82
20	5.7231	1868.10
30	8.5028	1850.69
40	11.28	1833.31

Here we see that optimum values of n and p are n = 15, p = 4 for which the maximum gain g is 1872.22

2.7 Another optimum procedure : -

Let us assume that the number of composite samples should be fixed at two and find optimum value of n and k which maxmimizes net gain 'g'. Since k is not fixed , c_4 is ~~the~~ function of kn. For simplicity, let it be

$c_4 = co_4 + dkn/2$ where d is constant

Then the net gain 'g' becomes

$$g = a-b\left[\frac{N-n}{Nn} S^2\beta + \frac{6^2 I}{n} + \frac{1-2k}{2(1-k)} 6^2 r + \frac{6^2+6^2 e}{2}\right]^{1/2}$$

$$- nc_1 - 2(c_2 + c_3 + co_4 + dkn/2 + c_5) \qquad [2.7.1]$$

The optimum value of n and k can be obtained by solving equations

$dg/dn = \emptyset$ and $dg/dk = \emptyset$

$dg/dn = \emptyset$ gives $(b/26\bar{y})[(S\beta^2/n^2)+6I^2/n^2] - c_1 - dk = \emptyset$ [2.7.2]

and

$dg/dk = \emptyset$ gives $(b/26\bar{y})[1/2(1-k)^2 6^2 r] - dn = \emptyset$ [2.7.3]

from equations (2.7.2) and (2.7.3) we get

$(b/26\bar{y})[(S^2\beta/n^2) + 6^2 I/n^2] = c_1 + dk$

and

$(b/26\bar{y})[1/(2(1-k)^2)6^2 r] = dn$

solving two equations

$$\frac{[(S^2\beta/n^2)+6^2 I/n^2]}{(1/2(1-k)^2)6^2 r} = \frac{c_1 + dk}{dn}$$

Therefore

$$n = \frac{2d(S^2\beta + 6^2 I)(1-k)^2}{(c_1 + dk)6^2 r} \qquad [2.7.3]$$

for above values

$\delta B = 0.2 \quad \delta I = 0.1 \quad \delta r = 0.12 \quad d = 2 \quad c_1 = 1$

equation (2.7.3) becomes

$$n = \frac{13.588\ (1-k)^2}{1 + 2k}$$

for different values of K the corrousponding values on n and net gain (g) are given in the following table, where a = 2000, b = 1000

k	n	net gain(g)
0.00001	13.58	1861.67
0.00001	13.53	1861.53
0.0001	13.06	1861.50
0.002	12.54	1861.33
0.003	12.06	1861.22
0.004	11.59	1861.12
0.005	10.59	1860.70
0.006	10.31	1860.50
0.007	9.915	1860.50
0.008	9.53	1860.25
0.009	7.17	1859.80
0.010	6.21	1854.97
0.020	4.16	1846.97
0.030	2.71	1838.42
0.040	1.69	1820.77

Hence we see that for any samll proporation the number of increment is at all the most 14 to get maximum net gain.