APPENDIX C

C.1 : The alias structures for 2^{7-2} design d_1 *and* d_2 as follows.

Design $\mathbf{d_1}$:

I = ABCF = ABDEG = CDEFG	A = BCF = BDEG = ACDEFG
B = ACF = ADEG = BCDEFG	C = ABF = ABCDEG = DEFG
D = ABCDF = ABEG = CEFG	E = ABCEF = ABDG = CDFG
F = ABC = ABDEFG = CDEG	G = ABCFG = ABDE = CDEF
AB = CF = DEG = ABCDEFG	AC = BF = BCDEG = ADEFG
AF = BC = BDEFG = ACDEG	AE = BCEF = BDG = ACDFG
AD = BCDF = BEG = ACEFG	AG = BCFG = BDE = ACDEF
BD = ACDF = AEG = BCEFG	BE = ACEF = ADG = BCDFG
BG = ACFG = ADG = BCDFG	CD = ABDF = ABCEG = EFG
CE = ABEF = ADCDG = DFG	CG = ABFG = ABCDE = DEF
DE = ABCDEF = ABG = CFG	DF = ABCD = ABEFG = CEG
DG = ABCDFG = ABE = CEF	EF = ABCE = ABDFG = CDG
EG = ABCEFG = ABD = CDF	FG = ABCG = ABDEFG = CDE
ACD = BDF = BCEG = AEFG	ACE = BEF = BCDG = ADFG
ACG = BFG = BCDE = ADEF	ADF = BCD = BDFG = ACDG
AEF = BCE = BDFG = ACDG	AFG = BCG = BDEF = ACDE

Design $\mathbf{d_2}$:

I = ABCD = CEFG = ABDEFG	A = BCD = ACEFG = BDEFG
B = ACD = BCEFG = ADEFG	C = ABD = EFG = ABCDEFG
D = ABC = CDEFG = ABEFG	E = ABCDE = CFG = ABDFG
F = ABCDF = CEG = ABDEG	G = ABCDG = CEF = ABDEF
AB = CD = ABCEFG = DEFG	AC = BD = AEFG = BCDEFG
AD = BC = ACDEFG = BEFG	CF = ABDF = EG = ABCDEG
CG = ABDG = EF = ABCDEF	CE = ABDE = FG = ABCDFG
AE = BCDE = ACEG = BDEF	AF = BCDF = ACEG = BDEG
AG = BCDG = ACEF = BDEF	BE = ACDE = BCFG = ADFG
BF = ACDF = BCEG = ADEG	BG = ACDG = BCEF = ADEF
DE = ABCE = CDFG = ABFG	DF = ABCF = CDEG = ABEG
DG = ABCG = CDEF = ABEF	ABE = CDE = ABCFG = DFG
ABF = CDF = ABCEG = DEG	ABG = CDG = ABCEF = DEF
ACE = BDE = AFG = BCDFG	ACF = BDF = AEG = BCDEG
ACG = BDG = AEF = BCDEF	ADE = BCE = ACDFG = BFG
ADF = BCF = ACDEG = BEG	ADG = BCG = ACDEF = BEF

□

LEMMA C.2 : When (n/u) is very small,

$$P(m,n) = 1 - \binom{u-m}{n} \Big/ \binom{u}{n} \approx \frac{mn}{u}$$

PROOF

Let us consider $m = 1$,

$$P(1,n) = 1 - (1 - \frac{n}{u})$$
$$P(1,n) = \frac{n}{u}$$

For $m = 2$

$$P(2,n) = 1 - (1 - \frac{n}{u})(1 - \frac{n}{u-1})$$

$$= 1 - \left\{1 - \frac{n}{u} - \frac{n}{u-1} + \frac{n^2}{u(u-1)}\right\}$$

$$= \left\{\frac{n}{u} + \frac{n}{u-1} - \frac{n^2}{u(u-1)}\right\}$$

Since (n/u) is not too large $\Rightarrow u$ is large

$\Rightarrow \frac{n}{u} \cong \frac{n}{u-1}$ and $\frac{n^2}{u(u-1)} \cong 0$. Therefore,

$$P(2,n) = \frac{2n}{u}$$

For $m = r$,

$$P(r,n) = 1 - (1 - \frac{n}{u})(1 - \frac{n}{u-1}). \; . \; .(1 - \frac{n}{u-r+1})$$

$$= 1 - \left\{1 - \frac{n}{u} - \frac{n}{u-1}. \; . \; .(1 - \frac{n}{u-r+1}) + \frac{n^2}{u(u-1)} + \frac{n^2}{u(u-1)(u-2)}. \; . \; . \; .\right\}$$

Neglecting higher order terms, we have

$$= \left\{\frac{n}{u} + \frac{n}{u-1} + . \; . \; . \; . \frac{n}{u-r+1}\right\}$$

$$\approx \frac{rn}{u}$$

Hence the proof. □