CHAPTER : IV
COMPUTATIONAL STUDIES

4.1 INTRODUCTION

In section 4.2 we describe the Stein's (1945) two-stage procedure for $N(\theta, \sigma^2)$ model. In section 4.3 we state the general two-stage procedure of Rattihalli and Shirke (unpublished) for $U(0, \theta)$ model and its performance with Cooke's (1973) two-stage procedure have studied in section 4.5. In section 4.4 we study the performance of Stein's (1945) and its modified form by reducing the second sample size by 1.

4.2 STEIN'S AND COOKE'S TWO-STAGE PROCEDURES

(1) Stein's two-stage procedure

Dantzig (1940) proved that there does not exist a fixed width confidence interval for the mean θ when the variance σ^2 is unknown in case of $N(\theta, \sigma^2)$ distribution. Later Stein (1945) obtained confidence interval for θ of width 2d by considering a two-stage sampling procedure and is described below :

Stage I : Take a sample of fixed size m (≥ 2) and compute

$$\bar{X}_m = \sum_{i=1}^{m} X_i/m \text{ and } S_m^2 = \sum_{i=1}^{m} (X_i - \bar{X}_m)^2/(m - 1).$$

Stage II : Take an additional sample of size N - m, where

$$N = \text{smallest integer} \geq S_m^2 \, t^2_{\alpha/2,m-1}/d^2. \tag{4.1}$$

If $N < m$ then $(1-\alpha)$ level confidence interval is obtained on the basis of first sample itself namely $\left(\bar{X}_m - S_m t_{\alpha/2,m-1}/\sqrt{m},\ \bar{X}_m + S_m t_{\alpha/2,m-1}/\sqrt{m}\right)$ whose width is $2S_m t_{\alpha/2,m-1}/\sqrt{m} \leq 2d$ else a $1 - \alpha$ level confidence interval for θ is given by,

$$\left(\bar{X}_N - S_m t_{\alpha/2,m-1}/\sqrt{N},\ \bar{X}_N + S_m t_{\alpha/2,m-1}/\sqrt{N}\right) \tag{4.2}$$

where $\bar{X}_N = \sum_{i=1}^{N} X_i/N$. The length of this confidence interval is $2S_m t_{\alpha/2,m-1}/\sqrt{N}$. If we choose N to be smallest integer satisfying (4.1) then the confidence interval has width $\leq$ 2d. For further details one may refer to Rohtagi (1986).

(II) Cooke's two-srage procedure

The Cooke's (1973) procedure is described in section 3.3.

4.3 A GENERAL TWO-STAGE PROCEDURE

We know that for various parametric models, well known two-stage sequential procedures for estimation of parameter of interest have been developed. A general two-stage sequential procedure is proposed by Rattihali and Shirke (unpublished).

Here we describe the proposed procedure for $U(0, \theta)$ model.

Let $Y_1, Y_2, \ldots, Y_m$ (indicating first sample) and $X_1, X_2, \ldots$ (which are used for second sample) be i.i.d. r.vs. from $U(0, \theta)$ and let $\hat{Y}_{(m)} = \max \{Y_1, Y_2, \ldots, Y_m\}$. Then a $(1-\alpha_1)$-level confidence interval for θ is $(\hat{Y}_{(m)}, \alpha_1^{-1/m} \hat{Y}_{(m)})$. From Rattihalli and Shirke (unpublished), the second sample size and $(1-\alpha)$-level confidence interval for θ are given by,

$$N(\underset{\sim}{y}_m) = \begin{cases} 0 & \text{if } \hat{Y}_m \alpha_1^{-1/m} \le d \\ [\log\alpha_2/(\log(1-d/\alpha_1^{-1/m}\hat{Y}_m))] + 1 & \text{if } \hat{Y}_m \alpha_1^{-1/m} > d \end{cases} \tag{4.3}$$

and $$(\hat{X}_{N(\underset{\sim}{y}_m)}, \hat{X}_{N(\underset{\sim}{y}_m)} + d). \tag{4.4}$$

where $1 - \alpha = (1 - \alpha_1)(1 - \alpha_2)$ and $\hat{X}_{N(\underset{\sim}{y}_m)}$ is the maximum of the second sample.

4.4 PERFORMANCE STUDY OF STEIN'S AND IT'S MODIFIED VERSION

Here we are discussing the Stein's procedure and its modified version that is by decreasing the second sample size by one. The comparison is done in terms of the coverage and ASN function by simulating 1000 confidence intervals for $N(0, \sigma^2)$. For fixed σ^2, α and d, we consider the following cases :

CASE (I) For σ^2 = 2.5, α = 0.1 and d = 1

TABLE (4.1)

FIXED SAMPLE-SIZE = 6.724000

SIZE OF FIRST SAMPLE	STEIN'S COVERAGE (%)	MODIFIED COVERAGE (%)	STEIN'S E(N)	MODIFIED E(N)
4	0.970000	0.955000	24.782000	23.833000
5	0.994000	0.991000	7.867000	7.234000
6	1.000000	1.000000	6.226000	5.919000
7	1.000000	1.000000	7.011000	6.962000
8	1.000000	1.000000	8.000000	8.000000
9	1.000000	1.000000	9.000000	9.000000
10	1.000000	1.000000	10.000000	10.000000

CASE (II) For σ^2 = 10, α = 0.1 and d = 1.

TABLE (4.2)

FIXED SAMPLE-SIZE = 26.89600

SIZE OF FIRST SAMPLE	STEIN'S COVERAGE (%)	MODIFIED COVERAGE (%)	STEIN'S E(N)	MODIFIED E(N)
24	0.962000	0.960000	42.695999	41.730999
25	0.973000	0.974000	42.202000	41.244999
26	0.979000	0.976000	43.528999	42.584999
27	0.972000	0.972000	42.764000	41.824001
28	0.968000	0.969000	42.661999	41.743000
31	0.963000	0.964000	43.394001	42.527000

Comments :- The following are some comments based on the simulation study.

(1) Both Stein's and its modified procedure attains the desired level. Further modified Stein's procedure has less ASN than Stein's procedure.

(2) The attained level for both Stein's and modified procedure is increasing with the first-sample size $(m) \geq n_o(\sigma^2)$.

(3) For $m \geq k \geq n_o(\sigma^2)$, we observe that $E_k(N) - E_m(N) \leq m - k$.

A C- program to obtain tables (4.1) and (4.2) is enclosed in Appendix - III.

4.5 PERFORMANCE STUDY OF COOKE'S AND GENERAL PROCEDURE

Consider Cooke's and general two-stage procedure described in section 3.2 and section 4.3 respectively. For fixed θ, α, d and α_1, we observe the following cases :

CASE (I) For $\theta = 2.0$, $\alpha = 0.01$, $d = 1$ and $\alpha_1 = 0.005$

TABLE (4.3)

REQUIRED SECOND-SAMPLE COVERAGE = 0.994975

FIRST SAMPLE SIZE	COOKE'S COVERAGE	GEN. METHOD COVERAGE	COOKE'S E(N)	GEN. METHOD E(N)
5	0.992000	1.000000	12.262000	28.396000
6	0.995000	1.000000	11.950000	25.666000
7	0.995000	1.000000	11.859000	24.511000

CASE (II) :- For θ = 2.0, α = 0.05, α_1 = 0.02 and d =1.0

TABLE (4.4)

REQUIRED SECOND-SAMPLE COVERAGE = 0.969388

FIRST SIZE	COOKE'S COVERAGE	GEN. METHOD COVERAGE	COOKE'S E(N)	GEN. METHOD E(N)
5	0.986000	1.001000	7.83700	16.302000
6	0.999000	1.000000	8.432000	16.214000
7	0.998000	1.000000	8.380000	16.351000
8	0.999000	1.000000	9.000000	16.761000
9	0.999000	0.999000	10.000000	17.262000
10	1.000000	1.000000	11.000000	17.995000

CASE (III) :- For θ = 3.0, α = 0.01, α_1 = 0.005 and d = 1.0

TABLE (4.5)

REQUIRED SECOND-SAMPLE COVERAGE = 0.994975

FIRST-SAMPLE SIZE	COOKE'S COVERAGE	GEN. METHOD COVERAGE	COOKE'S E(N)	GEN.METHOD E(N)
5	0.991000	1.000000	17.438000	24.251000
6	0.985000	0.999000	16.693000	36.286000

CASE (IV) :- For θ = 2.0, α = 0.05, α_1 = 0.02 and d = 1.0

TABLE (4.6)

REQUIRED SECOND-SAMPLE COVERAGE = 0.969388

FIRST SAMPLE SIZE	COOKE'S COVERAGE	GEN. METHOD COVERAGE	COOKE'S E(N)	GEN.METHOD E(N)
11	1.001000	1.001000	12.001000	18.706000
12	1.001000	1.001000	13.001000	19.527000
13	1.001000	1.001000	14.001000	20.398000

CASE (V) :- For given θ = 2.0, d = 1.0, α = 0.05 and the first sample size (m) = 5, we have the possible values of α_2 for the general two-stage procedure.

TABLE (4.7)

First Sample Cov.	Second Sample Cov.	E(N)	
0.951000	0.998948	22.654000	
0.952000	0.997899	20.981000	
0.953000	0.996852	20.037000	
0.954000	0.995807	19.384000	
0.955000	0.994764	18.904000	
0.956000	0.993724	18.507000	
0.957000	0.992685	18.182000	
0.958000	0.991649	17.913000	
0.959000	0.990615	17.662000	
0.960000	0.989583	17.446000	
0.961000	0.988554	17.311000	
0.962000	0.987526	17.164000	
0.630000	0.986501	17.040000	
0.960000	0.985477	16.888000	
0.965000	0.984456	16.769000	
0.966000	0.983437	16.661000	
0.967000	0.982420	16.599000	
0.968000	0.981405	16.543000	
0.969000	0.980392	16.498000	
0.970000	0.979381	16.445000	
0.971000	0.978373	16.401000	
0.972000	0.977366	16.359000	
0.973000	0.976362	16.321000	
0.974000	0.975359	16.283000	
0.975000	0.974359	16.275000	* indicates smallest diff.
0.976000	0.973361	16.265000	**indicates smallest ASN
0.977000	0.972364	16.265000	**
0.978000	0.971370	16.272000	
0.979000	0.970378	16.288000	
0.980000	0.969388	16.312000	
0.981000	0.968400	16.350000	
0.982000	0.967413	16.404000	
0.983000	0.966429	16.457000	
0.984000	0.965447	16.508000	

0.985000	0.964467	16.581000
0.986000	0.963489	16.656000
0.987000	0.962513	16.748000
0.988000	0.961538	16.866000
0.989000	0.960566	17.023000
0.990000	0.959596	17.195000
0.991000	0.958628	17.404000

Comments :- The following are some comments based on the simulation study.

(1) Both general and Cooke's procedure attains the desired level.

(2) The ASN for Cooke's procedure is much less than general procedure.

(3) Lastly we studied the best possible $1-\alpha_2$ indicated by (**) in the table (4.7), which minimizes ASN in case of general two-stage procedure.

The C - programs to obtain above tables (4.3) to (4.7) are enclosed in Appedix-IV (A) and IV (B).

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