APPENDIX A

PROOF OF LEMMA 2.4.2

Since the random variable U follows non-central chi-square distribution with n d.f. and non-centrality parameter λ, it is possible to express probability density function (pdf) of U as a weighted sum of pdf's of central chi-square variates with n + 2j d.f., j = 0, 1, 2, ... , with weights

$$A_j(\lambda) = (\lambda/2)^j \frac{e^{-\lambda/2}}{j!} .$$

That is, we have

$$f_U(x) = \sum_{j=0}^{\infty} A_j(\lambda) \, f_{\chi^2_{n+2j}}(x) \qquad (1)$$

(Refer Johnson and Kotz (1970b), pp 132, equation (3).)

Now the r^{th} moment about origin of a central chi-square random variable with v d.f. is given by

$$\mu'_r = 2^r \frac{\Gamma(U/2 + r)}{\Gamma(U/2)} \qquad (2)$$

(Refer Johnson and Kotz (1970a), pp 168, equation (10).)

Hence, from (1) and (2), we have

$$E(U^r) = 2^r \sum_{i=0}^{\infty} A_j(\lambda) \frac{\Gamma(n/2 + j + r)}{\Gamma(n/2 + j)} \blacksquare$$