CHAPTER 0

INTRODUCTION AND CHAPTERWISE SUMMARY

0.1 INTRODUCTION

Statistical techniques and their applications in quality improvement have had long history. Dr. walter A. Shewart of Bell Telephone Laboratories first time introduced the concept of control chart. The pioneering work of Dr. W. Edward Deming is one of the main reasons for Japanese manufacturing organizations to have broad capabilities. The philosophy of Dr. Deming is an important framework for improving quality and productivity with cost effectiveness.

Dr. Juran, who worked with Dr. W. A. Shewart is one of the founding father of statistical quality control.

Industrial organisations are now using statistical techniques and getting benifit in terms of quality and money. The basic 'Statistical Process Control' (SPC) tools are:

1. Histogram
2. Check Sheet
3. Parato Chart
4. Cause and Effect Diagram
5. Defect Concentration diagram
6. Scatter Diagram
7. Control Chart

Among these seven the most sophesticated is the control chart. A brief account of these tools can be found, for example, in Montgomery (1996). These tools are essentially used to see whether the proces is in statistical control or not. Once it is found that the process is in statistical control, next job of statistician is the Process capability Analysis.

0.2 CONCEPT AND LITERATURE SURVEY

For analyzing process capability statistical techniques like histogram and control charts have been applied on a large scale by a variety of industries since the early 1980's. Although a standard definition has not yet emerged for the term "Process Capability Analysis", there is a widespread agreement that its objective is to determine how well the output from a process meets specification limits set by engineering requirements or by the consumer. There is also agreement-unfortunately not as widespread- that a process must be in statistical control before its capability can be assessed. In other words, a stable, predictable distribution for the output is a prerequisite for the capability analysis.

A science of process capability analysis began as a comparison of the distribution of process output with product tolerances. Manufacturers simply used frequency histograms, log plots or control charts to compare process data to the specification limits. Using what is known as process capability studies, data from a statistically controlled process was systematically gathered and plotted on these charts. The judgement of the capability was then based on the visual relationship between the process distribution and the specification range. For example, consider the data sets of the two processes having same specification limits as follows:

Process A:

50.980	50.217	52.870	48.691	48.004	49.378
48.587	51.297	49.324	48.964	52.124	49.041
51.754	50.222	50.660	50.516	49.890	47.699
49.544	49.814	49.871	51.141	50.632	50.986
50.713	49.526	48.765	50.079	49.398	50.479
50.548	47.718	51.682	50.901	49.967	51.163
51.524	49.412	50.162	50.312	48.723	48.839
51.388	49.626	50.064	50.120	49.068	47.557
51.028	51.260				

Process B:

46.346	49.000	51.616	50.822	54.218	49.828
54.732	53.354	49.167	50.748	50.413	51.955
48.653	48.730	52.319	53.882	49.332	52.075
49.846	48.397	51.886	50.960	47.269	49.975
48.868	51.038	51.254	49.873	49.556	51.891
47.125	48.668	48.958	47.550	51.887	51.497
50.678	51.016	51.318	55.925	50.284	51.064
47.841	48.092	46.336	52.732	50.938	52.833
48.633	49.390				

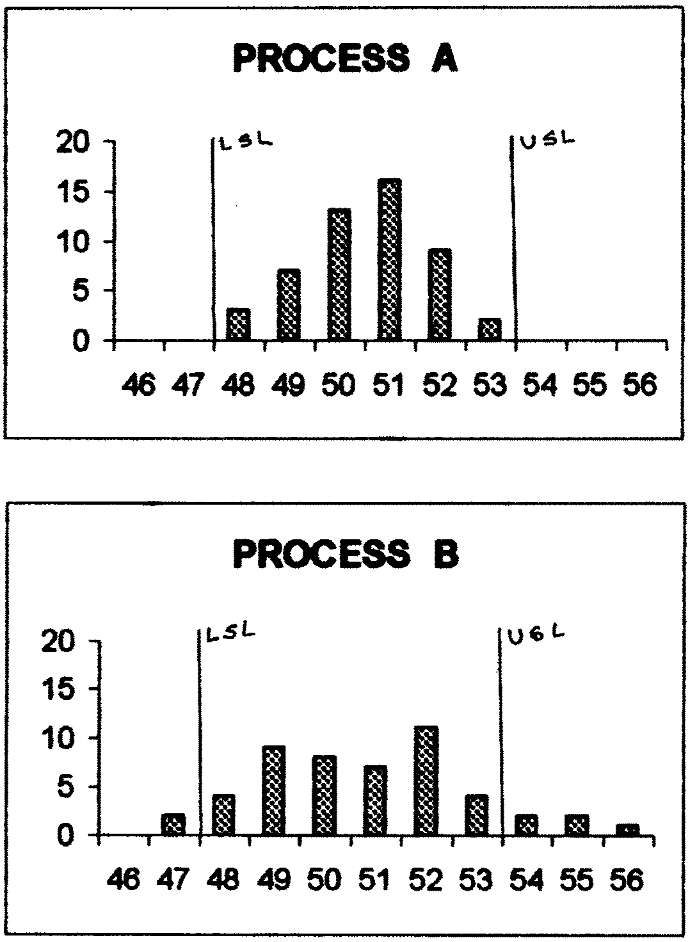

Figure 0.1

For both the processes, USL = 53 and LSL = 47.
The histograms of the two processes are as shown in Figure 0.1.

Observing the histograms it follows that process A is more capable than the process B. Note that the estimates of process standard deviations for process A and process B are 1.185 and 2.116 respectively. It follows that as the standard deviation increases the process becomes less capable. However the visual impression of the histogram does not give any quantification of the capability of the process. The managers within the organizations, clever at managing but not so skilled statistically, were quick to demand some type of index which would sum up in one number what the graph revealed. Thus the Process Capability Index (PCI) was born.

Process capability indices are used widely throughout the world to give a quick indication of process capability in a format that is easy to use and understand. Using process capability indices to express process capability has made the setting and communicating of quality goals much simpler, and their use is expected to continue to increase. The indices provide management with a single-number summary of what is happening on the production floor, which significantly simplies the supervision of production activities.

Kotz et al. (1993) defines process capability index as *the measure of the extent to which the output of a process satisfies a preassigned specification.* The first process capability index was introduced by Juran et al. (1974). Further it was modified to take an account of process shift. Kane (1986) provides a thorough discussion and lucid comparison of the various PCIs. In the literature these PCIs are referred to as the first generation indices. However, in early 1990's it has been proved that these indices are of limited use and are appropriate only with the measurements that are independent and reasonally normally distributed. Thus, a number of statistical questions are being asked.

1. What are appropriate interpretations for capability indices with or without normality?
2. Assuming normality, what are the distributional and sampling properties of the commonly used indices? How do these depend on sample size?
3. Assuming normality how can confidence bounds be obtained for capability indices?
4. Are there appropriate capability indices for non-normal process data?

The answers to above questions require statistical theory.

Since early 1990's many authors have tried to answer these questions. Before that Hsiang and Taguchi (1985) and later independently Chan et al. (1988) have introduced the second generation PCI. Further Pearn et al. (1992) proposed the third generation PCI. Chou et al. (1990), Bissell (1990), Boyles (1991), Kushler and Hurley (1992) and Subbaiah and Tamm (1993) among others consider the point and interval estimation of these indices in normal process enverionment. Franklin and Wasserman (1992) have developed the bootstrap confidence intervals for the PCIs. Chan et. al. (1990) discuss the asymptotic distributions of some commonly used indices. Clements (1989) has modified the first generation indices for non-normal processes. Further Munechika (1986), Pearn et al. (1992), Bai and Chou (1997) among others have proposed different approaches to assess the capability of non-normal process. Kotz and Johnson (1993) have summarized all these. Further Spiring (1997) tried to unify these indices into a single model, by giving C_{pw}. A very good survey on PCI has been given in Kotz and Lovelace (1998).

As corporate quality improvement efforts becomes standardized, and as evidence of capability becomes a requirement in supplier contracts, companies are reconsidering the value and reliability of single-number summeries for the behaviour of complex processes.

Some authors also have commented on the weakness of capability indices. Gunter (1989), for example, discusses limitations of C_{pk} with non-normal data and cautions that unless the process is in control and hence predictable, the use of C_{pk} "becomes a kind of mindless effort that managers confuse with real statistical process control efforts". Other writers have criticized standard capability indices as over-simplifications.

In spite of these misgivings, the popularity of capability indices has not diminished, largely because single-number summeries are irresistible to managers responsible for hunderds of processes running concurrently.

0.3 CHAPTERWISE SUMMERY

Chapter I introduces the PCIs C_p, C_{pk}, C_{pm} and C_{pmk}. Section 1.2 through Section 1.4, we discuss the developement of these PCIs, their interpretations (in terms of probability of nonconformance, whenever possible) and their weaknesses. Section 1.5 explains some relations among the wellknown indices. In Section 1.6 we discuss the unifying approach to the PCIs, due to Spiring (1997).

Chapter II is devoted to the point and interval estimates of the PCIs C_p, C_{pk}, C_{pm} and C_{pv}. For the index C_p exact confidence interval is available. For the PCIs C_{pk} and C_{pm} different approximate confidence intervals have been discussed. In Section 2.3 a simulation study is carried out to check the coverage of confidence interval due to Bissell (1990) for C_{pk}. Some numerical values of $E(\hat{C}_{pk})$ and $var(\hat{C}_{pk})$ are tabulated. Section 2.4 discusses two different point estimates and two confidence intervals for the PCI C_{pm}. Similarly in Section 2.5 a point estimate and a confidence interval for the unifying index C_{pv} are discussed. In Section 2.6 we discuss the nonparameteric bootstrap confidence intervals suggested by Franklin and Wasserman (1992) for the PCIs discussed in previous sections. A simulation study is carried out to check the coverage and the average width of these intervals. Also a comparison of these intervals is made with the confidence intervals for different PCIs discussed in previous sections.

Chapter III discusses the process capability analysis for non-normal process measurements. Section 3.1 explains how the PCIs discussed in the previous chapters are unsuitable for non-normal distributions. Section 3.2 discusses some wellknown tests for testing normality of process measurements. In Section

3.3 and Section 3.4 we discuss Clements' and Munechika's approachs to process capability estimation for non-normal process data. Section 3.5 compares the two approaches. Section 3.6 gives some further approaches.