CHAPTER - I

HISTORY OF BOUNDARY LAYER THEORY

Introduction:

In this chapter in Section 1 we give outline of the boundary layer theory. Section 2 consists major developments in axially symmetrical boundary layer theory. In Section 3 we gave the major developments of steady and unsteady two dimensional boundary layer theory. Lastly in Section 4 we define some basic concepts which are used for our problems to be discussed in Chapter II.

1. Outline of the boundary layer theory :

In 1904 Ludwing Prandtl introduced the concept of a boundary layer and analysed the flow in boundary layer subsequently the boundary layer equations have been well investigated for many engineering problems, and the results play a very important role in the fluid dynamics of viscous fluids and also play an important role in the practical treatment of a fluid.

The boundary layer theory is the foundation of all modern developments in fluid mechanics, and aerodynamics which have been classified by the study of boundary layer flow and its effects on the general flow around the body such as in the study of aircraft response to atmospheric gust, in further phenomenon involving wing etc. Although more than half a century old the subject of the boundary layer is still receiving considerable interest and these are still a number of unsolved problems baffling the investigators, the concept of thin region of quick transition near the boundary surface has solved many intricate practical problems and has enabled deep probing into the non-linear differential equation.

The strating point of this great physical concept was the well known D'Alembert's paradox in the late 19th century. D'Alembert observed that when a solid body moved through a fluid the flow pattern based on the inviscial theory agreed with the experimental results almost everywhere in the flow field, but strangely enough the resistance experienced by the body was found to be zero. Prandtl made an attempt to resolve the dilemma and suggested that the resistance to the body was caused by the viscosity of the fluid and that the flow fields near and away from the body were different in character.

Many other new results were obtained by research workers within the ten years after his research work with the help of Prandtl's boundary layer concept. At that time viscous fluid theory was studied in the two and three dimensional cases by using steady and unsteady flow of an incompressible or compressible medium. Also they considered one or more components with or without energy addition under the influence of magnetic forces. During the first 50 years of the boundary layer theory the fundamental mathematical connection to the Navier-Stokes' differential equations. And also there was no existance, uniqueness and goodness of a solution which was obtained. At that time numerical approximation method was not developed so that no one could show perfect error which was involved in the solution.

Transformation of the boundary layer equation into special forms so that the computation of the numerical results will be simplified or will be easier for special devices. The boundary layer equations may be transformed into the generalised heat conduction equation by Von Mises transformation. L.L. Moore [31] transformed the boundary layer equations into an integral forms that is particularly suitable for a differential calculation.

The mathematical difficulties encounted in the study of axially symmetrical boundary layer are considerably smaller and hardly exceed those in the two-dimensional case.

Prandtl (1904) published a paper entitled "on the motion of fluid with very small viscosity", considered the problem of an incompressible fluid and Blasius [1] investigated the same problem in detail in 1908. Blasius [2] studied the boundary layer flow over a flat plate and obtained explicit solution of the Prandtl boundary layer equation.

Due to the application of the theory of parabolic differential inequalities to the Prandtl's boundary layer equation, all the problem of existence, uniqueness etc. had been solved by considering case of two dimensional steady flow of an incompressible medium.

2. Major developments in axially symmetrical boundary layer theory :

There are two different kinds of axially symmetrical boundary layer theory. One is for the flow in jet or in the wake behind a body of revolution where the axis of the revolution is in the fluid and the other is the boundary layer is the boundary layer over a body of revolution of a large radius in comparison with the thickness of boundary layer.

The mathematical difficulties encounted in the study of axially symmetrical boundary layer are considerably smaller and hardly exceed those in the two dimensional case. Axially symmetrical boundary layer occur e.g. in flows past axially symmetrical bodies the axially symmetrical jet.

The process of boundary layer formation about an axially symmetrical body accelerated impulsively was investigated by E. Boltz [4] in Goettingen thesis.

U.T. Boedewadt [3] studied the problem in which the fluid at large distance from the stationary wall rotates likes a rigid body with constant angular velocity.

H. Schlichting [4] obtained solution for the Laminar circular jet which analogous to the one for a two dimensional jet. The process of the formation of a boundary layer on a rotating disk was studied by K.H.Thiriot [49] in his thesis presented to the University of Goettingen. He considered the case of the disk accelerated impulsively in a fluid at rest to a uniform angular velocity as well as the case of a disk rotating with the fluid and suddenly arrested in its motion. S.D.Nigam [35] computed the growth of a boundary layer on a disk strated impulsively. Belonasov S.M. [5] obtained the axially symmetrical flow of a viscous incompressible flow. Burns J.C. [6] generalised axially symmetrical flow past a circular boundary layer.

3. <u>Steady and unsteady two-dimensional laminar boundary layer theory</u> :

The two dimensional boundary layer flow over a flat plate of compressible fluid studied by E. Pohlhausen [36] in 1921 for a thermally insulated plate for a small flow velocity and small temperature difference, with constant density and viscosity.

Blasius [2] studied the boundary layer growth set impulsively from rest into translation motion by using successive approximation. Goldstein and Rosenhead [21] extended Blasius solution and gave a better estimate of the time required for separation at the rear stagnation point for circular cylinder. Schlichting [45] obtained small amplitude of oscillation of the body in a fluid at rest.

Busemann [8] first studied boundary layer for an incompressible fluid. Busemann [9] and Wada [56] obtained the solution for flow on a flat plate by keeping Prandtl number (Pr) constant. Howarth [22] studied the compressible and incompressible boundary layer at zero pressure gradient. Illingworth [26] investigated the transformation of both normal and streamwise co-ordinates and obtained the relation between them at non-zero pressure gradient for incompressible flow. Tani [50] extended the solution for the compressible flow by taking prandtl number different from unity. Poots [37] studied Tanis [50] problem by taking heat transfer at the wall.

Krishnan [27] obtained the non-linear wave propagation in steady tormonic flow. Britov [10] construct the absence of two dimensional flow between concentric rotating cylinder. Cebeci [14] studied the unsteady laminar and turbulent boundary layer with fluctuations in external velocity. Goldstein [19] constructed a singular solution containing an arbitrary constant in the neighbourhood of separation. Stewartson [46] obtained the general solution involving an infinite number of arbitrary constants. Landau and Lifshitz [30] made a discussion on flow near separation by postulating that the normal component of velocity tends to infinity at the separation point.

Hartree [23] and Stewartson [46] obtained the series solution for a linearly retarded free stream and Tani [50] extended this series solution to the more general case.

Prandtl [38] and Blasius [2] introduced the form of similarity solution, for flow on a flat plate. Falkner and Skan [16] extended this form in the case of free-stream velocity proportional to the x^m, representing irrotational flow around a corner found by two plane boundaries meeting at an angle $\pi/(m+1)$. Goldstein [20] applied the boundary layer approximation for a flow in awake and Schlichting [47] used the boundary layer approximation for a flow in a jet. Van Dyke [55] obtained the series solution for a flow past a parabolic cylinder.

During ten years after the Prandtl's paper, there are seven papers on boundary layer which were published at Gottingen. All these papers were written on the basis of Prandtl's original paper. Zhukovsheil [59] assumed that the fluid velocity is zero at the wall and rapidly increases until it becomes equal to the theoretical velocity of irrotational motion. Then he found that the thickness of layer is inversely proportional to the theoretical velocity. Mises [32] introduced the stream function and shown that the boundary layer equation was reduced to a form analogous to the heat conduction. Burgers [11] reported an experimental observation of the velocity distribution across the boundary layer on a flat plate. Froude [17] pointed out that the frictional forces must have its counter part in the Loss of momentum of the fluid that has passed along the surface of the plate. Rankine [43] in his paper on the prediction of required engine power of proposed ships, considered the frictional resistance is due to the direct and indirect effects of adhersion between the skin of ship and the particles of water which glide over it. Rankine showed that the formation of boundary layer takes place at the adjacent to the ship surface. Tollmisen [51] investigated the growth of the boundary layer on a circular cylinder impulsively set in rotation from rest.

Prandtl [39] expressed his opinion about the interest in boundary layer theory spread outside. Prandtl [40] obtained the boundary layer solution for flow through a two dimensional channel with the help of stream function. Prandtl [41] explained the change in flow pattern around a sphere on passing through the critical Reynold number. Toffer [52] refined the numerical computation of Blasius Eiffel [15] and observed the transition of the flow in the boundary layer from Laminar to turbulent. Hiemens [24] carried out the boundary layer calculation of pressure distribution on circular cylinder.

Blasius [2] and Boltz [13] submitted two papers on boundary layer under Prandtl's guidance at Gottingen. Blasius [2] studied the flow along a flat plate placed parallel to the uniform stream. Prandtl [42] applied the boundary layer concept to the heat transfer problem. Schubauer [48] observed the flow past an elliptic cylinder Millikan [33] applied Karman and Millikan [28] method to the Schabauer's [48] elliptic cylinder and obtained a successful solution. Walz [57], Mangler [34], Timman [53] studied the Karman [29] method by assuming a more adequate form of the velocity profile. Wada [56] Hudimoto [25] Tani [54] studied approximate methods of integrating the momentum integral equation.

4. Basic concepts required for our problems to be discussed in Chapter II.

(I) Fluid :

Under the action of the forces all material exhibit deformation. If the deformation in the material increases continuously without limit under the action of shearing forces, however, small the material is called a fluid. This continuous deformation under the action of forces is manifested in the tendency of fluid to flow.

Fluids are usually classified as liquids or gases. A liquid has intermolecular forces which hold it together so that it possesses volume but no definite shape, when it is poured into a container will fill the container upto the volume of the liquid regardless of the shape of the container. Liquids have but slight compressibility. For most purpose it is however sufficient to regard liquid as incompressible fluid. A gas, on the other hand, consists of molecules in motion which colloid with each other tending to disperse it so that a gas has no set volume or shape. This intermolecular forces are externally small in gases. A gas will in any container into which it is placed and is therefore known as a compressible fluid.

(II) <u>Thermal Conductivity</u> :

Difference of temperature in a fluid in the course of time, are reduced by heat flowing from higher to lower temperature. There are three basic models of heat transfer viz. conduction, convection and radiation. Heat radiation is neglected at high temperature. The transfer of heat by convection depends on the velocity field, hear we discussed conduction. Two parallel layers of fluid at a distance d apart, are kept at different temperature T_1 and T_2 (one of the layers may be solid surface). Fourier noticed that a flow of heat is set up through the layers such that the quantity of heat q transferred through unit area in unit time is directly proportional to the difference of temperature between the layers and inversely proportional to the distance d thus

$$q = K \frac{T_1 - T_2}{d}$$

where K is constant of proportionality and is known on the coefficient of thermal conductivity.

(III) <u>Thermal diffusivity</u> :

As we see, the effect of conductivity on the temperature field is determined by the ratio of K to the product of density ρ and specific heat C_p rather than by K alone. This ratio is known on the thermal diffusivity and is usually denoted by a

$$a = \frac{K}{\rho C_p}$$

IV) <u>Physical importance of non-dimensional parameters</u>

a) <u>Prandtl number</u> :

The ratio of kinematic viscosity to the thermal diffusivity of the fluid

$$\text{i.e.} \quad \frac{\text{Kinematic viscosity}}{\text{Thermal diffusivity}} = \frac{\gamma}{a} = \frac{\mu / \rho}{K / \rho C_p} = Pr$$

is designated as the Prandtl number.

b) <u>Reynold number</u>

The dimensionless quantity Redefined as

$$R_e = \frac{UL \rho}{\mu} = \frac{U L}{\gamma}$$

where U, L, ρ and μ are some characteristic values of the velocity, length, density and viscosity respectively is known on the Reynold number.

c) Grashoff number :

The dimensionless quantity Gr which characterises the free conduction is known as the Grashoff number and is defined as

$$Gr = \frac{gL^3 \; (T_w - T_\infty)}{\nu^2 \; T_\infty}$$

where g is acceleration due to gravity and T_w, T_∞ are two respective temperatures.

d) Local skin friction coefficient :

The dimensionless shearing stress on the surface of a body due to fluid motion is known as Local skin friction coefficient and is defined as

$$C_f = \frac{\tau_w}{\rho \; U^2/2}$$

where τ_w is the local shearing stress on the surface of the body.

e) Nusselt number :

In the dynamics of viscous fluid one is not much interested to know all the details of the velocity and temperature fields but would certainly like to know quantity of heat transfer can be calculated with the help of coefficient of heat transfer $\alpha(x)$ which is defined by Newton's Law of cooling.

If g(x) is the quantity of heat exchanged between the wall and the fluid, per unit area per time at a point x, then

$$q(x) = \alpha(x)\ (T_w - T_\infty)$$

where $(T_w - T_\infty)$ is the difference between the temperature of the wall and that of the fluid since at the boundary the heat exchanged between the fluid and the body is only due to conduction according to Fourier's law we have,

$$q\ (x) = -K\ \left(\frac{\partial T}{\partial \eta} \right)_{\eta=0}$$

where η is the direction of the normal to the surface of the body. From these two laws we defined Nusselt number is as follows

$$N_u = \frac{\alpha\ (x)\ L}{K} = \frac{L}{(T_w - T_\infty)} \left(\frac{\partial T}{\partial \eta} \right)_{\eta=0}$$

where L is some characteristic length.

V) <u>Forced and free convection</u> :

The problem of thermal boundary layer may be classified into two categories viz. (i) forced convection, (ii) free convection. By forced convection we mean the flow in which the velocity arising from the variable density (i.e. due to force of bouyancy) are negligible in comparison with the velocity of the main or forced flow, whereas the free convection, also known as the neutral convection the motion is essentially caused by the effect of gravity for the heated fluid of variable density.

VI) Point of separation :

It is defined as the limit between forward and reverse flow in the layer in immediate neighbourhood of the boundary wall. In otherwards the point of separation is the point of which $\left(\frac{\partial u}{\partial y} \right) = 0$ or $\tau_w = 0$.

VII) Separation of boundary layer :

There are two methods of study the separation of boundary layer

a) Physical approach

b) Analytical approach

a) Physical approach :

δ is the boundary layer thickness which is increase in the down stream direction a point comes after that the flow in the boundary layer become reversed. This cause the decelerated fluid particles to the forced outwards which means the boundary layer is separated from the wall we than sepak of the "boundary layer separation" and the point at which the boundary layer separates is known as "point of separation". The phenomenon is called the separation of boundary layer. This will occur on blunt bodies such as circular and elliptic cylinder or spheres.

The motion in the boundary layer is determined by the following factors :

i) It is retarded by friction at the boundary wall,

ii) It is pulled forward through the action of viscosity

iii) It is retarded by the adverse pressure gradient ($dp/dx > 0$)

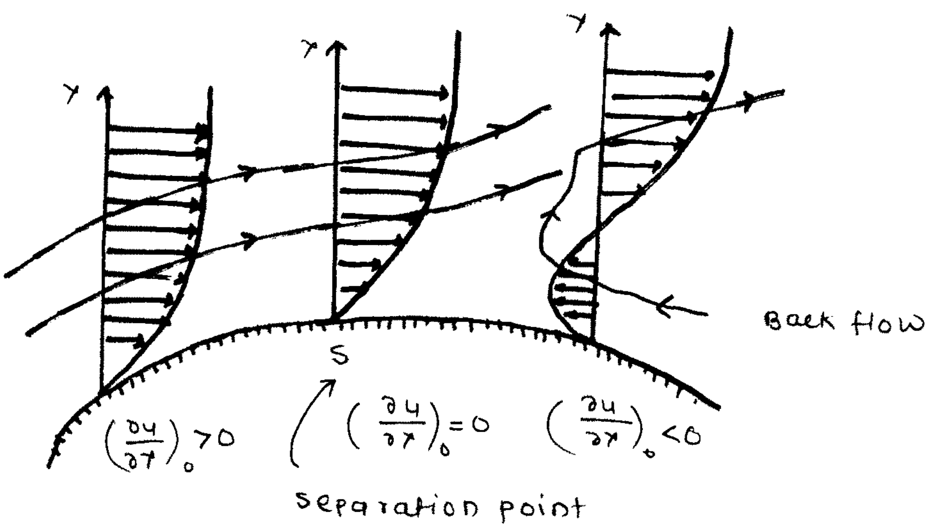


Fig.1. Boundary layer separation

b) Analytic approach :

Analytically the separation phenomenon may be explained by applying the prandtl boundary layer equation both outside the boundary layer and at the wall outside the boundary layer is

$$U \frac{du}{dx} = - \frac{1}{\rho} \frac{dp}{dx}$$

and at the wall, i.e. at $y = 0$ we have $u = v = 0$, the equation is

$$\mu \left(\frac{\partial^2 u}{\partial y^2} \right)_o = \frac{dp}{dx}$$

<u>Case i)</u> : $dp/dx = 0$ (zero pressure gradient)

In this case $\left(\frac{\partial^2 u}{\partial y^2} \right)_o = 0$ and hence, the velocity gradient decreases continually from a positive value at the wall to zero at the outer edge of the boundary layer. The velocity profile 'must therefore, have a steadily increasing. The point of inflexion occurs on the wall since $\left(\frac{\partial^3 u}{\partial y^3} \right)_o = 0$ but $\left(\frac{\partial^4 u}{\partial y^4} \right) \neq 0$ as can easily be verified by differentiating the boundary layer equation with respect to y and evaluating the value at $y = 0$. The fluid particles continue to move forward and therefore, the question of boundary layer separation does not arise.

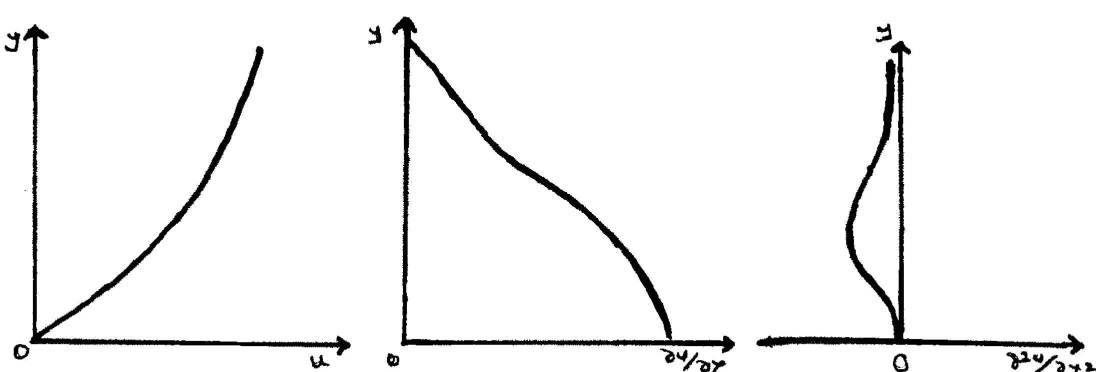
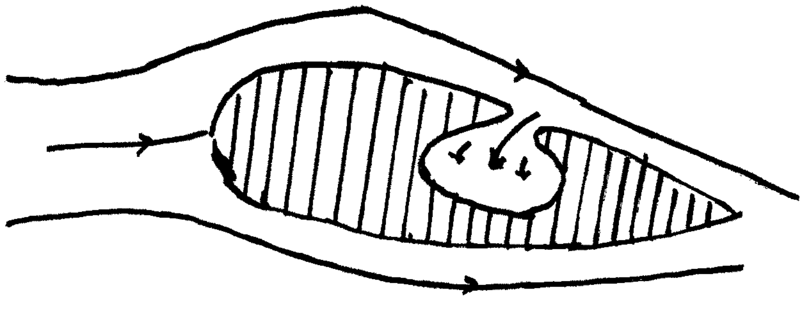


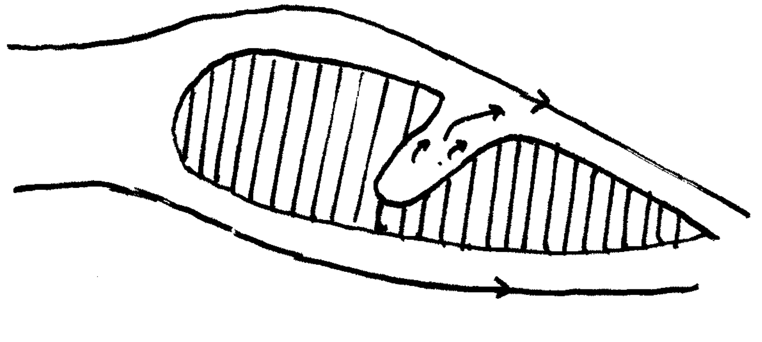
Fig.2 : velocity distribution in the case of zero pressure gradient (constant pressure)

VIII) Boundary layer control :

It is logical now to ask whether separation of boundary layer can be prevented or, more generally, can the growth of the boundary layer be controlled. Indeed, according to the physical approach of the boundary layer it should be possible to delay or even prevent separation by removing the decelerated fluid particle caused by the adverse pressure gradient in the region where separation is likely to be developed in figure. This possibility was first pointed out by L. Prandtl by applying suction through a small slit on the upper rear portion of circular cylinder, prandtl showed in his experiments that the flow adheres to the cylinder over a considerably larger portion of its surface instead of separating at 81° from the stagnation point as it does in the case when no suction is applied. The total drag is reduced considerably and simultaneously a large lifting force in the direction perpendicular to the free stream is induced. An alternative means of controlling the boundary layer is the supplying of the additional energy to the boundary layer by injecting fluid parallel to the surface in figure b, thus enabling the boundary layer to proceed further against an adverse pressure gradient. The ideal method of prevention of separation would be a means to eliminate the boundary layer. This can be accomplished in principle by having a solid wall moving with the stream. The logical way to obtain such a result is to place a rotating circular cylinder with its axis at the right angle to the flow. It is clear that on the upper surface of the cylinder where the cylinder moves in the same direction as the flow, no separation is possible. However, incomplete separation is developed on the Lower side of the cylinder. The resultant flow field does not vary appreciably from the potential flow theory this example was first applied by prandtl to illustrate the boundary layer theory.



(a) suction of fluid



(b) injection of fluid

Fig.3 : Suction and injection of the fluid

IX) Flow Through porous media :

Reynolds number flows are the flows through porous media such flows are very much prevalent in nature and therefore, these need through investigation. The study of flow through porous media is comparatively easy because in these flows the inertia forces are usually very small as compared to viscous forces.

Flow through porous media occur in filtration of fluids and seepage of water in river beds. Movement of underground water and oils are some other important example of flows through porous media. An oil reservoir mostly consists of porous sedimentary formation such as Limestone and sandstone in which oil is entrapped. Oil can be obtained from such reservoirs by drilling wells in oil bearing area dw down to the oil reservoir and then either allowing or causing the oil to flow through porous oil bearing rocks into the well. Same is the principle of obtaining underground water from wells.

To study the underground water resources also one need to investigate the flows of fluids through porous media. In fact the land along the side of the rivers is usually porous and the water goes underground due to the seepage in the rainy season the flow of water through rivers, especially seasonal rivers, is high while in other season it is pretty flow. One can study underground water can be pumped out in the off season for irrigation purposes. This will lower down the level of underground water in the river bed area and during the rainy season the river water will be gone underground with a greater seepage velocity and thus one can check floods also another important example of flow through porous media is the seepage under a dam.

..

REFERENCES

1. Blasius, H. : Generalised in flussigkeiten mit Kleiner Reibung Zeit Maths Phy 1908, P.L. Also NACA TM No. 1256.
2. Blasius, H. (1900) : Z. Maths Phy. 66 : 1-37.
3. Boedewadt, T. : Die Drehstromung uber festern Grande ZAMM 20, 241 (1940).
4. Boltz, E. : Grenzschichten an Rotutionkorpern in Flussigkeiten mit Kleiner Reibung Thesis Gottingen 1908.
5. Belonosov, S.M. : Vyncisl Math (Klev.) Vyp 30, (1976): 116-125.
6. Burns, J.C. : A sepectrum of Math. Auckland Univ. Press 1971 : 165-171.
7. Burgers, J.M. : Proc. Int. Congress Appl. Math. 1st Delft 1925 : 101-111.
8. Busemann, A. (1931) : In Hand buckeler Experimental Physik, ed. W. Wien. F. Harms 4(f): 341-360. Leipzing Akademigche.
9. Busemann, A. (1935) : Z.A. New Math. Mech. 15 : 23-25.
10. Britov, N.A. (1976) : Mat Fiz Vyp 17 : 85-89.
11. Burgers, J.M. (1925) : Proc. Int. congr. Appl. Mech. 1st Pelft 1924 : 113-28.
12. Blasius, H. : Z.Math. Phy. 55 : 5-25.
13. Boltz, E. (1908) : Gottingen dissertation.

14. Cebeci, T. : Proc. Roy Soc. London Ser. A 355, no. 1681 : 225-238.

15. Eiffel, G. (1912) : C.R.Acad Sci. Paris Ser. A. 155: 1597-99.

16. Falkneo, V.M. and Skan, S.W. (1930) : Aeronaut Res. Coun, Rep. Mem. No. 1314.

17. Froude, W. (1874) : Res. Br. Assoc. Adv. Sci. : 249-255.

18. Goldstein, S. and Rosenhead, L. (1936) : Proc. Cambridge Philos Soc. 32 : 393-401.

19. Goldstein, S. (1948) : G.J.Mech. Appl. Math 1 : 43-49.

20. Goldstein, S. (1933) : Proc. K. Soc. London Ser. A. 142 : 545-62.

21. Goldstein, S. and Rosenhead, L. : Proc. cambridge Philo. Soc. 32 : 390-401.

22. Howarth, L. (1948) : Proc. R.Soc. London Ser. A-194: 16-42.

23. Hartree, D.R. (1937) : Proc. Cambridge Philos. Soc. 33 : 223-39.

24. Hiemenz, K. (1911) : Dinglers J. 326 : 321-324, 344-48, 357-62, 372-76, 391-93.

25. Hudimoto, B. (1941) : J. Soc. Aeronaut Soc. Jpn. 8: 279-82.

26. Illingworth, C.R. (1949) : Proc. R.Soc. London Ser.A 199, : 533-57.

27. Krishnan, E.V. (1976) : J.Fluid Mech no.1 : 17-28.

28. Karman, Th. V. and Millikan, C.B. (1934) : NACA Tech. Rep. No. 504.

29. Karman, Th. V. (1921) : Z.A. New Math. Mech I : 233-252.

30. Landau, L.D. and Lifshitz E.M. (1959) : Fluid Mech Oxford : Pergaman.

31. Moore, L.L. : A solution of Laminare boundary layer equation for a compressible fluid with variable properties including dissociation Jour. Sci. 19, No.8, 1952 : 505-18.

32. Mises, R.V. (1927) : Z. Angew Math Mech 7 : 425-431.

33. Millikan, C.B. (1936) : J. Aeronaut Sci. 3 : 91-94.

34. Mangler, W. (1944) : Z. Angew. Math. Mech. 24:251-61.

35. Nigam S.D. : Quarterly Amer. Math. 9, : 89-91 (1951).

36. Pohlhausen, E. : Der warmeaustausch zwischen festen korper and Flussigkeiten mit kleiner Reibung and Kleiner Warmeleitung ZAMM 1, 1921 : 121.

37. Poots, G. (1960) : Q.J.Mech. Appl. Maths. 13 : 57-64.

38. Prandtl, L. (1905) : Verh. Int. Math. Kongr 3rd Heidelberg 1904 : 484-91, Transl. 1928, NACA Memo No. 452.

39. Prandtl, L. (1928) : Z. Angew Math. Mech. 8 : 249-51.

40. Prandtl, L. (1938) : Z. Angew Math Mech 18 : 77-82.

41. Prandtl, L. (1914) : Nachr. Ges Wiss Tottingen Math. Phys. Kl : 177-190.

42. Prandtl, L. (1910) : Phys. Z. 11 : 1072-78.

43. Rankine, J.M. (1864) : Trans. Inst. Nav. Archit. 3 : 316-33.

44. Schlichting, H. : Zaminare stralausbreitung-ZAMM 13, 260 (1933).

45. Schlichting, H. (1932) : Phys. Z. 33 : 327-35.

46. Stewartson, K. (1954) : Proc. Cambridge philos. Soc. 50 : 454-65.

47. Schlichting, H. (1933) : Z. Angew Math. Mech. 13 : 260-63.

48. Schubaber, G.B. (1935) : NACA Tech. Rep. No. 527.

49. Thiriot, K.H. : Uber die Laminare Anlaufstromun einer Flussigkeit Uber einens rotierenden Boden bei Plotzlicher Anderung des Drehunggzustandes ZAMM 20, 1 (1940).

50. Tani, I. (1954) : J. Aeronaut Sci. 21 : 487-95.

51. Tollmisen, W. (1924) : Gottingen dissertation.

52. Toffer, C. (1912) : Z. Math. Phys. 60 : 397-98.

53. Timman, R. (1949) : Rep. Trans Luchtvlab Amsterdam 15 : F: 29-45.

54. Tani, I. (1949) : J. Phys. Soc. Jpn 4 : 149-54.

55. Van Dyke, M. (1964) : Perturbation method in fluid Mechanics, New York and London Academic.

56. Wada, K. (1944) : Rep. Aeronaut Res. Inst Tokyo, Imp. Liniv No. 302.

57. Walz, A. (1941) : Ber Lilienthal Ges Luftfahrlf No.141: 8-12.

58. Wada, K., : J. Phys. Soc. Jpn 4 : 42-45.

59. Zhukorscheii, N.E. (1916) : Aerodynumique Paris, Gauthier, Villars.

oOo