CHAPTER-II

Solutions of the Navier-Stokes Equations in porous media

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Introduction :

In this chapter, in Section 1 we discussed some basic concepts required for our problems. In Section 2 we gave brief history and major developments in the flow problems through porous media. In Section 3 we studied the problems of flow in convergent and divergent porous channels. In Section 4 we discussed Generalized plane coutte flow between two porous plates. In Section 5 we studied Generalized plane coutte flow between two coaxial infinite porous cylinders, when inner cylinder is moving with constant velocity U and outer is at rest. In Section 6 we obtained the velocity distribution in the case of spiral flow between two coaxial cylinders when the outer and inner cylinders are rotating with constant angular velocities.

1. Basic concepts required for our problems to be discussed:

(a) Fluid :

All materials exhibit deformation under the action forces. If the deformation in the material increases continuously without limit under the action of shearing forces, however, small, the material is called a "fluid". This continuous deformation under the action of forces is manifested in the tendency of fluids to flow.

Fluids are usually classified as liquid or gases. A liquid has intermolecular forces which holds it together so that it possesses volume but no definite shape. When it is poured into a container will fill the container upto the volume of the liquid regardless of the shape of the container. Liquid have but slight compressibility for most purposes it is, however, sufficient to regard liquids as "incompressible fluids". A gas, on the other hand, consists of molecules in motion which colloid with each other tending to disperse it so that a gas has no set volume or shape. The intermolecular forces are extremely small in gases. A gas will fill any container into which it is placed and is therefore, known as (highly) "compressible fluid".

(b) Viscosity :

Viscosity of a fluid is that characteristic of real fluids which exhibits a certain resistance to alterations of form. Viscosity is also known as internal friction. All known fluids (gases or liquids) possess the property of viscosity in varying degrees.

(c) Newtonian Fluid :

Newton observed that in a simple rectilinear motion of fluid layers, one is moving over the other with the same relative property, will experience a tangential force proportional to the relative velocity between the two layers and inversely proportional to the distance between the layers. If the two neighbouring fluid layers are moving with velocities u and $u + \delta u$ and at a distance δy the shearing stresses is given by

$$T \; \alpha \; \frac{\delta u}{\delta y} \quad \text{or} \quad T = \mu \frac{du}{dy}$$

This is called "The Newtonian hypothesis" and a fluid satisfying this hypothesis is called Newtonian fluid.

The constant of proportionality μ is called as coefficient of viscosity and du/dy is the strain-rate of the fluid. Newtonian fluid are also known as viscous fluids.

(d) Non-Newtonian Fluids :

The fluids which do not satisfy the Newtonian hypothesis that fluids are called as non-Newtonian fluids.

(e) Reynolds number :

The dimensionless quantity 'Re' defined as

$$Re = \frac{UL\rho}{\mu} = \frac{U\,L}{\nu}$$

where U, L, ρ and μ are some characteristic values of the velocity, length, density and viscosity of the fluid respectively, is known as the Reynolds number.

(f) Prandtl Number :

The ratio of the kinematic viscosity to the thermal diffusivity of the fluid.

$$\text{i.e.} \quad \frac{\text{Kinematic viscosity}}{\text{Thermal diffusivity}} = \frac{\nu}{a} = \frac{\mu / \rho}{K/ \rho\, c_p} = \frac{\mu C_p}{K} = Pr.$$

is designated as the Prandtl number.

(g) Suction :

The retarded fluid in the boundary layer is sucked into the body. The point of suction is near the point of separation, either slightly ahead or behind so that no back flow will occur.

(h) Injection :

Fluid is injected from the body into the boundary layer so as to increase the kinetic energy of the fluid in the boundary layer and to delay the separation.

(i) Darcy's Law :

Darcy's (1856) performed a number of experiments on flows of water through porous media by making water to percolate vertically through sand filters. It was observed that the rate of percolation of water was directly proportional to the cross sectional area of the filter bed and the total force, say the sum of pressure gradient and the gravity force. These experiments gave an empirical law, known as Darcy's Law as

$$q = CA \left(\frac{P_1 - P_2}{h_1 - h_2} + \rho g \right) \qquad \ldots (1)$$

where A : the cross sectional area of filter bed

C : K/μ

where K : the coefficient of viscosity

q : the flux of the fluid.

2. Flow through porous media and major developments of the flow problems in porous media.

(a) Flow through Porous Media :

Another important class of small Reynolds number flows are the flows through porous media, such flows are very much prevalent in nature and therefore these need through investigation. The study of flows through porous media is comparatively easy because in these flows the Inertia forces are usually very small as compared to viscous forces.

Flows through porous media occur in filteration of fluid and seepage of water in river beds. Movement of underground water and acts are some other important examples of flows through porous media.

An oil reservoir mostly consists of porous sedimentary formation such as limestone and sandstone in which oil is entrapped.

Another important example of flow through porous media is the seepage under a dam, with the high pressure on the waterside of the dam the seepage of water through the soil in dam area lowers the water head and thus result in loss of energy. Therefore, the study of seedage of water under the dam is very important. There are numerous other practical uses of fluid flows through porous media.

A few words about porous media itself, and the porosity is definitely called, for a porous media is literally a solid which contains a number of small holes distributed throughout the solid these holes may be effective or ineffective by effective holes we mean these holes through which the fluid can actually pass, it is these holes which contribute towards the porosity of these material by ineffective holes may either be so fine that fluid can not move through them due to surface tension, or the holes may not be interconnected. If the holes are not interconnected then the fluid cannot pass through them and thus these become ineffective in figure by holes we shall mean only the effective holes. The holes may be distributed homogene-outly or having any set pattern or may be very small or moderately large, some of the examples of porous media are spunches, a book of sand, cotton and woolen packings.

The porosity of a material is defined as a fraction of the total volume of the material which is actively occupied by the holes to obtain the apparent density of a porous material we first calculate the density of the pore free material ρ_s and then the density ρ_α of the dry porous material form this porosity is defined as

$$\rho = 1 - \frac{\rho_\alpha}{\rho_s}$$

(b) Major Developments of the Flow problems through porous media :

Gunadhar, Paria. [9] studied the plain strain deformation of a porous visco-elastic body containing a field in the shape of a circular cylinder has been found.

Datta, S.K. [7] studied the problem of steady-state laminar flow in an annulus with porous walls for an ordinary viscous fluid, has been considered by Berman . He took the fluid injection rate at one wall to be equal to the fluid suction rate at the other walls.

Datta, S.K. [6] studied the laminar flow of a certain type of non-Newtonian fluid between two porous plane parallel boundaries has been discussed assuming constant suction at both the walls the type of fluid taken satisfies the strain-strain relation postulated by Rivilinn.

Arun Kumar Ghosh [10] the flow of a Reiner-Rivlin fluid between two co-axial porous circular cylinder has been studied for the particular condition when the inner cylinder starts moving from rest with constant velocity. He obtained solution of the problem in two extreme cases of very large and very small coefficient of viscosity. The longitudinal motion of a piston in a Reiner-Rivlin fluid contained in a fixed circular cylinder has solution reported for the specific cross flow Reynolds number equal to unity.

Mishra, S.P. and Acharya, B.P. [25] the flow of a Rivilin-Ericksen fluid between two co-axial circular cylinders has been studied for the particular condition when the inner cylinder starts moving from rest for a certain period and then comes to rest. The inverse laplace transform in the general case being complicated, the solution of the problem has been obtained when the gap between the cylinders is small. It is observed that the velocity and the shearing stress at the wall are considerably affected by the elasticity of the liquid.

Narang, H.N. and Lamba, R.C. [27] studied the transfer of heat and moisture in an infinite porous plate in contact with moving fluid. The equation of internal heat and mass transfer in the porous media have been solved in conjuction with the flow and energy equation of the fluid, taking into account the frictional heating. The fluid has been taken to be incompressible and the motion is assumed to have started impulsively, the problem solved with the help of Laplace transform and solution for small and large values of time have been obtained.

Gupta, M.C. and Goyal, M.C. [11] studied the flow of viscous incompressible fluid between two porous co-axial rotating circular cylinders with different radial velocities 'V_a' and 'V_b' at the walls has been studied with the help of perturbation technique. An exact solution of the Navier-Stokes equation reduced to second and third order non-linear differential equations with appropriate boundary conditions.

Gupta, M.C. and Goyal, M.C. [12] studied the unsteady flow of homogenous viscous incompressible fluid between two porous co-axial rotating cylinders has been studied and complete solution for all the three velocity distributions have been obtained in closed forms with the assumption that the rate of suction at the one wall is equal to the rate of injection at the other wall. It has been found that the axial velocity is maximum at the middle of the annulus and the velocity profiles are symmetrical about the maximum velocity for any finite value . The symmetry of temperature distribution profiles exhibit that the maximum of the temperature.

Khan, Mohd Abdul [20] studied the unsteady hydromagnetic flow of a viscous incompressible and electrically conducting fluid due to rotating vibration of a porous disks about a constant nonzero mean over a fixed has been analysed.

Venkateswara [40] studied the special problem of the motion of two immiscible liquids in a slightly dipping heterogenous porous media with the presence of an immiscible connate water phase.

Verma, P.D. and Gaur, Y.N. [43] studied the oscillating flow of a viscous incompressible fluid fixed porous sphere has been investigated using stokes approximation, the stream function and hence the velocity components for the flow outside and inside the sphere have been obtained in terms of porosity parameter. The extreme values of the porosity parameter have also been obtained and represented graphically for small values of the frequency of oscillations.

Tewari, V.D. [35] studied the effect of viscous dissipation has been studied for the case of the fully developed natural convection flow of a visco-elastic fluid on a porous vertical flat plate. Numerical calculations have been made to study the effect of elasticity; suction parameter and prandtl number on the velocity and temperature profiles.

Gaur, Y.N. [13] studied the problem of flow of a viscous incompressible fluid confined between two rotating co-axial infinite porous discs have been investigated with the assumption that the rate of injection of the fluid at one disc is equal to the rate of suction of the fluid at the other. The velocity components have been expressed in terms of three dimensionless function, which in two are obtained in ascending power of the Reynolds number (taken to be small) the effects of porosity on radial, transverse and axial velocities have been dopicted graphically for variousvalues of the ratio of the angular velocities of the two discs.

Thakur, P.J. and Sinha, K.D. [36] studied the steady state solution of the Navier-Stokes equations for the viscous incompressible flow between two infinite parallel porous plates, one executing linear oscillations and the other in uniform motion in its own plane has been obtained.

Captain, R.N. and Dube,shishirkumar [5] studied the flow of a viscous electrically conducting incompressible fluid over an oscillating non-conducting and non-magnetic infinite porous flat plate in the present of a transverse magnetic field has been analysed when the magnetic Reynolds number is equal to the viscous Reynold's number small uniform suction or injection has been imposed along the surface of the plate. Expressions for velocity, induced magnetic field, current density, electric field and skin friction are obtained particular case when the applied magnetic field is zero has also been considered.

Matkowsky, B.J. and Siegmann [21] investigated the Von Karman similarity equation for fluid flow between two infinite co-axial discs that rotate with equal rotation rates and in opposite direction. The non-linear singular perturbation problem for high Reynolds number is analysed by formal asymptotic method.

Siddappa, B. and Bujurk, N.M. [32] studied slow viscous flow between parallel surfaces with injection at one surface is considered the solution are given the stagnation flow in the neighbourhood of an opposed surface.

Siddappa and Patel, S. Gundappa [33] investigated an exact solution is obtained for the free convection laminar flow of an incompressible viscoelastic (Rivlin-Ericksen) fluid past a porous wall with constant suction. It is found that the rate of heat transfer decreases with an increasing frequency stress is also attered from the classical case.

Francis, M. Skalak and Chang-yi-Wang [34] studied the equation describing similarity solutions through a uniformly porous tube and channel with equal rates of injection or suction at the walls are analyzed. The number and character of the solutions possible for various values of suction and injection are found.

Pandey, [29] studied the solution of the Navier-Stokes equations for the slow steady motion of a viscous incompressible fluid between two porous walls at slightly variable distance from each other equal suction velocities $\pm V_o(x)$ have been imposed at the two walls and the solution has been obtained by using the fourior transform method.

Gupta, P.M. and Kulshreshtha, S.K. [8] has given the solution of Navier-Stokes equation for the case of steady state laminar flow in an annulus with porous walls under the assumption of constant influx through one wall equal to influx through the outer wall. Verma has also obtained the solution for the problem of flow of a viscous liquid under exponential pressure gradient superposed on the steady laminar motion of an incompressible liquid between two co-axial cylinder. Das has also obtained solution of the problem of steady flow of a viscous liquid in an annulus with uniform arbitrary injection and suction velocities.

Patni, G.C. and AtoLia, R.N. [30] studied the effect of elasticity of the walter's type B" liquid on pressure longitudinal and transverse velocities are discussed. In this case of plane couette flow between two parallel plates with uniform suction at the lower stationary plate. It is also found that the transverse velocity is independent of the axial distance x.

Mishra, Shankar Prasad and Mohapatra, Priti [24] investigated the unsteady free convection flow of an incompressible electrically conducting viscous liquid past a hot vertical plate in the presence of a transverse magnetic field. The flow problem phenomena has been characterized by non-dimensional number p (Prandtl number) G.(Grashoff number), m (Magnetic number and W (frequency parameter) the effect of these parameters on the velocity and temperature distributions, amplitude and phase of skin friction and the fluctuating parts of the velocity have been tabulated and represented graphically.

Gupta, Premchandra and Sharma, Ram Gopal [14] studied the unsteady flow of viscous incompressible fluid through porous media in a long rectilinear tube with impermeable boundary. Technique of Laplace transforms has been applied to solve the equation of motion and it is demonstrated with the help of graphs that the flow in porous media is slower than in an ordinary flows.

Verma, P.D. and Vyas H.K. [41] studied the slow steady flow of a viscous fluid past a porous sphere of variable permeability. For the porous material within the sphere Darcy's Law will hold good. The law of variation of permeability is taken to be proportional to the nth power of the radial distance from the centre of the sphere the boundary condition at the porous surface has been taken by Jones the drag experienced by the porous sphere is determined the variation of the drag with surface permeability has been shown graphically for different values of 'η'.

Gupta, Premchandra [15] made an attempt to study the unsteady laminar flow of a viscous incompressible fluid through porous media having uniform radial magnetic field in a channel whose cross section is a circular under the influence of arbitrary time varying pressure gradient studied with the help of the technique of Laplace, finite fourier cosine and the finite Hankel transforms when the pressure gradient is an absolute constant. It has been observed for a fixed value of Hartmann number. The flow increases as porosity increases and that for a fixed porosity, the flow decreases as Hartmann number increases.

Acharya, B.P. and Padhy, S. [1] studied an analysis of a free convective flow a viscous liquid past a hot vertical porous wall is presented under the assumption that the suction velocity is constant and normal to the wall and the wall temperature is spanwise cosmusoidal. Approximate solution of the equation of motion and energy have been obtained by the method of regular perturbation. The effect of the different flow parameter on the function affecting the mean velocity, temperature, skin friction and rate of heat transfer have been presented.

Nandlal Singh [27] studied the problem of flow of a visco-elastic fluid under unsteady pressure gradient in a region bounded by two parallel porous plates. It is assumed that at one plate fluid is injected with a certain constant velocity and that at other it is sucked off with the same velocity. An exact solution for the fixed injection Reynolds number.

Venkatachalapp, M.; Sekar, R and Rudriah [44] studied the propagation of finite amplitude tidal waves at the interface between two semi-infinite fluid saturate-d porous media of different densities with the external constraint of rotation is investigated using both analytical and numerical method. The set of non-linear partial differential equations governing the wave motion have been reduced to the set of non-linear ordinary differential equations using a suitable integral. From this set, the phase portails are obtained.

Jyotirmoy Sinha Roy and Nalin Kanta [17] studied a series solution for the steady, laminar visco-elastic flow produced by rotating disc with suction. The constants in the series are evaluated numerically. It is shown that this approach yields valid solution of high accuracy for all cases of suction at the disk surface. The effect of the elasticity on the flow field have been studied.

Varshney, C.L. and Kamal Kumar [19] studied a theoretical analysis of the three dimensional unsteady flow of an incompressible viscous fluid through a porous medium past an oscillating porous plate subjected to uniform suction/injection (blowing) in a rotating system. Whole system is in plate of solid body rotation with constant angular velocity about z axis normal to the plate. The effect of porous medium, the suction/injection (blowing) parameter on the velocity distribution has been graphically shown.

Megahed, A. A des [22] studied the two dimensional flow of viscous incompressible and electrically conducting fluid through a porous medium bounded by an infinite porous plate and subjected to a uniform external magnetic field is treated in two cases. Assuming the Magnetic Reynold's number small in both the cases. Some special cases are deducted and discussed.

Bhargava Rama and Meena Rani [4] studied the problem of MHD flow and heat transfer in a channel with porous walls of different permeability has been investigated by the method of quasilinearisation. Starting from the initial guessed values the velocity function along with the temperature function has been obtained for different sets of values of Reynolds number R suction parameter α and Hartmann number o .

Bhattacharjee, A. and Borkakali, A.K. [3] studied heat transfer in the flow of a conducting fluid between two non-conducting porous disks - one rotating and other at rest, in the presence of a transverse uniform magnetic field, the lower disk being adiabatic, is studied asymptotic, solution are obtained for different values of Hartmann number M and λ .

Verma, Vijaykumar, S. and Syam Babu, M. [42] studied A steady flow of a viscous incompressible fluid through a channel bounded by permeable media of different permeabilities is considered. A transverse magnetic field is applied to the wall. The flow inside the lower porous media is with high permeability. The upper porous media is of low permeability. They obtained the expression for velocity, magnetic field, mass flux and coefficient of spin friction in two cases corresponding to (i) when the upper wall is rigid and the lower wall is porous and (ii) when both the walls are porous. They discuss the effect of magnetic parameter σ on the velocity by introducing the magnetic field. They observe that the velocity decreases in both the cases as M increases there is further reduction in the velocity. The magnitude of the velocity in both the cases is larger than the corresponding values in the rigid case.

Singh, Tejwant and Gupta, R.K. [37] studied an elastico-viscous steadily flow in the annulus of two co-axial circular cylinders, rotating with different angular velocities together with translating motion of inner cylinder along the axis of rotating, cylinders being porous, It is found that the torodial and axial component of velocity decreases with an increases in elasticity of the fluid. The axial component increases with an increase in cross-viscosity of the fluid, while torodial component is independent of it. The behaviours of torodial and axial component of velocity, with an increase in Reynolds number.

Reghavacharyulu, N.CH. [31] studied the combined free and forced convection in a saturated vertical porous tube of uniform circular cross section with a uniform heat source. The governing equations are solved for velocity and temperature fields in the form of fourier Bessel series.

Gaur, Y.N. and Bhatnagar, V. [16] investigated the slow unsteady flow through a porous circular tube with axial waviness by perturbing the solution for flow through a porous smooth tube. Fourier transform technique has been used to get the expression for axial and radial velocities and pressure.

Agarwal, R.S. and Meena Rani [2] The numerical solution of energy equation for the temperature distribution in viscous flow past a porous flat plate is obtained. A uniform suction, that follows step function change, is applied normal to the plate. The result for different values of Prandtl number are found by finite difference technique, when the suction velocity doubles in the step change.

Takhar, H.S. and Soundalgekar, V.M. [38] studied the effects of suction and injection on the flow of a micropolar fluid past a semi-infinite porous plate. Similar solution to the boundary layer flow of this fluid are presented and the effects of suction and injection are discussed.

Purohit, G.N. and Sharma, R.D. [28] studied heat transfer in the flow of a conducting viscous incompressible fluid between two nonconducting rotating porous disks under the influence of an applied magnetic field perpendicular to the disc the temperature versus dimensionless distance graphs reveal that the maximum temperature occurring between the disks increases with the Hartmann number M and decreases with ϱ (the ratio of angular velocities of the disks).

Megahed, Abel, A. [23] studied the unsteady flow of a viscous incompressible and electrically conducting fluid between two infinite porous plates, one of them is fixed and the other is moving with uniform velocity is considered. When a uniform magnetic field acts perpendicular to the plates the transverse components of the velocity is expressed in terms of any arbitrary function of time. A method is introduced to obtain the exact solution for both the equation of motion and induced magnetic induction.

Komal Kumar, Sharma, H.C. and Gupta, P.C. [18] studied a theoretical analysis of two dimensional flow of conducting stratified viscous fluid through a porous medium bounded by a rigid plane in the slip flow regime in the present of a uniform transverse magnetic field has been presented. The effect of porous media, the magnetic field. The stratification parameter and the rarefication, parameter on the friction phase and amplitude are shown graphically.

Kuiry, D.R. and Thakur, P.J. [39] he obtained the solution for the flow of viscous incompressible fluid of small electrical conductivity part on infinite porous flat plate started impulsively from rest in the presence of a constant transverse magnetic field fixed relation to the fluid with an imposition of small uniform suction or injection over the plate some results and skin frictions are also calculated.

3. Flow in Convergent and Divergent Porous Channels :

Let us consider the steady flow of a viscous incompressible fluid between two non-parallel porous plane walls. Let the external forces be absent. We use the cylindrical polar coordinates (r, θ, z). We take z axis is the line of intersection of the planes and r is the distance from this line, the walls are in the planes $\theta = \pm \alpha$.

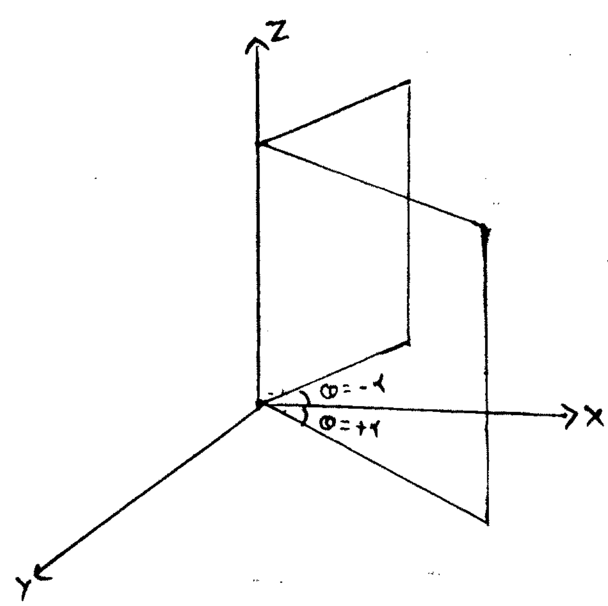


Fig.1 : Flow in cgt and Dvt porous channel.

If the motion is purely radial, then the only non-zero component of velocity is V_r and hence the equation of continuity and momentum reduce to

$$\frac{\partial}{\partial r}(r V_r) = 0 \qquad \ldots (3.1)$$

$$\rho V_r \frac{\partial V_r}{\partial r} = -\frac{\partial p}{\partial r} + \mu \left(\frac{\partial^2 V_r}{\partial r^2} + \frac{1}{r}\frac{\partial V_r}{\partial r} + \frac{1}{r^2}\frac{\partial^2 V_r}{\partial \theta^2} - \frac{V_r}{r^2} \right) \qquad \ldots (3.2)$$

and $$0 = -\frac{1}{r}\frac{\partial p}{\partial \theta} + \mu \frac{2}{r^2}\frac{\partial V_r}{\partial \theta} \quad \ldots (3.3)$$

The boundary conditions are

$$0 = \pm \alpha, \quad V_r = V_o \quad \ldots (3.4)$$

where V_o is constant suction Velocity.

The equation of continuity (3.1) gives

$$V_r = \frac{f(\theta)}{r} \quad \ldots (3.5)$$

where $f(\theta)$ is an arbitrary function of θ to be determined. Due to Equation (3.4), the equation (3.5) gives

$$V_o = \frac{f}{r} \quad \ldots (3.6)$$

Substituting Equation (3.6) in Equation (2.2) and (2.3) we get

$$-\rho \frac{f^2}{r^3} = -\frac{\partial p}{\partial r} + \mu \frac{f''}{r^3} \quad \ldots (3.7)$$

$$0 = \frac{-\partial p}{\partial \theta} + 2\mu \frac{f'}{r^2} \quad \ldots (3.8)$$

and the corresponding boundary conditions are

$$\theta = \pm \alpha, \quad f = r V_o \quad \ldots (3.9)$$

Differentiating (3.7) with respect to θ we have

$$-\frac{\rho}{r^3} 2ff' = \frac{-\partial^2 p}{\partial \theta \partial_r} + \mu \frac{f'''}{r^3} \quad \ldots (3.10)$$

Differentiating (3.8) with respect to '0' we get

$$0 = \frac{-\partial^2 p}{\partial r \partial \theta} - \frac{4\mu f'}{r^3} \qquad \ldots (3.11)$$

Substituting equation (3.10) and (3.11) we get

$$2ff' + \nu \; (f''' + 4f') = 0 \qquad \ldots (3.12)$$

where a prime denotes differentiation with respect to θ and kinematic viscosity $(\nu) = \mu / \rho$

Integrating equation (3.12) with respect to O , we get

$$f^2 + \nu \; (f'' + 4f) = K \qquad \ldots (3.13)$$

where K : Constant of integration.

Multiplying (3.13) by 2f and integrating once again, we get

$$f'^2 = \frac{2}{3\nu} (h + 3kf - 6 \nu f^2 - f^3) \qquad \ldots (3.14)$$

where h : second constant of integration.

The problem is to solve the equation (3.14) with the help of the boundary conditions given in equation (3.9) which are two number. But the equation of the equation(3.14) requires three boundary conditions and, therefore, an additional boundary condition is to be prescribed. When the flow is purely divergent or purely convergent the function 'f' will be symmetrical about $\theta = 0$, and in such a case the value of f at $\theta = 0$ may be prescribed.

Equation (3.14) can be written as

$$\left(\frac{\partial f}{\partial \theta}\right)^2 = \frac{2}{3\nu} (f_1 - f)(f_2 - f)(f_3 - f) \qquad \ldots (3.15)$$

where the constant f_1, f_2 and f_3 are connected by the relations -

$$f_1 + f_2 + f_3 = -6\nu \qquad \ldots (3.16)$$

$$f_1 f_2 + f_2 f_3 + f_3 f_1 = -3K \qquad \ldots (3.17)$$

and $$f_1 f_2 f_3 = h \qquad \ldots (3.18)$$

Integrating equation (3.15) between the limits $-\alpha$ to 0, we get

$$\alpha = \int_{-\alpha}^{0} d\theta = \pm\sqrt{\frac{3\nu}{2}} \int_{0}^{f_o} \frac{df}{(f_1-f)(f_2-f)(f_3-f)^{1/2}} \qquad \ldots (3.19)$$

where f_o : The value of f at $\theta = 0$. The solution of equation (3.16) may be expressed in terms of the elliptic function

(a) <u>Flow in a divergent porous channel</u> :

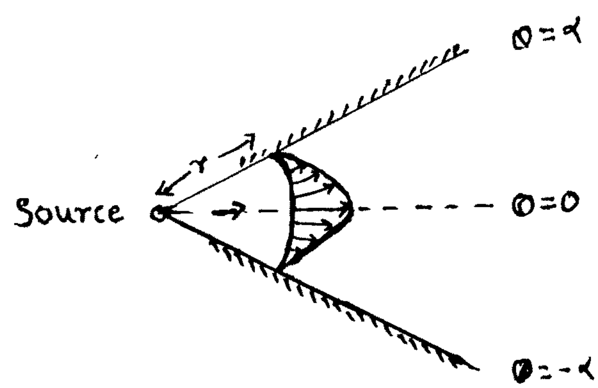


Fig.2 - Flow in a divergent porous channel.

For the flow in a divergent porous channel, the radial velocity will be positive and, therefore, from equation (3.5) we conclude that in this case $f(\theta)$ is positive ($f > 0$). Since the middle of the channel $f' = 0$ we find from equation (3.15) that f must be equal to f_1, f_2 or f_3.

Let $f = f_1 = f_o$

the equation (3.19), in the view of equations (3.16) and (3.18) may be written as

$$\alpha = \sqrt{\frac{3\nu}{2}} \int_0^{f_1} \frac{df}{\left[(f_1 - f)\left\{f^2 + (6\nu + f_1)\, f + \frac{h}{f_1}\right\}\right]^{1/2}} \quad \ldots(3.20)$$

Since, on the walls f vanishes, therefore, it follows from equation (3.14) that

$$f'^2 = \frac{2h}{3\nu} \quad \ldots (3.21)$$

Hence H is positive, therefore α has its greatest value for a given value of f_1 when $h = 0$, that is either f_2 or f_3 is zero.

$$\left.\begin{aligned} &\text{Let } f = f_1 \cos^2\psi \\ &Re = \frac{f_1}{\nu} = \frac{r(V_r)max}{\nu} \end{aligned}\right) \quad \ldots (3.22)$$

= Reynolds number.

Substituting (3.22) in the equation (3.20) and neglecting the term containing 6ν, when Re is large, we get

$$\alpha = \sqrt{\frac{3\nu}{2}} \int_0^{\pi/2} \frac{-2f_1 \operatorname{Cos}\psi \operatorname{Sin}\psi \, d\psi}{f_1 \operatorname{Sin}^2\psi \; f_1^2 \operatorname{Cos}^2\psi \; (1 + \operatorname{Cos}^2\psi)}$$

$$= \sqrt{\frac{3\nu}{2}} \; \frac{2f_1}{f_1\sqrt{f_1}} \int_0^{\pi/2} \frac{\operatorname{Cos}\psi \operatorname{Sin}\psi \, d\psi}{\operatorname{Sin}\psi \operatorname{Cos}\psi \sqrt{1 + \operatorname{Cos}^2\psi}}$$

$$\alpha = \sqrt{\frac{3\nu}{f_1}} \sqrt{2} \int_0^{\pi/2} \frac{d\psi}{\sqrt{1 + (1 - \operatorname{Sin}^2\psi)}}$$

$$\alpha = \sqrt{\frac{3\nu}{f_1}} \int_0^{\pi/2} \frac{d\psi}{\sqrt{1 - \frac{1}{2}\operatorname{Sin}^2\psi}}$$

$$\alpha = \sqrt{\frac{3}{Re}} \int_0^{\pi/2} \frac{d\psi}{(1 - \frac{1}{2}\operatorname{Sin}^2\psi)^{1/2}}$$

$$\alpha\sqrt{Re} = \sqrt{3} \int_0^{\pi/2} \frac{d\psi}{(1 - \frac{1}{2}\operatorname{Sin}^2\psi)^{1/2}} \qquad \ldots (3.23)$$

Thus $\alpha\sqrt{Re}$ has an upper limit when 'Re' is large and α is small. Hence if the angle α and Reynolds number Re are specified, then the velocity profiles may be calculated from equation (3.15) either f_2 or f_3 equal to zero.

(b) Flow in a convergent porous channel.

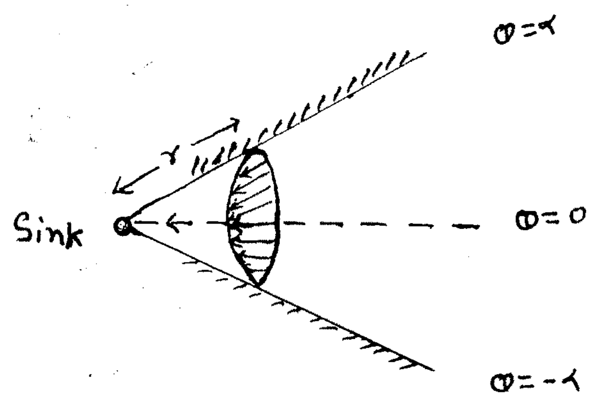


Fig.3 - Flow in a convergent porous channel.

For the flow in a convergent porous channel, f must be negative therefore, from equation (3.18) and (3.21), it follows that one of the root should be positive and other two must be negative.

Let f_1 be positive and f_2, f_3 be negative. Further $f_2 \leqslant f \leqslant 0$ (it is only in the middle of the channel, where $f = f_2$) and $f_3 \leqslant f_2$.

$$\left.\begin{aligned} &\text{Let } Re = -f_2/\nu \\ &f/f_2 = W \\ &-f_1/f_2 = W_1 \\ \text{and } \quad &f_3/f_2 = W_3 \end{aligned}\right\} \quad \ldots (3.24)$$

so that the W's are positive and W_3 lies between 0 and 1.

Substituting equation (3.23) in equation (3.16), we have

$$1 - W_1 + W_3 = \frac{6}{Re} \qquad \ldots (3.25)$$

with the help of equation (3.24), the equation (3.15) reduce to

$$\left(\frac{df}{d\theta}\right)^2 = \frac{2}{3\nu} (Re)^3 (W_1 + W)(1 - W)(W_3 - W)$$

$$\left[\frac{d}{d\theta}(-\nu W Re)\right]^2 = \frac{2}{3\nu} (Re)^3 (W_1 + W)(1 - W)(W_3 - W)$$

$$\left(\frac{dW}{d\theta}\right)^2 = \frac{2Re}{3} (W_1 + W)(1 - W)(W_3 - W)$$

$$d\theta = \sqrt{\frac{3}{2Re}} \frac{dW}{\sqrt{(W_1 + W)(1 - W)(W_3 - W)}} \qquad \ldots (3.26)$$

On integration we get

$$\int_0^\theta d\theta = \sqrt{\frac{3}{2Re}} \int_W^1 \frac{dW}{\left[(W_1 + W)(1-W)(W_3-W)\right]^{1/2}}$$

$$\theta = \sqrt{\frac{3}{2Re}} \int_W^1 \frac{dW}{\left[(W_1+W)(1-W)(W_3-W)\right]^{1/2}} \qquad \ldots (3.27)$$

On integration (3.25) between limit $-\alpha$ to 0 for θ and 0 and 1 for W we get

$$Re^{1/2}\,\alpha = \sqrt{\frac{3}{2}} \int_0^1 \frac{dW}{\left[(1-W)(W_3-W)(W_1+W)\right]^{1/2}} \qquad \ldots (3.28)$$

Now there is no restriction on the upper limit of $Re^{1/2}\alpha$. However, in order that $Re^{1/2}\alpha$ may be large W_3 must be nearly equal to 1 and therefore, if 6/Re is neglected from (3.25) we conclude that W_1 is nearly equal to 2. With these values of W_1 and W_2, we find from equation (3.27) and (3.28) that

$$\alpha - \theta = \sqrt{\frac{3}{2Re}} \int_0^W \frac{dW}{(1 - W)(2 + W)^{1/2}} \qquad \ldots (3.29)$$

Putting $2 + W = 3 \tan h^2 \psi$

and integrating equation (3.29) we get

$$\alpha - \theta = \sqrt{\frac{3}{2Re}} \int_{\tan h^{-1}\sqrt{2/3}}^{3\tan h^2\psi - 2} \frac{6 \tan h\psi \; Sec^2 h\psi}{3 \; Sec \; h^2\psi \cdot \sqrt{3} \tanh\psi} d\psi$$

$$= \sqrt{\frac{2}{Re}} \int_{\tan h^{-1}\sqrt{2/3}}^{3\tan h^2\psi - 2} d\psi$$

$$= \sqrt{\frac{2}{Re}} \left[3 \tan h^2\psi - 2 - \tan h^{-1} (\sqrt{2/3})\right]$$

$$(\alpha - \theta)\sqrt{\frac{Re}{2}} = 3 \tan h^2\psi - 2 - \tan h^{-1} (\sqrt{2/3})$$

$$= 3 \tan h^2\left[\tan h^{-1}\sqrt{\frac{W+2}{3}}\right] - 2 - \tan^{-1}\sqrt{2/3}$$

$$= 3 \left\{\tan h \tan h^{-1}\sqrt{\frac{W+2}{3}}\right\}^2 - 2 - 1.146.$$

$$= 3 \left\{ \sqrt{\frac{W+2}{3}} \right\}^2 - 2 - 1.146$$

$$= \frac{3(W + 2)}{3} - 2 - 1.146.$$

$$= W - 1.146.$$

$$(\alpha - \theta) \sqrt{\frac{Re}{2}} + 1.146 = W$$

$$W = \frac{f}{f_2} = \frac{V_r}{(V_r)_{max}}$$

$$= 3 \tan h^2 \left[\left(\frac{Re}{2} \right)^{1/2} (\alpha-\theta) + 1.146 \right] - 2 \qquad \ldots (3.30)$$

Satisfying the boundary condition (3.9) since tan hx is close to unity when x is close to 2.5, it follows that from equation (3.30) that for large Re. the velocity V_r will be equal to $(V_r)_{max}$ everywhere except in a thin layer near each wall of thickness proportional to $Re^{1/2}$

Further, from equation (8), on integration, we find

$$\frac{P}{\rho} = \frac{2 \nu f}{r^2} + F(r) \qquad \ldots (3.31)$$

Since from equation (3.7) we conclude that $r^3 \frac{\partial p}{\partial r}$ should be function of θ only, F(r) should have the following form -

$$F(r) = \frac{-K}{2r^2} \qquad \ldots (3.32)$$

Hence

$$\frac{p}{\rho} = \frac{2\nu f}{r^2} - \frac{K}{2r^2} \qquad \ldots (3.33)$$

now when $W_1 = 2$ and $W_3 = 1$, we find from (3.24) and (3.27) that

$$K = f_2^2 = r^2 (v_r)^2_{max}$$

Therefore,

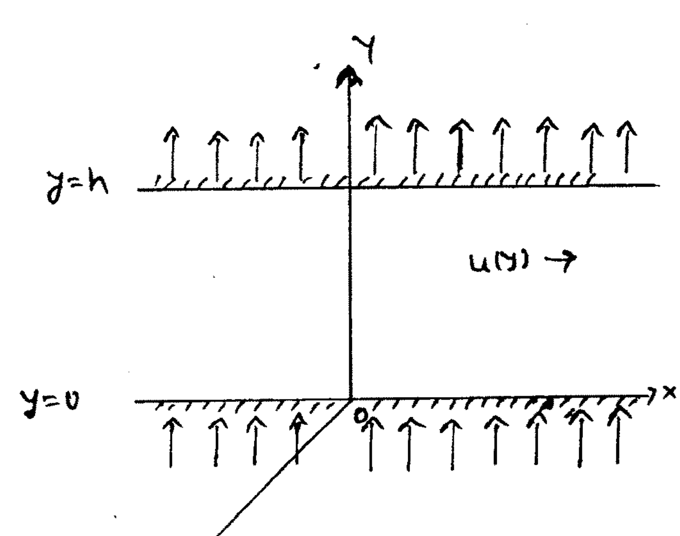
$$\frac{p}{\rho} = \frac{-f_2^2}{2r^2} + \frac{2\nu f}{r^2}$$

<u>or</u>
$$\frac{p}{\rho} + \frac{(v_r)^2_{max}}{2} = \frac{2\nu f}{r^2}$$

that is, for large Re the pressure at the walls, is equal to the pressure of the main flow.

The results obtained above concerning the velocity and pressure near the walls, for large Reynolds number, are in exact agreement with the basic assumption of the theory of boundary layers.

4. <u>Generalized plane couette flow between two parallel porous plates.</u>



Let us make the following assumptions :

(1) The two parallel infinite porous plates be situated at $y = 0$ and $y = h$

(2) The flow between two plates is steady incompressible one in the x direction. Under the above mentioned assumptions the Equation of continuity gives -

$$\frac{dv}{dy} = 0 \qquad \ldots (4.1)$$

That is, v does not depend upon y. This gives that the fluid is entering the flow region through one plate at some rate as it is leaving through the other plate. Further, let us take that the fluid is entering the flow region at a constant velocity V_o through the plate at $y = 0$ and leaving with same velocity through the plate at $y = h$, then the equation (4.1) gives that throughout the flow region, the velocity along y axis is V_o.

Then the momentum equations become

$$V_o \frac{du}{dy} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu \frac{d^2 u}{dy^2} \qquad \ldots (4.2)$$

$$o = -\frac{1}{\rho}\frac{\partial p}{\partial y} \qquad \ldots (4.3)$$

From the equation (4.3), we find that p does not depend on y. With this, equation (4.2) gives that

$$\frac{\partial p}{\partial x} = -P$$

is constant.

Then the equation (4.2) has the solution

$$u = A + Be^{\frac{V_o}{\nu} y} + \frac{P}{\rho V_o} y \qquad \ldots (4.4)$$

where A and B are constants of integration and are to be determined from the boundary conditions.

$$\left.\begin{array}{lll} y = 0 & : & u = 0 \\ y = h & : & u = U \end{array}\right) \qquad \ldots (4.5)$$

by using first boundary condition of (4.5) to equation (4.4) we get

$$0 = A + B \qquad \ldots (4.6)$$

by using second boundary condition of (4.5) to equation (4.4) we get

$$U = A + Be^{\frac{V_o}{\nu} h} + \frac{P}{\rho V_o} h$$

or $$U = -B + Be^{\frac{V_o}{\nu} h} + \frac{P}{\rho V_o} h$$

or $$U = B\left[e^{\frac{V_o}{\nu}} - 1\right] + \frac{P}{\rho V_o} h$$

or $$B = \frac{U - \frac{P}{\rho V_o} h}{e^{\frac{V_o}{\nu} h} - 1} \qquad \ldots (4.7)$$

Hence

$$A = -\left[\frac{U - \dfrac{P}{\rho V_o} h}{e^{\frac{V_o}{\nu} h} - 1}\right] \qquad \ldots(4.8)$$

Substituting values of A and B in equation (4.4) we get

$$u = \left[\frac{U - \dfrac{P}{\rho V_o} h}{e^{\frac{V_o}{\nu} h} - 1}\right]\left[e^{\frac{V_o}{\nu} y} - 1\right] + \frac{P}{\rho V_o} y \qquad \ldots (4.9)$$

The equation (4.9) gives the velocity distribution of Generalized plane coutte flow between two porous plate.

(a) Volume rate of flow

Volume rate of flow in the case of Generalized plane couette flow between two porous plates is given by

$$Q = \int_0^h u\,dy$$

$$= \left[\frac{U - \dfrac{P}{\rho V_o} h}{e^{\frac{V_o}{\nu} h} - 1}\right]\int_o^h \left(e^{\frac{V_o}{\nu} y} - 1\right) dy + \frac{P}{\rho V_o}\int_o^h y\,dy$$

$$= \left[\frac{U - \dfrac{P}{\rho V_o} h}{e^{\frac{V_o}{\nu} h} - 1}\right]\left[\frac{\nu}{V_o} e^{\frac{V_o}{\nu} y} - y\right]_o^h +$$

$$+ \frac{P}{\rho V_o}\left[\frac{y^2}{2}\right]_0^h$$

$$Q = \left[\frac{U - \frac{P}{\rho V_o}}{e^{\frac{V_o}{\nu}} - 1} \right] \left[\frac{\nu}{V_o} \; e^{\frac{V_o}{\nu} h} - h - \frac{\nu}{V_o} \right] + \frac{P}{\rho V_o} \frac{h^2}{2} \qquad \ldots (4.10)$$

(b) <u>Skin friction :</u>

Skin friction in the case of Generalized plane couette flow between porous plates is given by

$$\tau_w = \mu \left(\frac{du}{dy} \right)_{y=0}$$

$$= \mu \; \frac{\left(U - \frac{P}{\rho V_o} h \right)}{\left(e^{\frac{V_o}{\nu} h} - 1 \right)} \; \frac{d}{dy} \left[e^{\frac{V_o}{\nu} y} - 1 \right] + \frac{P}{\rho V_o}$$

$$= \mu \left[\frac{U - \frac{P}{\rho V_o} h}{e^{\frac{V_o}{\nu} h} - 1} \right] \frac{V_o}{\nu} \; e^{\frac{V_o}{\nu} y} + \frac{P}{\rho V_o} \qquad \ldots (4.11)$$

(c) <u>Coefficient of skin friction</u>

Coefficient of skin friction in the case of generalized plane couette flow between two porous plates is given by

$$C_f = \frac{\tau_w}{\rho \frac{U^2}{2}}$$

$$c_f = \frac{2\mu \left[\dfrac{U - \dfrac{P}{\rho V_o} h}{e^{\frac{V_o}{\nu} h} - 1} \right] \dfrac{V_o}{\nu} e^{\frac{V_o}{\nu} y} + \dfrac{P}{\rho V_o}}{\rho U^2} \qquad \dots (4.12)$$

5. Generalized plane coutte flow between two co-axial infinite porous cylinders, when inner cylinder is moving with constant velocity U and outer is at rest.

Let a viscous fluid be flowing axially between two co-axial infinite porous cylinders of redii a and b ($b > a$) under a pressure gradient $P = -\partial p/\partial x$.

Let the inner cylinder be moving with a constant velocity U and outer cylinder be at rest. Suppose the velocity of injection of the inner cylinder is V_o and that of suction at outer cylinder are inversely proportional to their radii.

We use the cylindrical polar coordinates (r, θ, z) to discuss the flow. Let the flow be occurring in radial and axial directions and depending on radial distance only, i.e. let the Velocity vector be $V(\, v(r),\, 0,\, u(r)\,)$

Then the equation of continuity becomes

$$\frac{dv}{dr} + \frac{v}{r} = 0$$

$$\text{or } V = \frac{A}{r} \qquad \dots (5.1)$$

where 'A' is a constant, and the Navier-Stokes equations of motion under the above hypothesis reduce to

$$\rho V \frac{dv}{dr} = \frac{-\partial p}{\partial r} \qquad \ldots (5.2)$$

$$\rho V \frac{du}{dr} = -\frac{\partial p}{\partial z} + \mu \left(\frac{d^2u}{dr^2} + \frac{1}{r} \frac{du}{dr} \right) \qquad \ldots (5.3)$$

The equation (5.2) gives that the pressure

$$P = P_o - \frac{1}{2} \rho A^2 \left(\frac{1}{r^2} - \frac{1}{a^2} \right) \qquad \ldots (5.4)$$

where P_o is the pressure at inner cylinder in equation (5.3) we find that $\partial p / \partial z = - P$ is a constant. Thus we can write this equation as

$$\frac{d^2u}{dr^2} + \frac{1}{r} \frac{du}{dr} - \frac{\rho A}{\mu r} \frac{du}{dr} = \frac{-P}{\mu} \qquad \ldots (5.5)$$

After solving the equation this equation we get the velocity distribution as

$$u = \frac{-P}{2(2-m)\mu} r^2 + C_1 r^m + C_2 \qquad \ldots (5.6)$$

where $m = \rho A/M$ and C_1 and C_2 are constant of integration which are to be determined from the boundary conditions

$$\left.\begin{array}{lll} u = 0 & : & r = b \\ u = U & : & r = a \end{array}\right) \qquad \ldots (5.7)$$

Using first boundary condition of equation (5.7) to equation (5.6) we get

$$0 = \frac{-P}{2\mu (2-m)} b^2 + C_1 b^m + C_2 \qquad \ldots (5.8)$$

Using second boundary condition of equation (5.7) to equation (5.6) gives

$$U = \frac{-P}{2\mu(2-m)} a^2 + C_1 a^m + C_2 \qquad \ldots (5.9)$$

Subtracting equation (5.8) from equation (5.9) we get

$$U = \frac{-P}{2\mu(2-m)} + \frac{P}{2\mu(2-m)} \frac{(a^2 - b^2)}{(a^m - b^m)}$$

Substituting this value in (5.8) we get

$$C_2 = \frac{P}{2\mu(2-m)} b^2 - \frac{Ub^m}{(a^m-b^m)} - \frac{P\ b^m}{2\mu(2-m)} \frac{(a^2 - b^2)}{(a^m - b^m)}$$

Substituting values of C_1 and C_2 in equation (5.6) we get

$$u = \frac{-P}{2\mu(2-m)} r^2 + \frac{Ur^m}{a^m-b^m} + \frac{P}{2\mu(2-m)} r^m \frac{(a^2-b^2)}{(a^m-b^m)} +$$

$$+ \frac{P}{2\mu(2-m)} b^2 - \frac{U}{(a^m-b^m)} b^m - \frac{P}{2\mu(2-m)} b^m \frac{(a^2-b^2)}{(a^m-b^m)} =$$

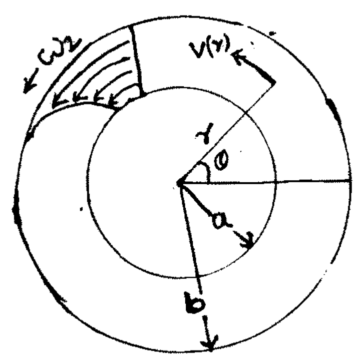
$$= \frac{P}{2(2-m)\mu} \left[-r^2 + b^2 + \frac{b^2-a^2}{b^m-a^m} (r^m - b^m) \right] + U \frac{r^m-b^m}{a^m-b^m}$$

$$u = -\frac{P}{2(2-m)\mu} \left[r^2 - b^2 - \frac{b^2-a^2}{b^m-a^m} (r^m - b^m) \right] +$$

$$+ U \frac{b^m-r^m}{b^m-a^m} \qquad \ldots (5.10)$$

Equation (5.10) gives the velocity distribution for Generalized plane couette flow between co-axial infinite porous cylinders.

6. Spiral flow between two co-axial cylinders when the outer and inner cylinders are rotating with constant angular velocities.



We consider spiral flow between two co-axial cylinders of radii a and b ($b > a$) such that the outer and inner cylinders are rotating with constant angular velocities ω_1 and ω_2 respectively and the inner cylinder is moving along the axis with velocity U. We use cylindrical co-ordinate system (r, θ, z) and assume that there is only θ and z components of velocity, which depend only on r i.e. the velocity vector is $\bar{V}$ (o, v(r), w(r)).

Then the equation of continuity is identically satisfied and the Navier-Stokes equations of motion reduce to

$$\rho \frac{v^2}{r} = \frac{\partial p}{\partial r} \qquad \ldots (6.1)$$

$$0 = -\frac{1}{r}\frac{\partial p}{\partial \theta} + \mu\left[\frac{d^2 v}{d r^2} + \frac{1}{r}\frac{d}{dr}\left(\frac{v}{r}\right)\right] \quad \ldots (6.2)$$

$$0 = -\frac{\partial p}{\partial z} \quad \ldots (6.3)$$

$$\mu\left(\frac{d^2 w}{dr^2} + \frac{1}{r}\frac{dw}{dr} + \frac{1}{r^2}\frac{d^2 w}{d\theta^2} + \frac{d^2 w}{d z^2}\right) = 0 \quad \ldots (6.4)$$

From equations (6.1) to (6.4) we conclude that $\frac{\partial p}{\partial \theta}$ is constant.

If $\frac{\partial p}{\partial \theta} = 0$, then the equations (6.2) and (6.4) reduce to

$$\frac{d^2 v}{dr^2} + \frac{d}{dr}\left(\frac{v}{r}\right) = 0 \quad \ldots (6.5)$$

and

$$\frac{d^2 w}{dr^2} + \frac{1}{r}\frac{dw}{dr} = 0 \quad \ldots (6.6)$$

On solving equations (6.5) and (6.6) we get

$$v = Ar + B/r \quad \ldots (6.7)$$

and

$$w = A \log r + B \quad \ldots (6.8)$$

where A and B are constants of integration which can be determined by using the boundary conditions

$$\left.\begin{array}{lll} r = a & : & v = a\,\omega_1 \\ r = b & : & v = b\,\omega_2 \end{array}\right) \quad \ldots (6.9)$$

and

$$\left.\begin{aligned} r = a \quad &: \quad W = U \\ r = b \quad &: \quad W = 0 \end{aligned}\right) \qquad \ldots (6.10)$$

The equation (6.7) can be written as

$$rv = Ar^2 + B \qquad \ldots (6.11)$$

Applying first and second boundary conditions of (6.9) to equation (6.11) we get

$$a^2 \omega_1 = Aa^2 + B \qquad \ldots (6.12)$$

$$b^2 \omega_2 = Ab^2 + B \qquad \ldots (6.13)$$

Substracting equation (6.12) from (6.13) we get

$$(b^2 \omega_2 - a^2 \omega_1) = A\,(b^2 - a^2)$$

Hence

$$A = \frac{b^2 \omega_2 - a^2 \omega_1}{b^2 - a^2} \qquad \ldots (6.14)$$

Substituting this value of A in (6.12) we get

$$B = \frac{-a^2 b^2}{b^2 - a^2} \left(\omega_2 - \omega_1 \right) \qquad \ldots (6.15)$$

Substituting values of A and B in equation (6.7) we get

$$V = \frac{1}{(b^2 - a^2)} \quad (b^2 \omega_2 - a^2 \omega_1) r - \frac{a^2 b^2}{r} \quad (\omega_2 - \omega_1) \qquad \ldots (6.16)$$

Applying the first and second boundary condition of (6.10) to equation (6.8) we get

$$U = A \log a + B \qquad \ldots (6.17)$$

$$O = A \log b + B \qquad \ldots (6.18)$$

Subtracting (18) from (17) we get

$$U = A (\log a - \log b)$$

$$A = \frac{U}{\log a - \log b}$$

or

$$A = \frac{U}{\log (a/b)} \qquad \ldots (6.19)$$

Substituting this value in (6.17) we get

$$U = \frac{U \log a}{\log(a/b)} + B$$

$$B = U - \frac{U \log a}{\log (a/b)}$$

$$B = U \left[\frac{\log (a/b) - \log a}{\log (a/b)} \right]$$

$$B = \frac{-U \log a}{\log (a/b)} \qquad \ldots (6.20)$$

Substituting values of A and B in equation (6.8) we get

$$W = \frac{U \log r}{\log (a/b)} - \frac{U \log b}{\log (a/b)}$$

$$= \frac{U (\log r - \log b)}{\log (a/b)}$$

$$W = \frac{U \log (r/b)}{\log (a/b)} \qquad \ldots (6.21)$$

The equation (6.16) and (6.20) gives velocity distribution in θ and z direction respectively.

o0o

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