CHAPTER - II

Study of some Boundary Layer Problems

1) Introduction:

In fluids flowing past heated or cooled bodies the transfer of heat takes place by conduction and convection. Heat radiation is negligible unless the temperature is very high. When the conductivity of the fluid is small, which is true in ordinary fluids, the heat transfer due to conduction is comparable to that due to convection only across a thin layer near the surface of the body. This means that the temperature field which spreads from the body extends, essentially, over a narrow zone in the immediate vicinity of its surface, whereas the fluid at a larger distance from the surface is not materially effected by the heated body. This narrow region near the surface of the body is known as thermal boundary layer. This is classified into forced and free convection.

Girishchandra Pande [6] investigated'effect of suction on unsteady free convection flow past a verticle flat plate! He studied the unsteady laminar free convection flow past a verticle infinite flat plate subjected to time dependent suction is considered when the plate temperature varies on some power of time. Series solution for velocity and temperature obtained in terms of known function when the prandtl number of the fluid is unity.

B.P.Acharya and S. Pandhy [1] studied free convective viscous flow past not verticle porous plate with periodic temperature. In this problem he obtained an analysis of a free convective flow of viscous liquid past a hot verticle porous wall is presented under the assumption that the suction velocity is constant and normal to the wall, and the wall temperature is spamwise cosinusoidal approximate solution of the equation of motion and energy equation have been obtained by the method regular perturbation.

J.L.Bansal [3] investigated the 'Asymptotic suction temperature profiles in laminar boundary layer over a porous flat plate'. In this note it has been shown that in the case of laminar boundary layer over a flat plate with homogeneous suction as we have the 'asymptotic suction velocity profiles' there exists also 'asymptotic suction temperature profile', for various values of the prandtl number (Pr). The recovery factor is such a case is to be independent of Pr and has a constant value one.

S. Prasad [13] obtained 'Boundary layer with suction along the porous wall in Tani's flow'. In this paper an investigation has been made into Laminar incompressible boundary layer with continuous suction along a porous wall in Tani's flow for which the potential flow velocity is given by $U(x) = U_0 \left(1 - \frac{x^2}{a^2}\right)$. Following M.R. Head [8] the momentum and the kinetic energy integral equation have been used by

S. Prasad with the aid of an eighth degree polynomial velocity profile to obtain a step-by-step numerical solution.

Krishna Lal [11] investigated 'free convection laminar boundary layer in unsteady flow'. In this paper he studied the effect of unsteady flow in the magnitude of surface temperature on the free convective laminar velocity and thermal boundary on a flat plate is studied. In Section one, the general equation of motion and temperature distribution are given. In Section two, the solution are obtained when the fluctuations in the velocity components and temperature distribution are in the form $(u, v, G) = (v_o, V_1, G_o) + \epsilon\ (u_1, v_1, G\ ) \exp\ (w_t)$ and lastly solution is given when the fluctuations is an exponentially decreasing function of time.

Ram Deo Matho [15] studied the 'Boundary layer with suction over a porous elliptic cylinder'. In this paper the method suggested by Head has been used, the momentum and the kinetic energy integral equation for two-dimensional boundary layer have been rederived and have been used with the aid of the schlicting's velocity profile to obtain a step-by-step, solution for the boundary layer with suction in the region of adverse pressure gradient over a porous elliptic cylinder with the ratio of the major to minor axis is four.

F.T.Smith and P.W.Duck [17] obtained 'The separation of Jets or thermal boundary layer from a wall'. In this paper consideration is given to the separation and subsequent reverse flow occurring when a Jet-like boundary layer on wall encounters a concave corner of finite angle α.

R. Sharma [18] explained 'A two parameter method for calculating the two-dimensional boundary layer with suction or injection'. In this paper he applied the method developed by eurle for flows without suction to the flows with suction or injection. Detailed calculation of the boundary layer parameters made by this method indicate that the error are within 5% of the exact values.

M.G.Palekar and D.P.Sharma [14] studied 'Approximate solution of the boundary layer equation with suction blowing'. according to him the problem under consideration is that of boundary layer flow along a flat plate with suction or blowing there are two types of surface suction namely V_w constant and $V_w \backsim x^{-1/2}$. An approximate integral method are obtained the principle merits of method is (1) solution are obtained in the closed analytic form (2) the similarity condition can be relaxed the velocity profile and skin friction are presented and compared with the result of the past investigators.

G.N.Sharma and D.P.Singh [19] investigated 'The effect of viscosity temperature law in unsteady boundary layer on flat plate'. They studied the effect of viscosity-temperature law, when the wall is in arbitrary motion with steady stream velocity. Prandtl number being unity. The dependence of coefficient of viscosity μ on temperature T is assumed to be

$$\mu = \text{constant} \frac{T^{w+1}}{T + C}$$

where w and C are constants.

This is the generalisation of the work of Sarma (1965) from a Linear law to the more realistic nonlinear law between viscosity and temperature.

N.C.Raghav Acharyulu [16] studied combined free and force convection in verticle circular porous channel. He obtained it in saturated verticle porous tube of circular cross-section with uniform heat source. The governing equations are solved for velocity and temperature fields in the form of fourier Bessel series.

D. Surma Devi and G. Nath [20] investigated 'Similarity solution of the unsteady boundary layer equation for a moving wall. In this problem he obtained the similarity solution of the unsteady laminar for two dimensional incompressible and of axisymmetric boundary layer equation for the case of surface which moves with a velocity which varies inversely as a linear function of time. The governing equation has been solved numerically. It is found that the effect of unsteadiness in the wall velocity and mass transfer on the skin friction and heat transfer parameters are applicable. The prandtl number strongly affects the heat transfer but the skin friction is unaffected by it.

Girish Chandra Pande [7] obtained 'unsteady thermal boundary layer flow past a porous flat plate'. He considered infinite flat plate subjected to the suction or injection and it is assumed that the normal velocity at the plate varies at $t^{-1/2}$. The expression in closed form for velocity temperature, skin friction and rate of heat transfer are obtained for two cases (i) when the plate temperature is the same as that of fluid at infinity and (ii) when the plate temperature varies as some power of time. The effect of suction and injection as these quantities is shown graphically.

R.P.Agarwal [2] studied 'Non-Linear two point boundary value problem'. In this problem he obtained existence and uniqueness of the solution of third order non-linear differential equation with boundary conditions prescribed at two points.

R. Sharma [21] obtained the 'exact solution of the incompressible laminar boundary layer equations with zero pressure gradient and variable suction'. In this paper a numerical solution of the boundary layer equations with zero pressure gradient and with general distribution of suction is obtained. A set of differential equation has been very accurately integrated numerically and principle characteristics of boundary layer flow have been determined.

Kizlov, L.F. and Losinskaja, T.I. [12] investigated 'Integration of an electronic computer of the boundary layer equation of Tani's flow in the presence of suction.' In this investigation they obtained results by using power series for the stream function. Numerical integration method and the system of differential equation of the boundary layer for flow of Tani type in the presence of suction.

Holt, M. and Modarress, D. [9] studied the 'Application of the method of integral relations to Laminar boundary layer in three-dimensions'. It has been traditional among fluid dynamics to employ some numerical means (such as the finite difference techniques) to solve the two-dimensional non-linear compressible boundary layer equation. But as an alternative to this numerical procedure, the boundary layer equation have been more successfully solved in an integral form, for example with the classical Karman-Pohlhausen momentum integral method. The main principle of the method of integral equation is based on the idea of representing the streamwise velocity gradient (normal to the wall) as a simple algebraic function of the streamwise velocity itself. These investigators extend the method of integral relations to the problem of three-dimensional compressible boundary layer flows with and without separation. By reducing the equation of motion to a quasi-incompressible form. They solved the resulting hyperbolic partial differential equation.

In this Chapter in Section 2 we have studied the approximate solution of the Pohlhausen's problem of forced convection in a laminar boundary layer on a flat plate. In Section 3 we have made an attempt to study the approximate solution of the Pohlhausen's problem of free convection from a heated verticle plate. In Section 4 of this chapter we investigated the boundary layer for Howarth's flow past a wedge. Section 5 deals with the study of boundary layer in Howarth's flow along the wall of convergent channel. Lastly in Section 6 we investigate the boundary layer with suction along porous wall in Tani's and Howarth's flow.

Before discussing in Section 2 we enlist some basic equations and the characteristic boundary layer parameters required for our problems to be discussed.

(a) <u>Free convection from a heated verticle flat plate</u> :

A flat plate is heated to a temperature T_w and placed vertically under gravity in a large body of fluid which otherwise at rest and has temperature T_∞ and density ρ_∞. Thus the equation governing the motion are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad \ldots (1)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial v}{\partial y} = \gamma \frac{\partial^2 u}{\partial y^2} + \frac{1}{\rho} G_x \qquad \ldots (2)$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = a \frac{\partial^2 T}{\partial y^2} \qquad \ldots (3)$$

where $G_x = g \rho_{\infty} - g \rho = \rho g \beta (T - T_{\infty})$ ... (4)

= resultant body force on the fluid since for any fluid

$$\frac{1}{\rho} = \frac{1}{\rho_{\infty}} \left\{ 1 + \beta (T - T_{\infty}) \right\} \qquad \ldots (5.)$$

where β is the coefficient of thermal expansion from equation of state we have

$$\frac{\rho_{\infty}}{\rho} = \frac{T}{T_{\infty}}$$

Thus from equation (5) we find that for gases,

$$\beta = \frac{1}{T_{\infty}}$$

Introduce $\theta = \dfrac{T - T_{\infty}}{T_w - T_{\infty}}$

Thus equation (2) and (3) become

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \gamma \frac{\partial^2 u}{\partial y^2} + g \left(\frac{T_w - T_{\infty}}{T_{\infty}} \right) \theta$$

$$u \frac{\partial \theta}{\partial x} + v \frac{\partial \theta}{\partial y} = a \frac{\partial^2 \theta}{\partial y^2} \qquad \ldots (6)$$

the boundary condition in the present case are

$$y = 0 \;:\; u = 0;\; v = 0;\; \theta = 1$$
$$y = \infty \;:\; u = 0\;;\; \theta = 0 \qquad \ldots (7)$$

by making the following substitution

$$\eta = \left\{ \frac{g\,(T_w - T_\infty)}{4\,\gamma^2\,T_\infty} \right\}^{1/4} \cdot \frac{y}{x^{1/4}} = C.\,\frac{y}{x^{1/4}}\,,$$

$$(x,y) = 4\,\gamma\,C\,x^{3/4}\,f(\eta) \qquad \ldots (8)$$

$$\theta\,(x,y) = \theta\,(\eta)$$

thus the equation (6) becomes,

$$f''' + 3ff'' - 2f'^2 + \theta = 0$$

and $\theta'' + 3Pr\,f_\theta' = 0$ ... (9)

with the boundary conditions

$$\eta = 0 \;:\; f = f' = 0\;;\; \theta = 1$$
$$\eta = \infty \;:\; f' = 0\;;\; \theta = 0 \qquad \ldots (10)$$

E. Pohlhausen solved these equation (9)

(b) <u>Karman momentum integral equation</u> :

The prandtl boundary layer equation for a steady two-dimensional incompressible flow are

$$u\,\frac{\partial u}{\partial x} + v\,\frac{\partial u}{\partial y} = U\,\frac{dU}{dx} + \gamma\,\frac{\partial^2 u}{\partial y^2} \qquad \ldots (11)$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad \ldots (12)$$

subjected to the boundary condition are

$$\begin{aligned} y = 0 &: \ u = v = 0 \\ y = \infty &: \ u = U(x) \end{aligned} \qquad \ldots (13)$$

where δ is the boundary layer thickness.

Due to equation (12) and (13) equation (11) can be reduced to the form

$$U^2 \frac{d\,\delta 2}{dx} + (2\delta 2 + \delta 1)\, U \frac{du}{dx} = \frac{\tau_0}{\rho} \qquad \ldots (14)$$

This equation is known as Karman momentum integral equation for two dimensional steady incompressible boundary layer

where

$$\delta 1 = \int_0^{\delta} \left(1 - \frac{u}{U}\right) dy, \text{ (displacement thickness)}$$

$$\delta 2 = \int_0^{\delta} \frac{u}{U} \left(1 - \frac{u}{U}\right) dy, \text{ (momentum thickness)}$$

and $\tau_0 = \mu \left(\frac{\partial u}{\partial y}\right)_0$ (shearing stress on the wall)

where ρ is density of the fluid.

(c) <u>Energy integral equation</u> :

By using equation (11), (12) and (13) we obtained the energy integral equation for two dimensional steady incompressible boundary layers in the form

$$\frac{d}{dx} \left(U^3 \delta_3 \right) = \frac{2D}{\rho} \qquad \ldots (15)$$

where

$$\delta_3 = \int_0^{\delta} \frac{u}{U} \left(1 - \frac{u^2}{U^2}\right) dy \quad , \text{ (energy thickness)}$$

and

$$D = \mu \int_0^{\delta} \left(\frac{\partial u}{\partial y} \right) dy \quad , \text{ (dissipation integral)}$$

(d) <u>Thermal energy integral equation :</u>

The method of thermal energy integral equation is calculated from thermal boundary layer equation.

The boundary layer equation

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad \ldots (16)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} \qquad \ldots (17)$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = a \frac{\partial^2 T}{\partial y^2} + \frac{\mu}{\rho C_p} \left(\frac{\partial u}{\partial y} \right)^2 \qquad \ldots (18)$$

with the boundary conditions

$$y = 0 \;:\; u = v = 0$$
$$y = \delta(x) \;:\; u = U(x) \qquad \ldots (19)$$

where $a = \frac{K}{\rho C_p}$ is the thermal diffusivity due to equation

(16), (17) and (19) can be reduced the form

$$\frac{d}{dx} \int_0^{\delta_t} u(T - T_\infty)\,dy = -a \left(\frac{\partial T}{\partial y} \right)_{y=0} + \frac{\mu}{\rho\, c_p} \int_0^{\delta_t} \left(\frac{\partial u}{\partial y} \right)^2 dy \quad \ldots(20)$$

(2) <u>Approximate solution of the Pohlhausen's problem of forced convection in a Laminar boundary layer on a flat plate</u> :

Taking the Pohlhausen's fourth degree velocity profile for a boundary layer over a flat plate.

$$\frac{u}{U_\infty} = f(\eta) = \sum_{i=0}^{4} a_i \eta^i \qquad 0 \le \eta \le 1 ,$$

$$\frac{u}{U_\infty} = 1 \qquad \text{as } n > 1 \qquad \ldots (1)$$

where $\eta = \frac{Y}{\delta}$ and δ is the thickness of the flat plate.

In order to determine the coefficients a_0 to a_4 we prescribed the following boundary condition and compatibility conditions.

$$y = 0 \;:\; u = 0 , \quad \frac{\partial^2 u}{\partial y^2} = 0$$

$$y = \delta \;:\; u = U_\infty , \quad \frac{\partial u}{\partial y} = \frac{\partial^2 u}{\partial y^2} = 0 \qquad \ldots (2)$$

$$\frac{u}{U_\infty} = f(\eta) = a_0 + a_1\eta + a_2\eta^2 + a_3\eta^3 + a_4\eta^4$$

Using the boundary conditions (2) we have calculate the coefficient a_0 to a_4 as follows -

$$a_0 = 0, \quad a_1 = 2, \quad a_2 = 0$$

$$a_3 = -2, \quad a_4 = 1 \qquad \ldots (3)$$

Now we calculate the following characteristic boundary layer parameters

i) <u>Displacement thickness</u> :

$$\delta_1 = \int_0^\delta \left(1 - \frac{u}{U_\infty}\right) dy$$

$$= \int_0^1 (1 - 2\eta + 2\eta^3 - \eta^4)\, \delta\, d\eta$$

$$= \delta \frac{3}{10}$$

$$= 0.3\,\delta \qquad \ldots (4)$$

ii) <u>Momentum thickness</u>

$$\delta_2 = \int_0^\delta \frac{u}{U_\infty}\left(1 - \frac{u}{U_\infty}\right) dy$$

$$= \delta \int_0^1 (2\eta - 4\eta^2 + 4\eta^4 - 2\eta^5 - 2\eta^3 + 4\eta^4 - 4\eta^6 + $$
$$+ 2\eta^7 + \eta^4 - 2\eta^5 + 2\eta^7 - \eta^8)\, d\eta$$

...

$$= \delta \; \frac{567 - 530}{315}$$

$$= 0.1176 \; \delta \qquad \ldots (5)$$

iii) <u>Shearing stress on the flat plate</u> :

$$\tau_o = \mu \left(\frac{\partial u}{\partial y} \right)_{y=0}$$

$$= \mu \frac{U}{\delta} \left(\frac{\partial f}{\partial \eta} \right)_{\eta=0}$$

$$= \mu \frac{U}{\delta} \left[2 - 6\eta^2 + 4\eta^3 \right]_{\eta=0}$$

$$= 2\mu \frac{U}{\delta}$$

$$= 2 \; \gamma \; \rho \; \frac{U}{\delta} \qquad \ldots (6)$$

iv) <u>Coefficient of skin friction</u>

$$C_f = \frac{\tau_o}{\frac{1}{2} \rho U^2}$$

$$= \frac{4 \; U \mu}{U^2 \rho \; \delta}$$

$$= \frac{4 \; \gamma}{U \; \delta} \qquad \ldots (8)$$

Now we have studied two cases for this problem (i) solution for cooling problem and (ii) solution for adiabatic wall.

a) <u>Solution for the cooling problem</u> :

We introduce the flow of viscous incompressible fluid at flat plate at constant temperature T_w which is placed along the direction of uniform stream velocity U_∞ and temperature T_∞.

Introduce the dimensionless temperature θ_1

$$\theta_1 = \frac{T - T_\infty}{T_w - T_\infty} \qquad \ldots (8)$$

The heat flux equation in the present case may be written as

$$\frac{d}{dx} \int_0^{\delta_t} (\theta_1 \frac{u}{U_\infty}) dy = \frac{-a}{U_\infty} \left(\frac{\partial \theta_1}{\partial y} \right)_{y=0} \qquad \ldots (9)$$

for the temperature distribution we consider the following polynomial in η_t

$$\theta_1 = L(\eta_t) = 1 - \sum_{i=0}^{4} a_i \eta^i \quad 0 \le \eta_t \le 1$$

$$\theta_1 = 0 \qquad \text{as } \eta_t \to 1 \qquad \ldots (10)$$

Satisfying the following boundary and compatibility conditions

$$\eta_t = 0 \; : \; \theta_1 = 1, \quad \left(a \frac{\partial^2 \theta_1}{\partial \eta_t^2} \right) = 0$$

$$\eta_t = 1 \; : \; \theta_1 = 0, \quad \frac{\partial \theta_1}{\partial \eta_t} = \frac{\partial^2 \theta_1}{\partial \eta_t^2} = 0. \qquad \ldots (11)$$

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The compatibility condition are obtained from the thermal boundary layer equation after neglecting heat due to dissipation. Moreover the form of the temperature distribution is so selected as to ensure identical velocity and temperature distribution in the case of Pr = 1 for the existence of Croco's first integral.

Substitute (10) and (11) in (9) we get

$$\frac{d}{dx} \int_0^{\delta_t} \left[(1 - \sum_{i=0}^{4} a_i \eta^i)(\sum_{i=0}^{4} a_i \eta^i) \right] dy =$$

$$= \frac{-a}{U_\infty} \left(\frac{\partial \theta_1}{\partial y} \right)_{y=0}$$

$$\frac{d}{dx} \left\{ \delta_t H(\Delta) \right\} = \frac{2a}{U_\infty \delta_t} \qquad \ldots (12)$$

$$\text{where} \quad \Delta = \frac{\delta_t (x)}{\delta (x)} \qquad \ldots (13)$$

$$\text{and} \quad H(\Delta) = \int_0^1 f(\eta) \, L(\eta_t) d\eta \qquad \ldots (14)$$

It may be noted that

$$\eta = \frac{Y}{\delta} = \frac{Y}{\delta_t} \cdot \frac{\delta_t}{\delta} = \eta_t \Delta \qquad \ldots (15)$$

Performing the indicated integration in (14) we get

$$\Delta \leq 1 : \quad H(\Delta) = \int_0^1 f(\eta_t \Delta) L(\eta_t) \, d\eta_t$$

$$= \int_0^1 \left[(2\eta_t \Delta - 2\eta_t^3 \Delta^3 + \eta_t^4 \Delta^4)(1 - 2\eta_t + 2\eta_t^3 - \eta_t^4) \right] d\eta_t$$

$$= \Delta \left[1 - \frac{4}{3} + \frac{4}{5} - \frac{1}{3} \right] + \Delta^3 \left[-\frac{1}{2} + \frac{4}{5} - \frac{4}{7} + \frac{1}{8} \right] +$$

$$+ \Delta^4 \left[\frac{1}{5} - \frac{1}{3} + \frac{1}{4} - \frac{1}{9} \right]$$

$$H(\Delta) = \frac{2}{15} \Delta - \frac{3}{140} \Delta^3 + \frac{1}{180} \Delta^4 \qquad \ldots (16)$$

$$\Delta \geq 1 : \quad H(\Delta) = \frac{1}{\Delta} \int_0^\Delta f(\eta) \, L\left(\frac{\eta}{\Delta}\right) d\eta$$

$$= \frac{1}{\Delta} \int_0^1 f(\eta) L\left(\frac{\eta}{\Delta}\right) d\eta + \frac{1}{\Delta} \int_1^\Delta L\left(\frac{\eta}{\Delta}\right) d\eta$$

Keeping in view that $f(\eta) = 1$ for $n \geq 1$

$$= \frac{1}{\Delta} \int_0^1 (2\eta - 2\eta^3 + \eta^4) \left(1 - \frac{2\eta}{\Delta} + \frac{2\eta^3}{\Delta^3} - \frac{\eta^4}{\Delta^4} \right) d\eta +$$

$$+ \frac{1}{\Delta} \int_1^\Delta \left(1 - \frac{2\eta}{\Delta} + \frac{2\eta^3}{\Delta^3} - \frac{\eta^4}{\Delta^4} \right) d\eta$$

$$H(\Delta) = \frac{3}{10} - \frac{3}{10} \frac{1}{\Delta} + \frac{2}{15} \frac{1}{\Delta^2} - \frac{3}{140} \frac{1}{\Delta^4} + \frac{1}{180} \frac{1}{\Delta^5}$$

to find δ^2 we use Karman momentum integral equation

$$\frac{d}{dx} \left(\frac{\delta_2^2}{\gamma} \right) = \frac{L(\lambda)}{U_\infty} \qquad \ldots (17)$$

where $L(\lambda) = 2\left\{I - \lambda\ (H + 2)\right\}$

But for flat plate $\lambda = 0$

$L(0) = 2I$

$$= \frac{2\,T_o\,\delta_2}{\mu\,U_\infty}$$

But $T_o = 2\mu\,\frac{U_\infty}{\delta}$

$$= \frac{4\delta_2}{\delta}$$

$$= \frac{4 \times 37}{315}$$

because $\delta_2 = \frac{37}{315}\,\delta$

therefore, $\frac{d}{dx}\left(\frac{\delta_2^{\,2}}{\gamma}\right) = \frac{4 \times 37}{315} \cdot \frac{1}{U_\infty}$

$$\delta_2^2 = \frac{4 \times 37}{315} \cdot \frac{\gamma\,x}{U_\infty}$$

Put the value δ_2

$$\delta_2^2 = \frac{1260}{37} \cdot \frac{\gamma\,x}{U_\infty} \qquad \ldots (18)$$

Equation (12) on integration gives

$$\delta_t^2 \; H^2(\Delta) = \frac{4a}{U_\infty} \int_0^x H(\Delta) \; dx \qquad \ldots (19)$$

with the help of equation (18) may be written as,

$$\Delta^2 H^2 \; (\Delta) = \frac{37}{315} \cdot \frac{1}{x} \cdot \frac{1}{Pr} \int_0^x H(\Delta) \; dx \qquad \ldots (20)$$

Startig the initial value Δ = constant the exact solutioh of equation (20) is

$$\Delta^2 \; H(\Delta) = \frac{37}{315} \cdot \frac{1}{Pr} \qquad \ldots (21)$$

Now it can easily checked that $\Delta = 1$, $Pr = 1$ is the solution of the above equation therefore when $Pr = 1$, $\delta_t = \delta$ and $\eta_t = \eta$ thus form of (9)

$$\theta_1 = 1 - \sum_{i=0}^{4} a_i \eta^i = 1 - f(\eta). \; ; \; Pr = 1$$

which is known as Croco's first integral.

Equation (21) is the algebraic equation in Δ for the prescribed Pr it can easily be solved. It will be convenient to prescribe and determine the corresponding Pr. It is found that for following cases -

i) For moderate values of the prandtl number the expression

$$\Delta = Pr^{-1/3} \qquad \ldots (22)$$

Constitute a very good approximation to the solution of equation (21).

ii) For very small Prandtl number (i.e. for large values of Δ),

$$H(\Delta) = \frac{3}{10} \qquad \text{(approximated)}$$

therefore from (21) as $Pr \rightarrow 0$

$$\Delta^2 = 0.3915343\, Pr^{-1/2}$$

$$\Delta = 0.625\, Pr^{-1/2} \quad \text{as } Pr \rightarrow 0 \qquad \ldots (23)$$

iii) for very large prandtl number (i.e. for small values of Δ),

$$H(\Delta) = \frac{2}{15} \qquad \text{(approximated)}$$

therefore from equation (21)

$$\Delta = 0.9385\, Pr^{-1/3} \quad \text{as } Pr \rightarrow \infty \qquad \ldots (24)$$

The temperature gradient at the wall, from equation (10) is given by

$$\left(\frac{\partial \theta_1}{\partial \eta_t} \right)_{\eta_{t=0}} = -2 \qquad \ldots (25)$$

Therefore the local Nusselt number for the heat transfer at the wall is calculated as

$$N_u(x) = \frac{-\left(\frac{\partial T}{\partial y}\right)_{y=0} \cdot x}{(T_w - T_\infty)}$$

$$= -\left(\frac{\partial \theta_1}{\partial \eta_t}\right)_{\eta_t=0} \cdot \frac{x}{\delta_t} ,$$

$$= 2 \cdot \frac{1}{\Delta} \left(\frac{37}{1260}\right)^{1/2} \cdot \sqrt{\frac{U_\infty x}{\nu}} ,$$

$$N_u(x) = \frac{0.342}{\Delta} \; Re_x^{1/2} \qquad \ldots (26)$$

where $Re_x = \frac{U_\infty x}{\gamma}$ is the local Reynold number

substituting the values of Δ from equation (22), (23) and (24) we find

i) for moderate values of Pr

$$N_u(x) = 0.342 \, Pr^{1/3} \; Re_x^{1/2} \qquad \ldots (27)$$

ii) For very small prandtl number

$$N_n(x) = 0.5472 \, Pr^{1/2} \, Re_x^{1/2} \qquad \ldots (28)$$

for $Pr \to 0$

iii) for very large Prandtl number

$$N_u(x) = 0.3644 \, Pr^{1/3} \, Re_x^{1/2} \text{ for } Pr \to \infty \qquad \ldots (29)$$

(b) Solution for adiabatic wall :

Introducing $\theta_2 = \dfrac{T - T_\infty}{U_\infty^2 / 2C_p}$ ... (30)

and keeping in view that for the adiabatic wall $\left(\dfrac{\partial T}{\partial y} \right)_{y=0} = 0$,

the thermal energy integral equation may be written as

$$\frac{d}{dx} \int_0^{\delta_t} \left(\theta_2 \frac{u}{U_\infty} \right) dy = \frac{2\,\nu}{U_\infty} \int_0^{\delta_t} \left[\frac{\partial}{\partial y} \left(\frac{u}{U_\infty} \right) \right]^2 dy \quad \text{... (31)}$$

Let $\theta_2 = r - \left(\sum_{i=0}^{4} C_i \, \eta_t^{\,i} \right)^2$ ... (32)

where the coefficients C_o to C_4 are to be determined with the help of following boundary and compatibility conditions

$$\eta_t = 0 \; : \; \frac{\partial \theta_2}{\partial \eta_t} = 0, \quad \frac{\partial^2 \theta_2}{\partial \eta_t^2} = -8 \, Pr \, \Delta^2 \, ,$$

$$\eta_t = 1 \; : \; \theta_2 = 0, \quad \frac{\partial \theta_2}{\partial \eta_t} = \frac{\partial^2 \theta_2}{\partial \eta_t^2} = 0 \quad \text{... (33)}$$

and $\gamma = \dfrac{Tr - T_\infty}{U_\infty^2 / 2C_p}$ (recovery factor)

For an adiabatic wall the compatibility conditions are obtained from the thermal boundary layer equation in the usual manner. Also the form of θ_2 is so selected as to ensure the Crocco's second integral when $P_r = 1$.

With the help of condition (33) we have calculated the coefficients C_0 to C_4 as follows

$$C_0 = 0$$

$$C_1 = 2\Delta \sqrt{P_r} \quad , \quad C_2 = 6\sqrt{\gamma} - 3C_1$$

$$C_3 = -8\sqrt{\gamma} + 3C_1 \text{ and } C_4 = 3\sqrt{\gamma} - C_1 \quad \ldots (34)$$

Substituting (1) and (32) in (31) we get,

$$\frac{d}{dx} (\delta_t G) = \frac{2\gamma\Delta}{U_\infty \delta_t} J \quad \ldots (35)$$

$$\text{where } G = \int_0^1 (\theta_2 \frac{u}{U_\infty}) d\eta_t \quad \ldots (36)$$

$$\text{and } J = \int_0^\Delta \left\{ \frac{\partial}{\partial \eta} \left(\frac{u}{U_\infty} \right) \right\}^2 d_\eta \quad \ldots (37)$$

Now we find

$$J = \sum_{i=1}^{4} \sum_{j=1}^{4} \frac{ij}{i+j-1} \Delta^{i+j-1} a_i a_j$$

$$= 4\Delta - 8\Delta^3 + 4\Delta^4 + \frac{36}{5}\Delta^5 - 8\Delta^6 + \frac{16}{7}\Delta^4$$

$$\text{and } J = \frac{52}{35} \quad \ldots (38)$$

Integrating equation (35) and taking the value of δ from

(18) we have obtained

$$G\Delta = \frac{37}{315} J \qquad \ldots (39)$$

Now it can easily checked that when $Pr = 1$ and $\Delta = 1$ then $r = 1$ is a solution of the above equation. Therefore, in such a case $\Gamma_i = a_i$ and $\eta_t = \eta$ thus from equation (32)

$$\theta_2 = 1 - \left(\sum_{i=0}^{4} a_i \eta_i \right) = 1 - f^2 (\eta)$$

which is the Croco's second integral.

Equation (39) indicates that r is function of Δ and Pr. But we know Δ is a function of Pr therefore the recovery factor r will be a function of Pr only. Hence equation (32) is an algebraic equation

It is found that

i) for moderate values of the prandtl number the expression

$r = Pr^{1/2}$

Constitute a very good approximation to the solution of equation (32).

ii) For very large prandtl number

$J = 4\Delta$ (approximated)

(3) Approximate solution of the Pohlhausen's problem of free convection from heated verticle plate :

The equations governing the motion of incompressible viscous fluid in the neighbourhood of a heated verticle plate is given by

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad \ldots (1)$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \gamma\frac{\partial^2 u}{\partial y^2} + g\alpha\theta \qquad \ldots (2)$$

$$u\frac{\partial \theta}{\partial x} + v\frac{\partial \theta}{\partial y} = a\frac{\partial^2 \theta}{\partial y^2} \qquad \ldots (3)$$

where $\theta = \dfrac{T - T_\infty}{T_w - T_\infty}$ and $\alpha = \dfrac{T_w - T_\infty}{T_\infty}$ $\qquad \ldots (4)$

subject to the boundary conditions

$$\begin{aligned} y = 0 &: \; u = 0 ; \; v = 0 ; \; \theta = 1 \\ y = \delta &: \; u = 0 ; \; \theta = 0 \end{aligned} \qquad \ldots (5)$$

For the second boundary condition, we have assumed here that the thickness of the thermal boundary layer is the same as that of the velocity boundary layer, although they are different in general. This assumption has its justification in that it simplifies the computational work and the result obtained very near to the experimental result.

Integrating equation (2) and (3) with respect to y between o to δ, we get

$$\frac{d}{dx} \int_0^{\delta} u^2 dy = g\alpha \int_0^{\delta} \theta dy - \gamma \left(\frac{\partial u}{\partial y} \right)_o \qquad \ldots (6)$$

and

$$\frac{d}{dx} \int_0^{\delta} u\theta \, dy = - a \left(\frac{\partial \theta}{\partial y} \right)_o \qquad \ldots (7)$$

We use the following polynomials in η (= y/δ) for the distribution of u and θ which satisfying the respective boundary conditions

$$u = u_1(x) \, \eta \, (1 - \eta)^3. \qquad \ldots (8)$$

$$\theta = (1 - \eta)^3, \qquad \ldots (9)$$

where the arbitrary function $u_1(x)$ has the dimension of velocity and is to be determined.

With these expression of u and θ the equation (6) and (7) become,

$$\frac{1}{252} \frac{d}{dx} (u_1^2 \delta) = \frac{1}{3} g \alpha\delta - \gamma \frac{u_1}{\delta} \qquad \ldots (10)$$

$$\frac{1}{56} \frac{d}{dx} (u_1 \delta) = \frac{2a}{\delta} \qquad \ldots (11)$$

Let us try the solution of the above equation in the forms

$$u_1 = c_1 x^m \text{ and } \delta = c_2 x^n \qquad \ldots (12)$$

Then by substituting these equation in (10) and (11) the two equations take the form

$$\frac{2m + n}{252} C_1^2 C_2 x^{2m+n-1} = \frac{1}{4} g_\alpha C_2 x^n - \frac{C_1}{C_2} \gamma x^{m-n} \quad ..(13)$$

and $$\frac{m+n}{56} C_1 C_2 x^{m+n-1} = \frac{3a}{C_2} x^{-n}$$

must be identically satisfied, this gives

$2m + n - 1 = n = m - n;\ m + n - 1 = -n$

$m = 1/2$ and $n = 1/4$

and therefore

$$C_1 = 8.64\ \gamma \left(Pr + \frac{112}{101} \right)^{-1/2} \left(\frac{g\ \alpha}{\gamma^2} \right)^{1/2} \quad ... (14)$$

$$C_2 = 5.09\ Pr^{-1/2} \left(Pr + \frac{112}{101} \right)^{1/4} \left(\frac{g\ \alpha}{\gamma^2} \right)^{-1/4} \quad ... (15)$$

From equation (12), (14) and (15) we have

$$\frac{\delta}{x} = 5.09\ \ Pr^{-1/2}\ (Pr + 1.1089)^{1/4}\ (Gr)^{-1/4}$$

where

$$Gr = \frac{g(T_w - T_\infty)\ x^3}{\gamma^2\ T_\infty} = \frac{g\alpha x^3}{\gamma^2}$$

(Grashoff number)

The temperature gradient at the wall is

$$\left(\frac{\partial\ \theta}{\partial\ \eta} \right)_0 = -2$$

The local Nusselt number for the heat transfer in the present case is given by

$$N_u(x) = \frac{-\left(\frac{\partial T}{\partial y}\right)_{y=0} \cdot x}{(T_w - T_\infty)} = -\left(\frac{\partial \theta}{\partial \eta}\right)_0 \cdot \frac{x}{\delta}$$

$$= 0.589\ Pr^{1/2}\ (Pr + 1.1089)^{-1/4}\ (Gr)^{1/4}$$

For air $Pr = 0.733$ and this expression gives,

$$N_u(x) = 0.589 \times 0.856154\ (0.8583877)\ Gr^{1/4}$$

$$= 0.433\ (Gr)^{1/4}.$$

(4) <u>Boundary layer for Howarth's flow past a wedge</u> :

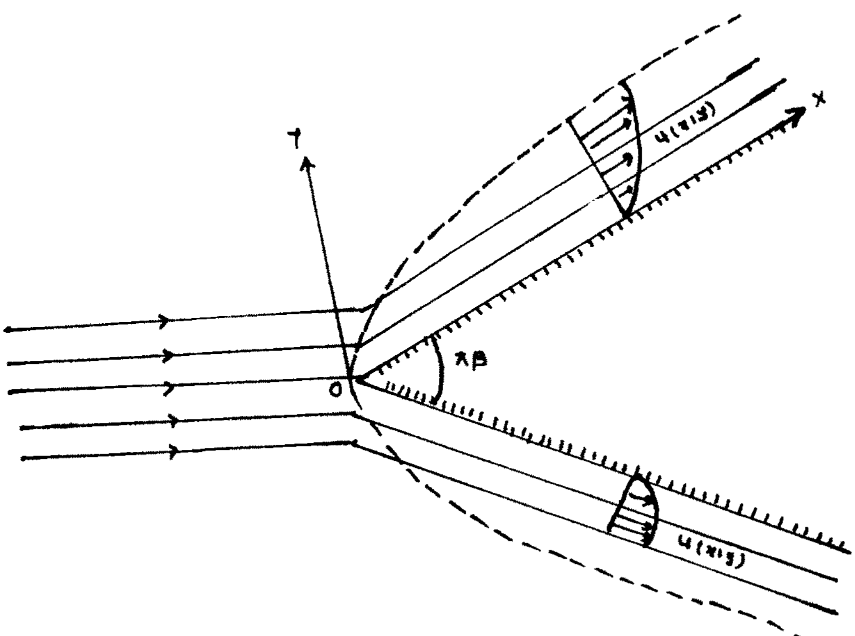

Fig.4: Boundary layer flow past a wedge of incompressible fluid.

Let origin be taken at the stagnation point. The x-axis along the wall and y-axis perpendicular to the wall. Thus we have the boundary layer equations

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = U \frac{d U}{d x} + \nu \frac{\partial^2 u}{\partial y^2} \qquad \ldots (1)$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad \ldots (2)$$

With the boundary condition

$$y = 0; \quad u = 0 = v$$
$$y \to \infty; \; u = U\,(x) \qquad \ldots (3)$$

where ν is the kinematic viscosity and $U(x)$ is the Howarth's flow potential velocity. Now we introduce the stream function $\psi(x, y)$ which satisfy the continuity equation (2). Thus we take

$$u = \frac{\partial \psi}{\partial x} \; , \quad v = - \frac{\partial \psi}{\partial y} \qquad \ldots (4)$$

by using equation (4) equation (1) take the form as

$$\frac{\partial \psi}{\partial y} \frac{\partial^2 \psi}{\partial x \partial y} - \frac{\partial \psi}{\partial x} \cdot \frac{\partial^2 \psi}{\partial y^2} = U \frac{dU}{dx} + \nu \frac{\partial^3 \psi}{\partial y^3} \qquad \ldots (5)$$

The corresponding boundary condition are

$$y = 0 \; ; \quad \psi = \frac{\partial \psi}{\partial x} = 0$$
$$y = \infty \; ; \quad \frac{\partial \psi}{\partial y} = U\,(x) \qquad \ldots (6)$$

In the neighbourhood of the stagnation point the Howarth potential flow velocity U may be written as,

$$U(x) = U_o \left(1 - \frac{x}{L}\right)$$

Hence η takes the form

$$\eta = - y \left(1 . \frac{U_0}{L \gamma} \right)^{1/2} x^{1/2} \qquad \ldots (7)$$

and

$$\psi(x,y) = - 1 \left(1. \frac{\gamma U_0}{L} \right)^{1/2} x. \, f(\eta) \qquad \ldots (8)$$

Using equation (7) and (8) in equation (4) we get the following

$$u = \frac{\partial \psi}{\partial y} = U(x) \, f'(\eta) = \frac{- U_0}{L} x f'(\eta)$$

$$v = - \frac{\partial \psi}{\partial x} = - \left(1. \gamma \frac{U_0}{L}\right)^{1/2} x^{\frac{M-1}{2}} f(\eta) \qquad \ldots (9)$$

and the boundary equation (1) reduces to an ordinary differential equation in $f(\eta)$ as,

$$f''' + ff'' + 1. (1 - f''^2) = 0 \qquad \ldots (10)$$

With the boundary conditions

$$\eta = 0 \; : \; f = 0 , f' = 0$$

$$\eta = \infty \; : \; f' = 1 \qquad \ldots (11)$$

Equation (10) is similar to Hartree's equation, therefore, we are using Hartree's solution

$$\text{Let } f(\eta) = \sum_{n=2}^{\infty} \frac{a_n}{n!} \eta^n \qquad \ldots (12)$$

which satisfies the first two b oundary conditions of (11) substituting (12) in equation (10) we find the coefficient expressed in terms of where $a_2 = \alpha$ (unknown)

$$a_2 = \alpha, \quad a_3 = -1, \quad a_4 = 0, \quad a_5 = \alpha^2, \quad a_6 = -2\alpha, \quad a_7 = 2,$$

$$a_8 = -\alpha^3, \quad a_9 = 4\alpha^2, \quad a_{10} = -16\alpha.$$

It may be noted that $\alpha = f''(0)$ is still unknown we have to find the value of α by substituting the values of f and f' from equation (12) we get a linear differential equation in f'' (n)

$$\text{Therefore } f''(\eta) = e^{-F(\eta)} \phi(\eta) \qquad \ldots (13)$$

$$\text{where } F(\eta) = \int_0^{\eta} F(\eta)\, d\eta$$

$$\text{and } \phi(\eta) = \alpha.\ 1 \int_0^{\eta} (1-f'^2)\, e^{F(\eta)}\, d\eta$$

Equation (13) satisfies the boundary condition at $\eta = 0$. Then unknown parameter α is found from the equation

$$e^{-F(\eta)} \phi(\eta)\, d\eta = 1.$$

which is obtained by integrating (13) and applying the boundary condition of (11) from the solution of (10) we obtained the boundary layer parameters as follows :

(a) Displacement thickness :

$$\delta_1 = \int_0^{\delta} (1 - \frac{u}{U}) dy$$

$$= - (\gamma . \frac{L}{U_0})^{1/2} . A(\beta),$$

where $A(\beta) = \int_0^{\infty} (1 - f') d\eta$

and $A(\beta) = \lim_{n \to \infty} (\eta - f(\eta))$

(b) Momentum thickness :

$$\delta_2 = \int_0^{\infty} \frac{u}{U} (1 - \frac{u}{U}) dy$$

$$= - (\gamma . \frac{L}{U_0})^{1/2} B(\beta)$$

where $B(\beta) = \int_0^{\infty} f' (1-f') d\eta$

(c) Shearing Stress :

$$\tau_w = \mu (\frac{\partial u}{\partial y})_{y=0}$$

$$= \mu \frac{U_0}{L} (\frac{1}{\gamma} . \frac{U_0}{L} x^2)^{1/2} f'' (0)$$

5. **Boundary layer Howarth's flow along the wall of a convetgent channel :**

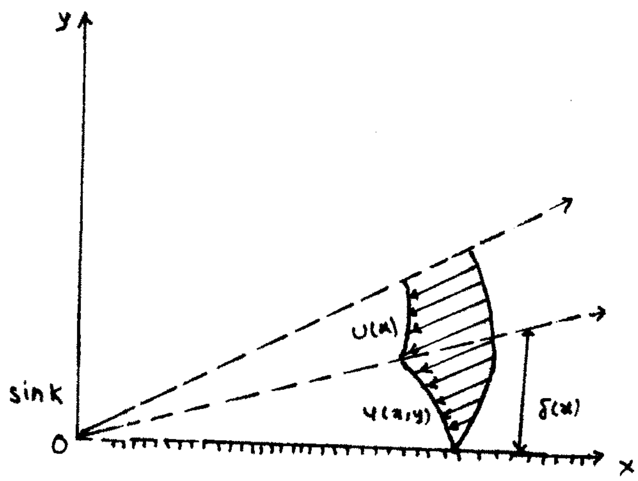

Fig. 5: B-oundary layer flow along the wall of a convergent channel.

The potential flow velocity is given by

$$U(x) = U_0 \left(1 - \frac{x}{L} \right) \qquad \ldots (1)$$

where x is measured along the wall of channel and U_0 is stream velocity, L is the reference length. Now we can introduce

$$\eta = \frac{y}{x} \sqrt{\frac{U_0}{L\gamma}} \qquad \ldots (2)$$

and stream function

$$\psi = \left(\gamma \frac{U_0}{L} \right)^{1/2} f(\eta) \qquad \ldots (3)$$

and

$$u = Uf'(\eta)$$

$$v = \left(\gamma \frac{U_0}{L}\right)^{1/2} \frac{\eta}{x} f'(\eta) \qquad \ldots (4)$$

and the function $f(\eta)$ satisfies the differential equation

$$f''' + 1 - f'^2 = 0 \qquad \ldots (5)$$

the boundary condition are,

$$\eta = 0; \quad f = 0, \; f' = 0$$
$$\eta = \infty: \; f' = 0 \qquad \ldots (6)$$

(6) Boundary layer with suction along porous wall in Tani's and Howarth's flow :

A solution has been obtained for the Laminar boundary layer of an incompressible fluid along a porous wall in Howarth's flow with uniform suction along the wall. The momentum and kinetic energy integral equation and the wall compatibility condition have been used to find an approximate solution with the help of Pohlhausen's fourth degree velocity profile.

(a) Introduction :

Curle [4] has suggested that for Howarth's flow along a porous wall separation can be completely avoided if the suction velocity v_s is given by $\frac{v_s}{\sqrt{-\nu U'}} > 1.55$.

Tani [23] used the series expansion method for the stream function and obtained that separation occurred at $x/a = 0.271$ in the case of potential flow is given by $u(x) = U_o \left(1 - \frac{x^2}{a^2}\right)$. M.R. Head [8] calculated the momentum integral equation for two-dimensional Laminar incompressible boundary layer. Howarth [10] studied the boundary layer of an incompressible and non-conducting fluid for the potential flow is given by $U(x) = U_0 \left(1 - \frac{x}{L}\right)$ and found that separation occurred at $\frac{x}{L} = 0.120$.

Equations :

Head [8] calculated the momentum integral equation and the kinetic energy integral equation as follows :

(i) Momentum Integral Equation :

With x as the co-ordinate along the wall and y the coordinator perpendicular to it, the two dimensional boundary layer equation for an incompressible fluid

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial y} + \nu \frac{\partial^2 u}{\partial y^2} \qquad \ldots (1)$$

The equation of continuity is

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad \ldots (2)$$

and for uniformly porous walls the boundary conditions are

$$y = 0 \; : \; u = 0, \; v = v_s \; ;$$
$$y = \infty \; : \; u = U(x) \qquad \ldots (3)$$

Because of the condition at $y = \infty$ we have from equation (1)

$$\frac{-1}{\rho} \frac{\partial p}{\partial x} = U \frac{dU}{dx} \qquad ..$$

Hence equation (1) becomes,

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = U \frac{dU}{dx} + \nu \frac{\partial^2 u}{\partial y^2} \qquad \ldots (4)$$

Integrating equation (4) with respect to y from $y = 0$ to $y = \infty$ under the boundary condition (3) we get

$$\frac{d\theta}{dx} = \frac{\tau_0}{\rho U^2} - \frac{\theta}{U} \frac{dU}{dx} \left(2 + \frac{\delta^*}{\theta}\right) + \frac{v_s}{U} \qquad \ldots (5)$$

where

$$\theta = \int_0^\infty \frac{u}{U} \left(1 - \frac{u}{U}\right) dy,$$

$$\delta^* = \int_0^\infty \left(1 - \frac{u}{U}\right) dy$$

Hence, the momentum integral equation in dimensionless form is

$$\frac{dt^*}{d\bar{x}} = \frac{2}{\bar{U}} \left[\ell - (2 + H) \Lambda + \lambda \right] \qquad \ldots (6)$$

where

$$\bar{x} = \frac{x}{L}, \; \bar{U} = \frac{U}{U_0}, \; H = \frac{\delta^*}{\theta}$$

$$\tau = \frac{\theta}{U} \left(\frac{\partial u}{\partial y} \right)_{y=0}$$

$$\Lambda = \frac{\theta^2}{\gamma} \frac{dU}{dx}$$

$$\lambda = \frac{V_s \theta}{\gamma}, \qquad \lambda > 0 = \text{injection}$$

$$\lambda < 0 : \text{suction}$$

where x is co-ordinate along the wall

L is reference length

U_0 is free stream velocity

v_s is normal velocity at the wall

(ii) Kinetic energy integral equation :

Adding $\frac{u}{2} \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right)$ to the left hand side of equation (4), multiplying through by u and integrating with respect to y from y = 0 to y = ∞ under the boundary conditions (3) we have

$$\frac{d \epsilon}{dx} = \frac{2 \gamma}{\theta U} D - \frac{3 \epsilon}{U} \frac{dU}{dx} + \frac{V_s}{U} \qquad \ldots (7)$$

where $\epsilon = \int_0^\infty \frac{u}{U} \left[1 - \left(\frac{u}{U} \right)^2 \right] dy,$

$$D = \int_0^\infty \left(\frac{\theta}{U} \right)^2 \left(\frac{\partial u}{\partial y} \right)^2 \frac{dy}{\theta}$$

i.e. $\frac{d}{dx} \left(\frac{\epsilon^2}{\gamma} = \frac{2H\epsilon}{U} \right) \left[2D - 3H_\epsilon \right] \qquad \ldots (8)$

where $H_{\epsilon} = \dfrac{\epsilon}{\theta}$

The direction variation of H with respect to x is given by

$$\frac{d H_{\epsilon}}{dx} = \frac{1}{\theta} \frac{d \epsilon}{d x} - \frac{\epsilon}{\theta^2} \frac{d \theta}{d x} \qquad \ldots (9)$$

Substituting for $\dfrac{d\epsilon}{d x}$ and $\dfrac{d \theta}{d x}$ from equation (7) and (5) respectively into equation (9) we have the kinetic energy, integral equation in dimensionless form,

$$\frac{d H_{\epsilon}}{d x} = \frac{1}{\bar{U}_{t^*}} \left[2D - H_{\epsilon} \left\{ \mathcal{L} - (H-1) \Lambda + \lambda \right\} + \lambda \right] \qquad \ldots (10)$$

(iii) <u>Wall compatibility condition</u> :

At the wall,

$y = 0 : u = 0, \; v = v_s ;$

Hence equation (4) gives,

$$\gamma \left(\frac{\partial^2 u}{\partial y^2} \right)_{y=0} = - U \frac{d U}{d x} + v_s \left(\frac{\partial u}{\partial y} \right)_{y=0} \qquad \ldots (11)$$

i.e. $m = - \Lambda + \lambda \mathcal{L}$

$$m = \frac{\theta^2}{U} \left(\frac{\partial^2 u}{\partial y^2} \right)_{y=0}$$

The two integral equation (6) and (10) and the wall compatibility condition (11) may be used with the help of presupposed family of velocity profiles for the approximate calculation of the boundary layer along porous wall.

(iv) <u>Velocity Profiles</u> :

The local boundary layer thickness would now depend on the rate of suction with Pohlhausen's fourth degree polynomial velocity profile may be used to investigate the boundary layers with suction

The profiles are given by,

$$\frac{u}{U} = F(\eta) + k\, G(\eta) \qquad \ldots (12)$$

$$\eta = \frac{y}{\delta(x, \bar{v}_s)}$$

$$F(\eta) = 2\eta + 2\eta^3 + \eta^4$$

$$G(\eta) = \frac{1}{6}\left[\eta - 3\eta^2 + 3\eta^3 - \eta^4\right]$$

and k is the free parameter which is used to satisfy the additional condition at the wall. For this system of velocity profiles the wall compatibility conditions (11) becomes

$$K\left(\frac{\theta^2}{\delta^2}\right) = - \lambda\,\ell + \Lambda \qquad \ldots (13)$$

The compatibility condition (11) will be satisfied in course of solution with the help of the profile parameter.

(b) Howarth's flow :

It is proposed to investigate the boundary layer with suction along a porous wall in Howarth's flow for which the potential flow velocity is given by $U(x) = U_0 \left(1 - \frac{x}{L} \right)$

$$\frac{U(x)}{U_0} = 1 - \frac{x}{L}$$

$$\bar{U} = 1 - \bar{x}$$

$$\Lambda = t^* \frac{d\bar{U}}{dx} = -t^*$$

and $$\lambda = \frac{V_s \theta}{\gamma} = t^{*1/2} \bar{V_s}$$

where $$\bar{V_s} = \frac{V_s}{U_0} \sqrt{\frac{U_0 L}{\gamma}}$$

Then equation (6), (10) and (11) reduce to

$$\frac{dt^*}{d\bar{x}} = f(x, t^*, H_\epsilon)$$

where

$$f(\bar{x}, t^*, H_\epsilon) = \frac{2}{1-\bar{x}} \left[\alpha + (2 + H) t^* + \bar{V}_s t^{*1/2} \right] \ldots (14)$$

$$\frac{dH_\epsilon}{d\bar{x}} = 9 \left(\bar{x}, t^*, \bar{H}_\epsilon \right)$$

where

$$g\,(x,\ t^*,\ H_\epsilon) = \frac{1}{(1-\bar{x})\,t^*}\Big[\,2D - H_\epsilon\ \Big\{\alpha + (H-1)\,t^* + V_s t^{*1/2}\Big\} + $$

$$+ V_s\ t^{*1/2}\Big] \qquad \ldots (15)$$

(c) <u>Tani's Flow</u> :

Boundary layer with suction along a porous wall in Tani's flow for which the potential flow velocity is given

$$U\ (x) = U_0\ \left(1 - \frac{x^2}{L^2}\right)$$

$$\frac{U(x)}{U_0} = 1 - \frac{x^2}{L^2}$$

$$\bar{U} = 1 - \bar{x}^2$$

$$\Lambda = t^*\ \frac{d\ U}{d\ \bar{x}} = -\ t^*$$

$$t^*\ \frac{d\ \bar{U}}{d\ \bar{x}} = -\ 2\bar{x}\ t^*$$

Now the momentum integral equation (6) and kinetic energy integral equation and the wall compatibility condition

$$\frac{d\ t^*}{d\bar{x}} = f\ (\bar{x},\ t^*,\ H_\epsilon)$$

where

$$f(\bar{x},\ t^*,\ H_\epsilon) = \frac{2}{1 - \bar{x}^2}\ \left[\,\alpha + 2\bar{x}\ t^*\ (H+2) + \frac{V_s\theta}{\gamma}\,\right]$$

$$\frac{d\,H_\epsilon}{d\bar{x}} = g(\bar{x}, t^*, H_\epsilon)$$

$$g(\bar{x}, t^*, H_\epsilon) = \frac{1}{t^*(1-\bar{x}^2)}\left[2D - H_\epsilon\left\{\alpha + (H-1)2\bar{x}\,t^* + \frac{V_s\,\theta}{\gamma}\right\} + \frac{V_s\theta}{\lambda}\right] \quad \ldots (16)$$

Table - Boundary layer characteristic for Pohlhausen's velocity profile against the profile parameter K

K	H	H_ϵ	α	D
0	2.554	1.571	0.235	0.1745
- 1	2.604	1.566	0.217	0.1718
- 2	2.647	1.561	0.199	0.1690

Point of separation along a wall given by different authors and present method

Reference	Point of separation $\bar{x}$
Tani[1]	0.271
Pohlhausen[2]	0.318
Walz[3]	0.248
Thawites[4]	0.262
Tani	0.265
Howarth	0.1248

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