CHAPTER I

C H A P T E R - I

History of Boundary Layer Theory

In this Chapter we give the brief history of the boundary layer theory and listed the major developments in this theory by various investigators.

1. Introduction :

In 1904 Ludwing Prandtl originated the boundary layer theory. He published a paper entitled "On the motion of a fluid with very small viscosity" in the proceedings of the International Congress held at Heidelberg in 1905 . The Prandtl boundary layer is very important and invaluable device for the practical treatment of a fluid. Blasisus [3] studied the boundary layer flow over a flat plate and obtained explicit solution of the Prandtl boundary layer equations.

With the help of Prandtl's boundary layer concept many other new results were obtained by research workers within the ten years after his research work. At that time viscous fluid theory was studied in the two and three-dimensional cases by using steady and unsteady flow of an incompressible or compressible medium. Also they considered one or more components, with or without energy addition, under the influence of magnetic forces. During the first 50 years of the boundary layer theory the fundamental mathematical questions were not answered, because at that time it was not possible to establish a sound mathematical connection to the Navier Stokes differential equations. And also there was no existence, uniqueness and goodness of a solution which was obtained. At that time numerical approximation method was not developed so that no one could show perfect error which was involved in the solution.

Due to the application of the theory of parabolic differential inequalities to the Prandtl's boundary layer equations, all the problems of existence, uniqueness etc. had been solved by considering case of two-dimensional steady flow of an incompressible medium. From the suggestion of Görtler 1950 ; Nickel 1958 solved many other problems who was the first man to use this new method. Then by using the theory of differential inequalities lot of problems were solved. Later on this new method is called as the "Well - rounded theory".

2. Prandtl's Paper :

Prandtl [33] studied the most important questions corresponding to the flow of fluid of small viscosity in the behavior of fluid at the wall of the solid boundary. From this it is clear that the flow is mostly irrotational near the solid wall. At that time the change in value of velocity takes place, from irrotational motion to the zero velocity which is given by the no slip condition (i.e. no relative tangentional velocity, at the surface of the solid body) which is happened at the wall of solid. Because of this a thin small layer is developed at the adjacent to the wall. If smaller the viscosity then thinner is the transition layer. The velocity gradient instead of small viscosity produces number of effects which have equal in magnitude to the inertia force. If the thickness of transition layer is proportional to the square-root of the kinematic viscosity. The effects of viscosity have importance only within a thin transition layer that transition layer is named as the boundary layer. Outside of this boundary layer, the flow got free viscosity and degree of accuracy of irrotational motion is increases.

If the thickness of boundary layer is small then it gives certain approximations for governing equations of boundary layer. The variation of pressure normal to the wall is negligibly small and variation of velocity along the wall is much smaller than its variation along normal to it. In the case of two dimensional flow the effect of the curvature of wall is negligibly small. Hence x and y may be taken as the distances along and normal to the wall respectively. Let u and v be the corresponding velocity components. Hence x component of the Navier-Stokes equations is written as

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + \frac{1}{\rho}\frac{\partial p}{\partial x} = \gamma\frac{\partial^2 u}{\partial y^2}$$

where ti is the time, p is pressure, ρ is density, γ is the kinematic viscosity. The pressure p is taken as function of x and t. It is shown that there is irrotational motion outside of the boundary layer. The above equation is parabolic but the original Navier-Stokes' equations are elliptic. Hence it can be integrated step by step in the direction of x when u is known. At the fixed value of x and for all y and t the obtained solution is supposed to be the order of approximation.

Prandtl obtained the solution of equation by considering p = constant. Which is the case of semi-infinite thin flat plate placed parallel to a stream of uniform velocity U which he obtained, as a rough estimate $1.1\rho\gamma^{1/2}l^{1/2}U^{3/2}$ for the two sided of unit width of a plate of length 'l'. This was the first theoretical analysis of the frictional resistance. Blasius [3] corrected the numerical coefficient 1.1 to as 1.33.

In certain cases the separation of the flow from the surface was determined by the external conditions. Without going into mathematical analysis Prandtl explained the true reason for the separation of flow with the increase in the pressure in the streamwise direction. Lastly Prandtl closed the paper by making sure that the theory by photographs of flows obtained in small hand operated water tank.

3. Prototype Concepts Of Prandtl's Paper :

Prandtl's paper is an extraordinary paper. It has three aspects. Firstly it is an extra-ordinary because it is an invaluable novel. It contained all new ideas in a single paper. It briefly explained the existance of a boundary layer and its connection with the fractional resistance. In the following papers there had been no boundary layer equation and no explanation of flow of separation.

Rankine [42] in his paper on the prediction of required engine power of proposed ships, considered the frictional resistance is due to the direct and indirect effects of adhesion between the skin of ship and the particles of water which glide over it. Rankine showed that the formation of the boundary layer takes place at the adjacent to the ship's surface. Loitsiankii [26] distinguished between smooth and rough surfaces in his paper entitled 'On the Resistance of fluid and the problem of Flight'. He recognized that the important role played by 'a thin layer of fluid adjacent to the solid surface and carrying along the neighbouring layers'.

Froude [11] pointed out that the frictional forces must have its counterpart in the loss of momentum of the fluid that has passed along the surface of the plate. Prandtl [39] quoted that Froude was the "first English author to refer the frictional resistance of a flat plate to the layers of fluid in intense shear near the surface."

4. Slow Acceptance of Prandtl's paper :

In the second way Prandtl's paper is extra-ordinary because of its very slow acceptance and growth. Prandtl's paper contained only eight pages. Because he had been given ten minutes for his lecture at the Congress.

Blasius [3] and Boltze [4] submitted two papers on boundary layer under Prandt's guidance at Göttingen.

Blasius [3] studied the flow along a flat plate placed parallel to uniform stream. Boltz [4] investigated a flow around a body of revolution (particularly a sphere). Prandtl [34] applied his boundary layer concept to the heat-transfer problem. Hiemenz [17] carried out boundary layer calculation of pressure distribution on a circular cylinder. Topfer [52] refined the numerical computations of Blasius. Prandtl [35] explained the change in flow pattern around a sphere on passing through the critical Reynolds number. Eiffel [10] observed the transition of the flow in the boundary layer from laminar to turbulent.

During ten years after the Prandtls paper, there were seven papers on boundary layer were published as Gottingen. But these were slowly accepted. All these papers were written on the basis of Prandtl's original paper. Gumbel [13] calculated frictional resistance of ship. Zhukovskii [60] assumed that the fluid velocity is zero at the wall and rapidly increases until it becomes equal to the theoretical velocity of irrotational motion. Then he found that the thickness of the layer is inversely proportional to the theoretical velocity. Kármán [24] had given the momentum integral equation which was obtained by integrating the momentum equation across the boundary layer.

Tollmien [53] investigated the growth of the boundary layer on a circular cylinder impulsively set in rotation from rest. Burgers [8] reported on experimental observation of the velocity distribution across the boundary layer on a flat plate. Mises [29] introduced the stream function and showed that boundary layer equation was reduced to a form analogous to the heat conduction.

Prandtl [36] expressed his opinion about the interest in boundary layer theory spread outside. Prandtl [37] obtained the boundary layer solution for flow through a two-dimensional channel with the help of stream function. This result was not published by him.

There were many reasons for the slow acceptance of the boundary layer theory because of first world war, Prandt's first paper was so short hence it was not appreciated by any other mathematician.

5. Sowing Seeds of Prandtl's paper :

Thirdly Prandtl's paper is extra-ordinary because it created the interest about the boundary layer theory. The investigation due to Blasius [3] , Boltze [4] and Hiemenz [17] are the outgrowth of the earlier development of the boundary layer. They opened new fields for other research workers.

At that time another new problem was born; how could the boundary layer be controlled ? Prandtls paper contained an experimental demonstration of preventing separation by removing boundary layer fluid by suction. Prandtl [38] and [39], Ackert [1] and [2], Schrenk [44, 45] and [46] carried out the experiments on boundary layer control. After lapse of half century the same problem was considered independently by Batchelor [5]. Hence Prandtl's paper is found to be seed of the subject.

6. Developments in Major Branches :

(a) Steady Two-Dimensional Laminar Boundary Layer :

Prandtl [33] and Blasius [3] introduced the form of similarity solutions, for flow on a flat plate. Falkner and Skan [12] extended this form in the case of the free-stream velocity proportional to x^m, representing irrotational flow around a corner found by two plane boundaries meeting at an angle $\pi/(m+1)$. Hartree [22] and Stewartson [47] obtained the series solution for a linearly retarded free stream and Tani [56] extended this series solution to the more general case Van Dyke [57] obtained the series solution for a flow past a parabolic cylinder. Chen et al. [9] considered the series solution for flow past a blunt nosed wedge. Goldstein [14] applied the boundary layer approximation for a flow in awake and Schlichiting [48] used the boundary layer approximation for flow in a jet.

Schubauer [49] observed the flow past an elliptic cylinder. Millikan [30] applied Karman and Millikan [25] method to the Schabauer's [49] elliptic cylinder and obtained a successful solution. Walz [60], Mangler [31] Timman [55] studied the Karman [24] method by assuming a more adequate form of the velocity profile, Walz [59.] Hudimoto [20], Tani [54] and Thwaites [55] studied approximate method of integrating the momentum integral equation. Hartree [22] obtained the numerical solution for linearly retarded free stream and suggested the presence of a singularity at the point of separation.

Goldstein [15] constructed a singular solution containing an arbitrary constant in the neighbourhood of separation. Stewartson [47] obtained the general solution involving an infinite number of arbitrary constant. Landau and Lifshitz [27] made a discussion on flow near separation by postulating that the normal component of velocity tends to infinity at the separation point.

(b) Unsteady Two-Dimensional Laminar Boundary Layer :

Blasius [3] studied the boundary layer growth set impulsively from rest into translational motion by using successive approximations. Goldstein and Rosenhead [16] extended Blasius solution and give a better estimate of the time required for separation at the rear stagnation point for circular cylinder. Schlichting [51] obtained small amplitude oscillation of body in a fluid at rest. Moore [32] considered the case in which a semi-infinite flat plate oves with a gradual change but with arbitrary time dependent velocity. Rott and Rosenzwing [43] and Lam and Rott [28] extended Light hill's [26] problem.

(c) Boundary Layers Incompressible Fluids :

Busemann [6] first studied boundary layer for an incompressible fluid. Busemann [7], Wada [59] obtained the solutions for flow on a flate plate by keeping Prandal number (Pr) constant. Howarth [21] studied the compressible and incompressible boundary layer at zero pressure gradient. Illingworth [23] investigated the transformation of both normal and streamwise coordinates and obtained the relation between them at non-zero pressure gradient for incompressible flow. Tani [56] extended the solution for the compressible flow by taking Prandtl number different from unity. Poots [41] studied Tani's [56] problems by taking heat transfer at the wall.

There are many other branches developed like -

i) Three - Dimensional Laminar boundary layers,

ii) Instability and Transition of Turbulence

iii) Boundary - Free Turbulent shear flow.

iv) Wall-Bounded Turbulent shear flow, etc.

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REFERENCES

1 Ackeret, J. (1925); Z. Flugtechn Motorluftschiff, 16: 44-52.

2 Ackeret, J. (1926); Z. Ver. Dtsch. Ing. 70 : 1153-58.

3 Blasius, H. (1908); Z. Math. Phys. 66 : 1-37.

4 Boltze, E. (1908); Gottingen dissertation.

5 Batchelor (1956); J. Fluid Mech. 1 : 177-90.

6. Busemann, A. (1931); In Handbuckeler Experimentalphysik, ed. W.Wien, F. Harms 4(f): 341-460. Leipzing : Akademiscne.

7 Busemann A. (1935); Z. Anew. Math. Mech. 15 : 23-25.

8 Burgers J.M. (1925); Proc. Int. Congr. Appl. Mech. 1st, Delft 1924, 113-28.

9 Chen <u>et al</u>. (1969); Z. Angew. Math. Phys. 20 : 919-27.

10 Eiffel, G. (1912); C.R.Acad. Sci. Paris Ser.A. 155 : 1597-99.

11 Froude, W. (1874); Res. Br. Assoc. Adv. Sci., 249-55.

12 Falkner, V.M. and Skan, S.W. (1930); Aeronaut. Res. Coun, Rep. Mem. No. 1314.

13 Giimbel, L. (1913); Jb. Schiffbahtechn Ges. 14 : 393-509.

14 Goldstein, S. (1933); Proc. R. Soc. London Ser.A. 142: 545-62.

15 Goldstein, S. (1948). G.J. Mech. Appl. Math. 1:43-69.

16 Goldstein, S. and Rosenhead, L. (1936); Proc. Cambridge Philos. Soc. 32 : 392-401.

17 Hiemenz, K. (1911); Dinglers J. 326 : 321-24, 344-48, 357-62, 372-76, 391-93, 407-10.

18 Hawarth, L. (1938); Proc. R.Soc. London Ser. A, 164: 547-79.

19 Hartree, D. (1939); Aeronaut. Res. Coun. Rep. Mem. No. 2426.

20 Hudimoto, B. (1941); J. Soc. Aeronaut Soci. Jpn. 8: 279-82.

21 Howarth, L. (1948) ; Proc. R.Soc. London Ser.A. 194: 16-42.

22 Hartree, D.R. (1937); Proc. Cambridge Philos. Soc. 33 : 223-39.

23 Illingworth, C.R. (1949); Proc. R.Soc.London Ser.A. 199 : 533-57.

24 Karman, Th. V. (1921); Z.A new Math. Mech. 1 : 233-52.

25 Karman, Th. V. and Millikan, C.B. (1934); NACA Tech. Rep. No. 504.

26 Loitsianskii, L.G. (1870); Mech. of Liquid and Gas. Moscow: Nauka (In Russian).

27 Landau, L.D. and E.M. Lifshitz (1959); Fluid Mechanics, Oxford: Fergaman.

28 Lam, S.H. and Rott, N. (1962); Proc. Int. Congr. Appl. Mech. 10th, Stresa, 1960, p. 239.

29 Mises, R.V. (1927); Z. Angew Math. Mech. 7 : 425-31.

30 Millikan, C.B. (1936); J. Aeronaut. Sci. 3 : 91-94.

31 Mangler, W. (1944); Z. Angew. Math. Mech. 24: 251-56.

32 Moore, F.K. (1951); NACA Tech. Note No. 2471.

33 Prandtl, L. (1905); Verh. Int. Math. Kongr. 3rd Heidelberg, 1904, pp. 484-91. Transl. 1928, NACA Memo No. 452.

34 Prandtl, L. (1910); Phys. Z. 11 : 1072-78.

35 Prandtl, L. (1914); Nachr. Ges. Wiss. Tottingen. Math. Phys. Kl. pp. 177-90.

36 Prandtl, L. (1928); Z. Angew. Math. Mech. 8 : 249-51.

37 Prandtl, L. (1938); Z. Angew. Math. Mech. 18: 77-82.

38 Prandtl, L. (1925); Natur Wissenschaften 13 : 93-108.

39 Prandtl, L. (1927); In Ergeb. Aerodyn. Versucsant. Gottingen 3 : 6-9. Munich and Berlin: Oldenbourg.

40 Proudman, I. and Johnson, K. (1962); J. Fluid Mech. 12: 161-68.

41 Poots, G. (1960); Q.J.Mech. Appl. Math. 13 : 57-84.

42 Rankine, W.J.M. (1864); Trans. Inst. Nav. Archit 5 : 316-33.

43 Rott, N. and Rosenzweing, M.L. (1960); J. Aeronaut. Sci. 27 : 741-47.

44 Schronk, O. (1928); Luftfahrtforschung 2 : 49-62.

45 Schronk, O. (1931); Z. Flugtechn. Motorluftschiff, 22 : 259-64.

46 Schronk, O. (1935) ; Luftfahrtforschung, 12 : 10-27.

47 Stewartson, K. (1954); Proc. Cambridge Philos. Soc. 50: 454-65.

48 Schlichiting, H. (1933); Z. Angew Math. Mech. 13 : 260-263.

49 Schubaher, G.B. (1935); NACA Tech. Rep. No. 527.

50 Stewartson, K. (1958); Q.J.Mech. Appl. Math. 11:399-410.

51 Schlichiting, H. (1932); Phys. Z. 33 : 327-35.

52 Topfer, C. (1912); Z. Math. Phys. 60: 397-98.

53 Tollmien, W. (1924); Goftingen dissertation.

54 Tani, I. (1949); J. Phys. Soc. Jpn. 4 : 149-54.

55. Timman, R. (1949); Rep. Trans. Luchtvlab. Amsterdam 15: F 29-45.

56 Tani, I. (1954); J. Aeronaut. Sci. 21 : 487-95.

57 Van Dyke, M. (1964); Perturbation Methods in Fluid Mechanics, New York and London Academic.

...

58 Walz, A. (1941); Ber. Lilienthal Ges. Luftfahrtf. No. 141, pp. 8-12.

59 Wada K. (1944); Rep. Aeronaut. Res. Inst., Tokyo, Imp. Univ. No. 302.

60 Zhukovskii, N.E. (1916); Aerodynumique Paris; Gauthier - Villars.

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